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COMPLETION REPORT

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MATHEMATICAL MODELS FOR UNGAGED WATERSHEDS WITH POTENTIAL FOR QUANTIFYING THE EFFECT OF LAND USE CHANGES ON STREAMFLOW

By

VIJAY P. SINGH

Prepared for

Office of Water Policy
United States Department of the Interior
Washington, D.C. 20240

LOUISIANA WATER RESOURCES RESEARCH INSTITUTE
Louisiana State University
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ABSTRACT

There has been a growing need in the United States to quantify the impact on watershed hydrology of such land use changes as urbanization, forest clearing, surface mining, construction and agricultural practices. This is required to satisfy federal legislation and anticipate the potential impacts so that ecological and environmental detriment can be minimized. This study was undertaken to evaluate existing mathematical models for simulation of streamflow from watersheds with insufficient hydrologic data. Response time and hydrograph characteristics were included in evaluation. On recognizing limitations of these models and taking advantage of experience with their use, a mathematical model based on drainage network characteristics was developed. Because it contains parameters which can be determined from physically measurable watershed characteristics, the model is eminently suitable for ungaged basins. Its parameters are affected by land use changes. Therefore, the effect of these changes can be quantified. Paucity of data did not allow this quantification on natural watersheds.

1. INTRODUCTION

Major river basins in the United States are being continously encroached by man in his quest for increased energy development, food production, industrial expansion, development of recreational facilities, construction of hydraulic works, and orderly urban and rural development. Consequently, substantial land use changes such as urbanization, forest cutting, surface mining, construction and changing agricultural practices are taking place in these basins. A question arises: What is the impact of these changes on ecology and environment of the basins? To quantify this impact is required to satisfy federal legislation and to minimize detrimental environmental effects. The impact of these changes on basin hydrology can be significant. For example, removal of a forest stand may increase water yield some 50 percent or more. Urbanization enhances potential for flooding which, in turn, may reduce property value. Thus, there has been a growing need to quantify the impact of major land use changes on streamflow both from the standpoint of anticipating the potential impacts, and in response to federal legislation.

Land use changes affect the entire spectrum of streamflow characteristics. Of particular interest are (a) response time characteristics, (b) hydrograph characterisitcs, and (c) water yield characteristics. Mathematical models are commonly employed for determining these characteristics. As a result there exists a plethora of models for synthesis of streamflow. An excellent exposition of mathematical modeling of streamflow is contained in Haan, Johnson and Brakensiek (1982), and Singh (1982). There is, however, a practical problem in employing most of these models for quantifying the effect of land use changes on streamflow. This problem arises because pertinent hydrologic data are often not available in areas undergoing these changes. In a recent survey of a broad range of hydrologic models which can be used to determine the effect of land use changes on basin hydrology, an ASCE Task Committee (ASCE, 1983) found most of these models to be inadequate. Even less adequate were these models when little to no hydrologic data were available. This means that potential real impacts must be predicted with little to no data. Therefore, predictive mathematical models requiring little to no data must be developed to evaluate the consequences of land use changes on hydrology. If these models are found adequate and reliable, their use may reduce the need to collect expensive hydrologic site data which normally have to be collected for a number of years.

This study was therefore directed toward evaluation of existing models and development of appropriate mathematical models applicable to ungaged basins. More specifically, the objectives were:

1. To survey and evaluate existing mathematical models for ungaged basins and show their inadequacy for quantifying the effect of land use changes on streamflow.
2. To develop mathematical models for ungaged basins.
3. To show applicability of the developed models for evaluating the impact of land use changes on streamflow.
4. To suggest directions for further research.

In this study response time and hydrograph characteristics of streamflow were dealt with in considerable detail but water yield characteristics only summarized briefly.

2. RESPONSE TIME CHARACTERISTICS

There are several measures of watershed response time characteristics as shown in figure 2.1. The time parameters most frequently used in hydrology are the time of concentration, lag time, time base, time to equilibrium, time to peak, time of travel, residence time, infiltration opportunity time, and time to ponding.

These characteristics constitute an important part of hydrologic modeling and design (Schulz and Lopez, 1974). For example, the design of urban drainage systems using the rational formula requires an estimate of the time of concentration to determine the critical rainfall intensity (Rossmiller, 1980; Ben-Zvi, 1984). The Snyder method of unit hydrograph synthesis requires an estimate of the lag time (Snyder, 1938). The kinematic wave flood routing uses the time to equilibrium which is closely associated with the time of concentration (Overton and Meadows, 1976; Singh, 1976; Singh and Agiralioglu, 1980). The Muskingum method of flood routing in channels makes use of the time of travel (Singh and McCann, 1979, 1980a, 1980b; Singh and Chowdhury, 1980). The SCS-TR-20 computer model (Soil Conservation Service, 1969), which is used for spillway design and other hydrologic analyses, requires an estimate of the time of concentration. The Clark method of hydrograph synthesis (Clark, 1945) also requires the time of concentration. The design of surface irrigation systems makes use of the infiltration opportunity time (Singh and Ram, 1983). The infiltration-runoff analyses require an estimate of the time to ponding (Smith, 1972; Prasad and Romkens, 1983). Flood frequency analyses use the time to peak

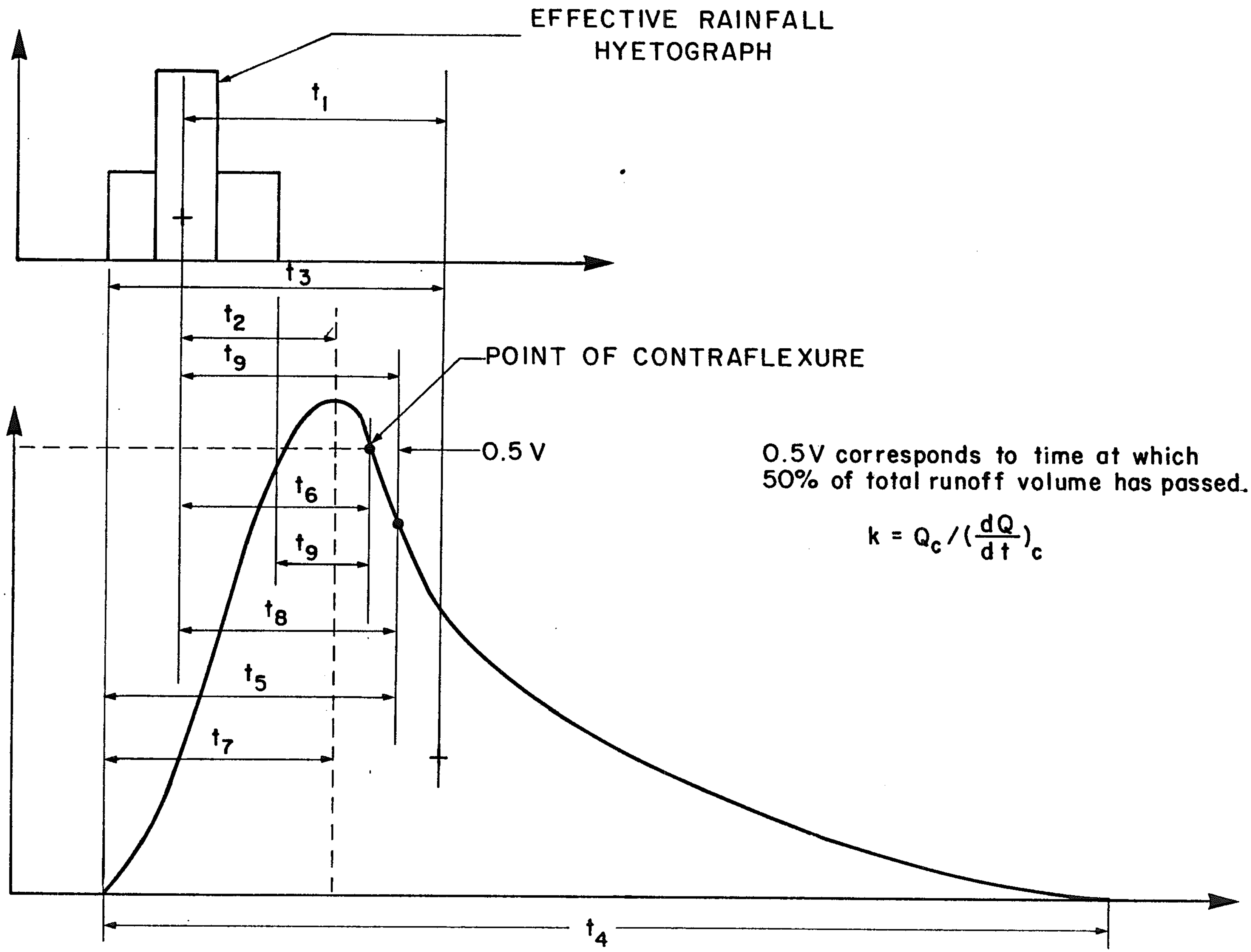


Figure 2.1 Definition sketch for various measures of time characteristics.

(Rodriguez-Iturbe, Sanabria and Camano, 1983). Wycoff and Singh (1976) provide a method for estimating the required volume of stormwater detention. Their method uses as input both the time to peak and the time base of a hydrograph. In summary, the bulk of hydrologic analyses requires the value of one time parameter or another as input, with the time of concentration and the lag time being used most frequently.

The accuracy of the design flood estimate is directly related to the accuracy with which an estimate of watershed response time is made. Meynink (1978) showed that varying the time of concentration from one half to twice the estimated value changed the peak discharge from 1.64 to 0.48 times the estimated value for a typical 5 km^2 watershed on the Darling Downs in Australia (Table 2.1). The range of half to twice the estimated time of concentration is probably consistent with the error associated with most empirical estimates of response times on ungaged watersheds. For a specified recurrence interval, a first approximation of the design flood estimate is inversely proportional to the response time. For example, if the response time is half the real value then the flood will be twice the real value and vice versa. Indeed little is known about the accuracy of estimates of response times on ungaged watersheds. Empirical expressions are usually used to estimate response time characteristics.

2.1 SYMBOLISM OF RESPONSE TIME CHARACTERISTICS

Referring to figure 2.1, we catalog the symbols designating various watershed response time characteristics. Similar but less comprehensive catalogs have been presented by Rao and Delleur (1974), Schulz and Lopez (1974), and Meynink (1978).

Table 2.1 Effect of time of concentration on peak discharge (after Meynink, 1978).

Time of Concentration (hours)	Rainfall Intensity (mm/hr)	Runoff Coefficient	Design Peak Discharge m^3/s
1	54	0.71	21.3
2	37	0.63	13.0
4	24	0.51	6.2

Symbol	Explanation	Reference
t_1	Lag Time - time interval between the centroid of effective rainfall and the centroid of direct runoff.	Horner and Flynt (1936); Mitchell (1948)
t_2	Lag Time - time interval between the centroid of effective rainfall and peak of direct runoff.	Snyder (1938); Taylor and Schwarz (1952); Eagleson (1962)
t_3	Lag Time - time interval between the beginning of effective rainfall and the centroid of direct runoff.	Wilson (1972); Schulz and Lopez (1974)
t_4	Lag Time - time interval between the beginning of direct runoff and the end of direct runoff or the time base of direct runoff hydrograph.	Snyder (1983); Meynink (1974)
t_5	Lag Time - time interval between the beginning of effective rainfall and the time when 50 percent of direct runoff has passed the gaging station.	Rao and Delleur (1974)
t_6	Lag Time - time interval between the centroid of effective rainfall and the point of contraflexure on direct runoff recession.	Rao and Delleur (1974)
t_7	Lag Time or Time to Peak - time interval between the beginning of effective rainfall and the peak of direct runoff.	Linsley, Kohler and Paulhus (1958)
t_8	Lag Time - time interval between the centroid of effective rainfall and the time when 50 percent of direct runoff has passed the gaging station.	Wilson (1972); U.S. Bureau of Reclamation (1965)
t_9	Time of Concentration - time interval between the end of effective rainfall and the point of contraflexure on direct runoff recession.	Viessman, et al. (1978); McCuen, Wong and Rawls (1983)
t_{10}	Time interval between the beginning of rainfall (not effective rainfall) and the peak of runoff (not direct runoff).	Lopez (1973)
t_{11}	Time interval between the centroid of effective rainfall and the peak of unit hydrograph	Lopez (1973)

Symbol	Explanation	Reference
t_{12}	Time interval between the beginning of effective rainfall and the peak of unit hydrograph.	Lopez (1973)
t_{13}	Time interval required for the hydrograph to rise from low flow to the maximum stage (might be equivalent to t_7).	Ramser (1927); Kirpich (1940); Gray (1961); Wu (1969)
t_{14}	Equilibrium Time - time interval required for the runoff rate to become equal to the supply rate (rainfall intensity).	Izzard (1946); Wei and Larson (1971)
t_{15}	Lag Time - time interval between 50 percent of effective rainfall and 50 percent of direct runoff.	Overton (1971); Singh (1975); Agiralioglu and Singh (1981); Singh and Agiralioglu (1982)
t_{16}	Time to Ponding - time interval between the start of rainfall and the start of direct runoff.	Smith (1972)
t_{17}	Infiltration Opportunity Time - the time period for which water is available for infiltration at a given point in space.	Singh and Sherman (1983)

2.2 FACTORS AFFECTING TIME CHARACTERISTICS

The factors affecting response time characteristics can be classified as (1) physiographic factors including areal extent, form, slope, and surface topographic characteristics; (2) rainfall characteristics including intensity in time and space, rainfall duration and direction of the storm; and (3) other factors such as antecedent soil moisture condition, infiltration characteristics, wind velocity and weather condition. A time characteristic is generally not a constant for a given watershed. It is constant only by assumption. Following McCuen, Wong and Rawls (1984), these factors can be classified into five groups: slope, watershed size, flow resistance, channel, and flow. These are either for overland flow, pipe flow or channel flow portion of the watershed. Thus, the methods of estimating time characteristics can be distinguished by both their input data requirements and the dominant flow regime as shown in Table 2.2.

If overland flow dominates on a watershed then the input data should represent this feature. The length and slope of overland flow are used frequently to characterize it. Measures of flow resistance may include Manning's roughness coefficient n, the runoff coefficient of the rational method C, the SCS curve number C_N, the percentage of imperviousness C_u, or the land cover type. A rainfall intensity, such as 2-year, 2-hour rainfall intensity, is often used as input to include the effect of flow on the time of travel.

The channel characteristics should be included in the input data if the channel flow makes a significant contribution to the total travel time of runoff. The length L_c and slope S_c of the main channel are used most frequently to incorporate the effect of slope and size. Manning's or Chezy's roughness coefficient is the most widely used index of flow resistance. Espey and Winslow (1968) used a coefficient C_f to indicate the degree of channelization which can also be considered as an indicator of flow resistance. The rainfall intensity, volume of direct runoff or hydraulic radius R corresponding to full flow can be used to represent the flow characteristics.

In many small watersheds the pipe flow plays a significant role in governing runoff characteristics. The input data then should reflect this feature. Flow resistance can be estimated by Manning's, Chezy's or Darcy-Weisbach friction factor. Slope and size can be represented in a

Table 2.2 Criteria for classifying time parameters and variables commonly used to represent the input data (adapted from McCuen, Wong and Rawls, 1984).

Flow Regime	Input Date				
	Flow Resistance	Watershed Size	Slope	Channel	Flow
Overland	n, C, C_N, C_u	L, A	S	-	I
Channel	c, n_m, C_f	L_c, L_{10-85}, L_{ca}	S_c, S_{10-85}	R	I, Q
Pipe	n	L	S	R	Q_p

A = Drainage area

c = Chezy's roughness coefficient

C = Runoff coefficient of the rational method

C_f = Channelization factor

C_N = Runoff curve number

C_u = Percentage of imperviousness

I = Effective rainfall intensity for a duration equal to the time of concentration

L = Length of overland flow

L_c = Length of the main channel

L_{ca} = Length to center of area

L_{10-85} = Length of the main channel between points 10 and 85 percenter of the total length from the watershed outlet

n_m = Manning's roughness coefficient

Q = Flow rate

Q_p = Maximum flow rate

R = Hydraulic radius

S = Slope

S_c = Slope of main channel

variety of ways. The flow characteristics can be reflected by a maximum flow rate Q_p, the largest or smallest pipe diameter, or mean hydraulic radius.

It should be remarked that in a real-world situation more than one flow regime may exist. For example, in an urbanized watershed all three flow regimes may exist: overland, channel and pipe flows. Consequently, a method of estimating a time characteristic must include factors representing these flow regimes. In other words, the input data must reflect these features.

2.3 TIME OF CONCENTRATION, T_c

The time of concentration T_c is the most frequently used time parameter. Like other parameters it is usually defined in terms of either the physical characteristics of a watershed or the effective rainfall hyetograph and direct runoff hydrograph. There are two commonly accepted definitions of T_c.

First, T_c is the time required for water to travel from the most remote portion of the watershed to its outlet or design point. The methods of estimation based on this definition use watershed characteristics, and sometimes a precipitation index such as the 2-year, 2-hour rainfall intensity. A number of empirical equations based on this definition will be given in the ensuing discussion.

The second definition of T_c is based on a rainfall hyetograph and the resulting runoff hydrograph. The effective rainfall hyetograph and direct runoff hydrograph are computed from these observations. The time of concentration is the time between the center of mass of effective rainfall and the inflection point on the recession of direct runoff hydrograph. Alternatively, T_c is often considered as the time from the end of effective rainfall to the point of inflection on direct runoff hydrograph recession (Horton, 1939) as shown in figure 2.1.

It should be noted that neither definition yields a true value or even a reproducible value of T_c. This is due to the difficulty of uniquely defining and then measuring the factors affecting it.

2.4 METHODS OF ESTIMATING T_c

There exists a multitude of methods for estimating T_c. Most of these methods can be classified by the input data required and the flow regime considered by them. These methods and their input data are shown in Table 2.2. Some of these methods are designed primarily for overland flow, some primarily for channel flow, and some for both channel and overland flows. Some of these methods are presented briefly here.

Most of the equations for estimating T_c can be expressed as a function of a length measure L_p, a slope measure S_p, and a coefficient C_p which is a measure of surficial watershed characteristics and which may or may not be a constant,

$$T_c = C_p \, L_p^{\,a} \, S_p^{\,b} \tag{2.1}$$

in which a and b are exponents varying usually from one watershed to the other. These equations have been derived either empirically or by using equations of open channel hydraulics.

2.4.1 HYDRAULICALLY DERIVED EQUATIONS

Various forms of equations of hydraulics can be applied to estimate T_c provided the watershed geometry can be represented in a simple manner and the friction characteristics are known (Singh, 1976; Chen and Evans, 1977; Singh and Agiralioglu, 1981; Hirano, 1980; Gregory, 1982). For example, Henderson and Wooding (1964) derived the time of concentration for overland flow on a plane using kinematic wave theory, which can be stated as

$$T_c = I^{\frac{1-m}{m}} (\frac{L}{\alpha})^{\frac{1}{m}} \tag{2.2}$$

where m and α are parameters in kinematic depth-discharge relationship. If we use the Chezy relation then

$$\alpha = c_z\ S^{0.5} \quad \text{and} \quad m = 1.5 \tag{2.3}$$

If the Manning relation is used then

$$\alpha = \frac{1}{n_m} S^{0.5} \quad \text{and} \quad m = 5/3 \tag{2.4}$$

where c_z is the Chezy friction coefficient, n_m the Manning friction coefficient, L the length of the plane, and S the ground slope. The overland flow lengths ranged from 50 to 100 feet. Ragan and Durue (1972) developed a nomograph to compute T_c using equation (2.2) in conjunction with equation (2.4). Agiralioglu and Singh (1981) developed nomographs for equation (2.2) with α estimated by Manning's, Chezy's or Darcy-Weisbach's roughness coefficient. Figure 2.2 is a nomograph with the Manning coefficient.

Singh (1976) derived T_c, using kinematic wave theory, for plane, and converging geometries for different conditions imposed on effective rainfall and friction parameters. For a converging geometry as shown in figure 2.3, the relation for T_c can be stated as

$$T_c = \frac{1}{m} I^{\frac{1-m}{m}} (\frac{L_0}{2\alpha})^{\frac{1}{m}} [B(a;b) - B_w(a;b)] \tag{2.5}$$

where a = 1 - (0.5/m), b = 1/m, $w = r^2$, r is the convergence parameter and L_0 the length of the section. The quantity B, without a subscript, denotes complete Beta function and can be expressed in terms of Gamma function as

$$t_c = 2.782 \frac{L^{0.6} n_m^{0.6}}{q^{0.4} S_o^{0.3}}$$

EXAMPLE:

$L = 200$ m
$n_m = 0.10$
$q = 0.5$ cm/hr
$S_o = 0.05$
(read) $t_c = 55$ min

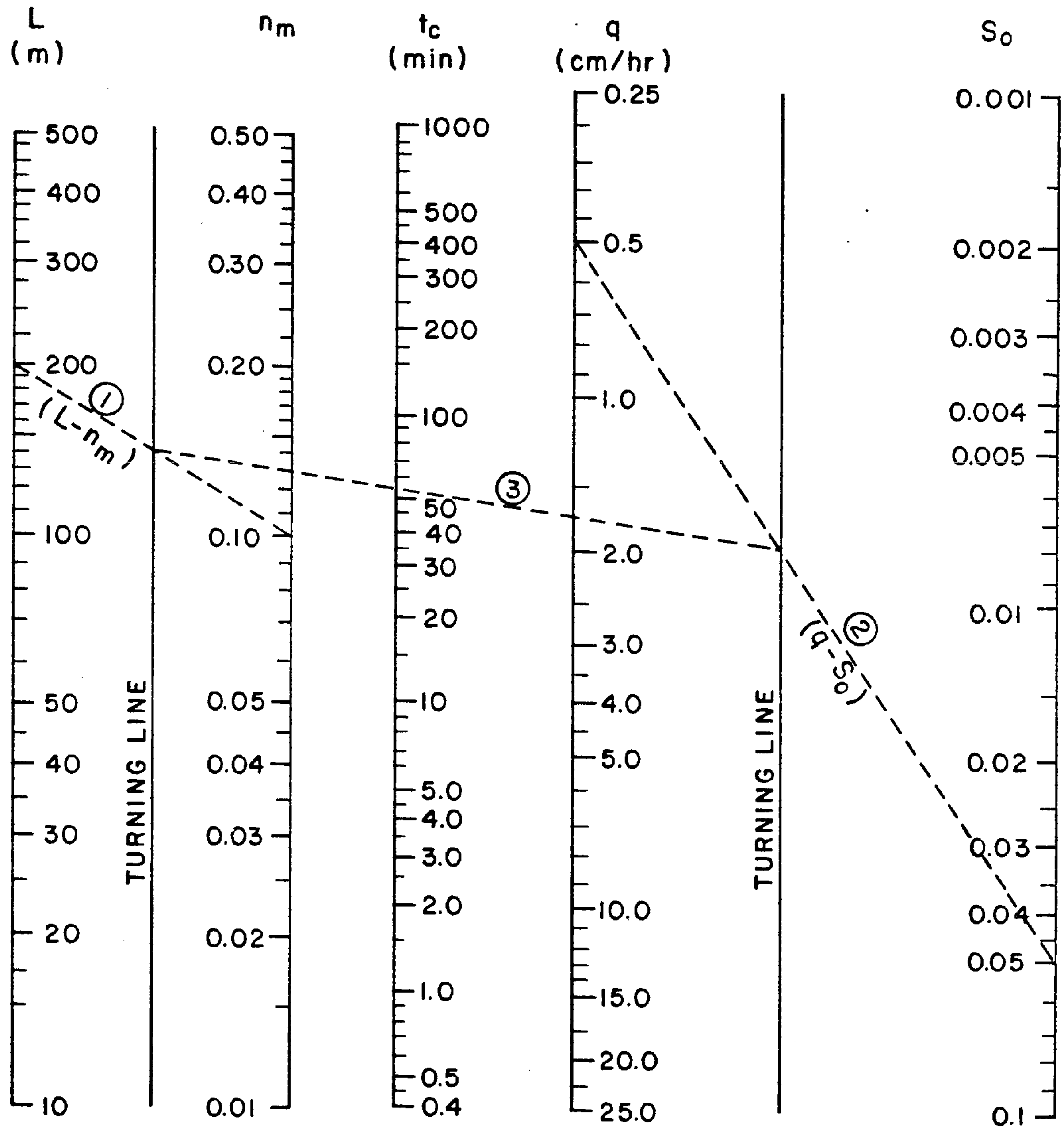
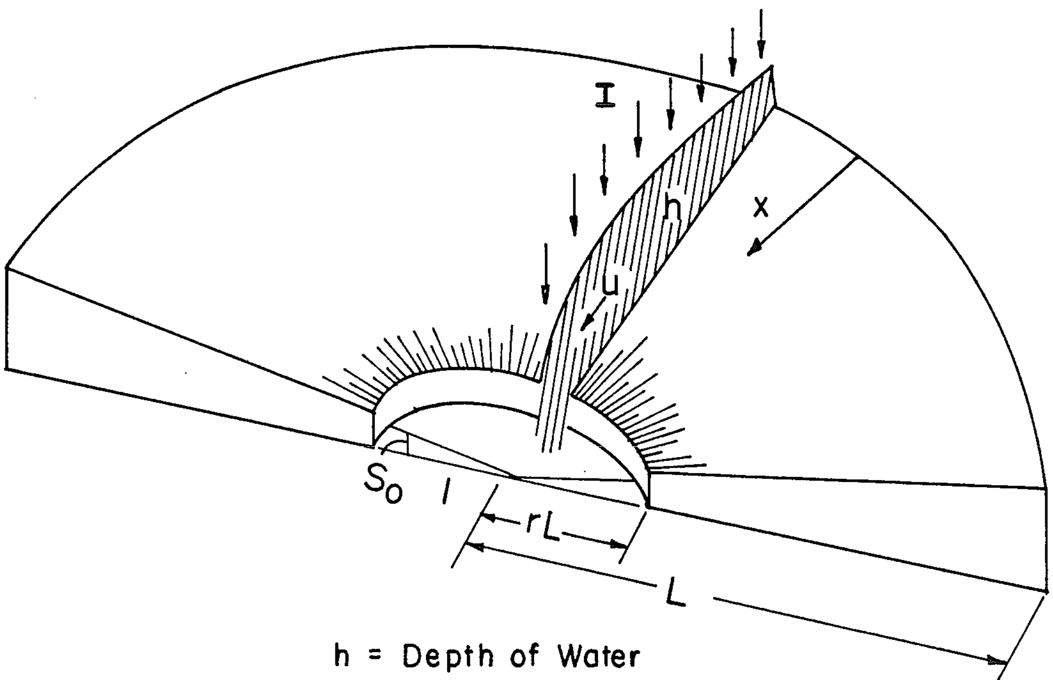


Figure 2.2 Nomograph for time of concentration for a plane using Manning's rough-ness coefficient (after Agiralioglu and Singh, 1981).



h = Depth of Water
u = Velocity of Flow

Figure 2.3 Geometry of a converging section.

$$B(a,b) = \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(a+b)} \tag{2.6}$$

However, B with a subscript denotes incomplete Beta function and has the following mathematical connotation,

$$B_w(a,b) = \int_a^w x^{a-1}\ (1-x)^{b-1}\ dx,\ a > 0,\ b > 0,\ x\varepsilon(0,1) \tag{2.7}$$

$$= \frac{w^a\ (1-w)^b}{a}\ \{1 + \sum_{j=0}^{\infty} \frac{B(a+1;j+1)}{B(a+b;j+1)}\ w^{j+1}\}$$

Equation (2.5) is considerably simplified when r can be assumed to be small,

$$T_c = \frac{1}{m}\ I^{\frac{1-m}{m}}\ (\frac{L_0}{2\alpha})^{\frac{1}{m}}\ [\frac{\Gamma(1 - \frac{1}{2m})\ \Gamma(\frac{1}{m})}{\Gamma(1 + \frac{1}{2m})}] \tag{2.8}$$

If the watershed is linearly diverging as shown in figure 2.4 then its time of concentration, as derived by Singh and Agiralioglu (1980), is given by

$$T_c = (\frac{T}{2})^{\frac{1-m}{m}}\ (\frac{1}{\alpha})^{\frac{1}{m}} \int_{rL}^{L} (\frac{x^2 - r^2\ L^2}{x})^{\frac{1-m}{m}}\ dx \tag{2.9}$$

where L is the length of the diverging section and r is the divergence parameter. If r is assumed to be small, equation (2.9) can be considerably simplified:

$$T_c = m(\frac{I}{2})^{\frac{1-m}{m}}\ (\frac{1}{\alpha})^{\frac{1}{m}}\ L^{\frac{1}{m}} \tag{2.10}$$

A nomographic solution of equation (2.9), with α represented by the Manning roughness coefficient, is shown in figure 2.5.

In a similar vein, using hydraulic considerations, Izzard (1946) derived an expression for T_c as

$$T_c = \frac{2}{60}\ (\frac{0.007\ I + c}{S^{1/3}})\ L(\frac{IL}{43200})^{-2/3} \tag{2.11}$$

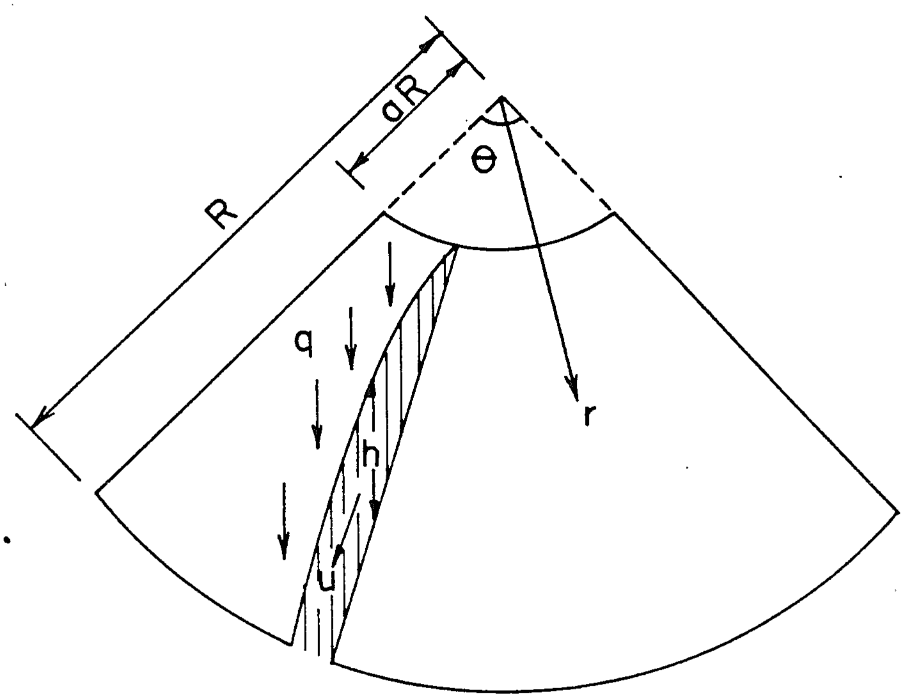


Figure 2.4 Geometry of a diverging section.

$$t_c = 3.878 \frac{R^{0.6} \; n_m^{0.6}}{q^{0.4} \; S_o^{0.3}}$$

EXAMPLE:
$R = 90\,m$
$n_m = 0.3$
$q = 5.0$ cm/hr
$S_o = 0.03$
(read) $t_c = 42$ min

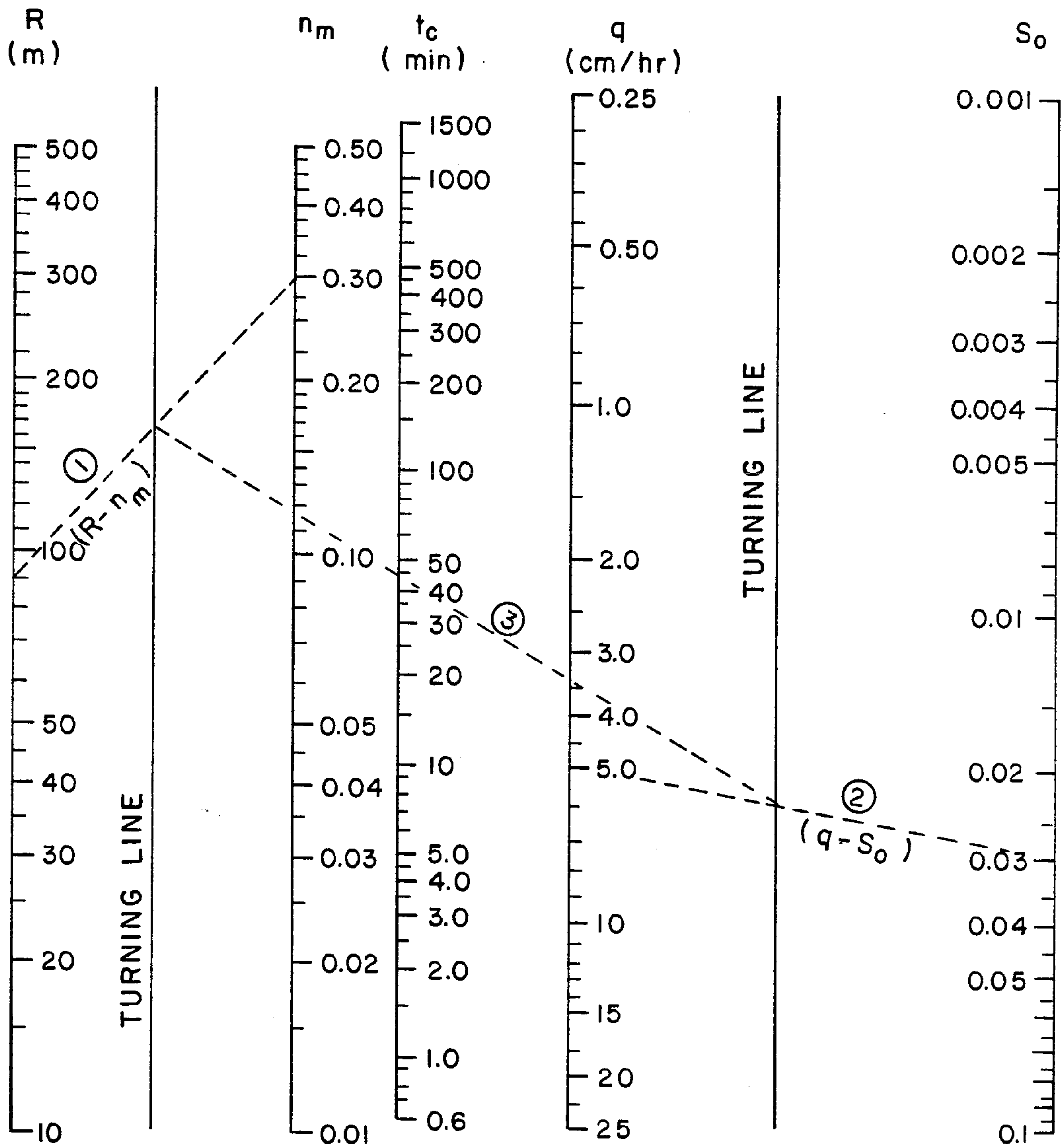


Figure 2.5 Nomograph for time of concentration for diverging surface using Manning's roughness coefficient (after Agiralioglu and Singh, 1981).

where T_c is overland flow time in minutes, L watershed length in feet, S slope of overland flow path in feet/feet, I effective rainfall in inches/hour, and c surface roughness coefficient. The values of c are given in Table 2.3. Equation (2.11) is valid for laminar overland flow for which I L < 500, and was derived for developed surfaces. This equation should be used cautiously for slopes steeper than 0.04. Equations similar to equation (2.11) were also derived by Butler (1977a, 1977b, 1980). Gill (1977) argued in a discussion of a paper by Butler (1977a) that the assumption of considering slope to be invariant with distance was unjustified. However, in a reply Butler (1977b) showed by using experimental data that replacing friction slope with bed slope caused an error of less than one percent, and was thus reasonable. Butler extended his derivations to incorporate both laminar and turbulent flows taking place on the watershed plane. It may be noted that equation (2.11) is similar to equation (2.2). Indeed the former can be derived from the latter.

2.4.2 EMPIRICALLY DERIVED EQUATIONS

Since T_c is most extensively used in the rational method whose basis is empirical, a simple relation for T_c may be perhaps more appropriate. The Bransby-Williams formula (Williams, 1922) is one such relation which can be written for T_c in hours as

$$T_c = \frac{L\ A^{0.4}}{D\ S^{0.2}} \tag{2.12}$$

where L is the longest distance in miles from the edge of the drainage basin to its outfall, A the area of the basin in square miles, D the diameter in miles of a circular basin whose area is A, and S the average fall in feet per 100 feet of the ground measured along the most direct route to the outfall. The drainage area can be up to 50 square miles.

Table 2.3 Values of c for Izzard's formula.

Surface	Value
Smooth asphalt surface	0.007
Concrete pavement	0.012
Tar and gravel pavement	0.017
Closely slipped soil	0.046
Dense bluegrass turf	0.060

Another simple relation is the Kirpich formula (Kirpich, 1940) which he calibrated for small watersheds in Pennsylvania and Tennessee. His formula for the Tennessee watersheds (ranging in size from 1 to 112 acres; slope varying from 3 to 10 percent) is

$$T_c = 13 \times 10^{-5} \frac{L^{0.77}}{S^{0.385}} \tag{2.13}$$

and for the Pennsylvania watersheds is

$$T_c = 21.67 \times 10^{-6} \frac{L^{0.77}}{S^{0.5}} \tag{2.14}$$

where L is the length of the watershed in feet, measured along the channel from the outlet and in a direct line from the upper end of the channel to the farthest point on the whatershed, and S the dimensionless ratio, the elevation difference between the outfall and the farthest point or approximately the average slope of the basin. If the slope is nonuniformly changing along L, a weighted S may be desirable. T_c should be multiplied by 0.4 and 0.2 for watersheds where the overland flow path is either concrete or asphalt and the channel is concrete-lined respectively. This would be classified as a channel flow method. However, given the relatively small drainage areas, it should reflect significant portions of overland flow travel time.

Based on an analysis of characteristics of 19 watersheds having areas between 25 and 1,624 square miles in the Scotie and Sandusky River watersheds, Johnstone and Cross (1949) developed the following relationship for T_c,

$$T_c = \frac{4.7}{r^2} \left(\frac{L}{S}\right)^{0.5} \tag{2.15}$$

where T_c is in hours, L length of the principal stream in the watershed in miles, S average slope of the mainstream in feet per mile, and r a branching factor based on the stream pattern. They found a good approximation of equation (2.15) by neglecting r as

$$T_c = 5 \left(\frac{L}{S}\right)^{0.5} \tag{2.16}$$

A similar relation for T_c was developed by Eaton (1954) who performed a correlation study for seven Tasmanian rivers having watershed areas from 48 to 322 square miles. His equation follows:

$$T_c = 1.35 \left(\frac{AL}{r}\right)^{0.37} \tag{2.17}$$

where the notations have the same meaning as above.

Kerby (1959), following the work of Hathaway (1945), proposed an empirical relation and developed a nomograph for determining the time of concentration for overland flow. His relation can be expressed as

$$T_c = \left(\frac{2}{3}\frac{L\ N}{S^{0.5}}\right)^{\frac{1}{2.14}} \tag{2.18}$$

where T_c is the time of concentration for overland flow in minutes, L the length in feet of overland flow defined as the distance from the extremity of the drainage area in a direction parallel to the slope until a defined channel is reached, S the difference in elevation between the extreme edge and the end of overland flow length, divided by L, and N the average surface retardance coefficient of overland flow. This would be classified as surface flow method. Watersheds, ranging in areas less than 10 acres, slopes less than one percent and N less than 0.8, were used. Table 2.4 gives values of N for some surfaces. Kerby (1950) noted that L would be within 1200 feet. Similar relations and nomographs have also been developed by Mockus (1961), Seelye (1968) and Denver Regional Council of Governments (1969) as shown in figure 2.6.

Table 2.4 Values of N for Kerby's formula.

Type of Surface	N
Smooth impervious surface	0.02
Smooth bare packed soil	0.10
Poor grass, cultivated row crops or moderately rough bare surfaces	0.20
Deciduous timberland	0.60
Pasture or average grass	0.40
Conifer timberland, deciduous timberland with deep forest litter or dense grass	0.80

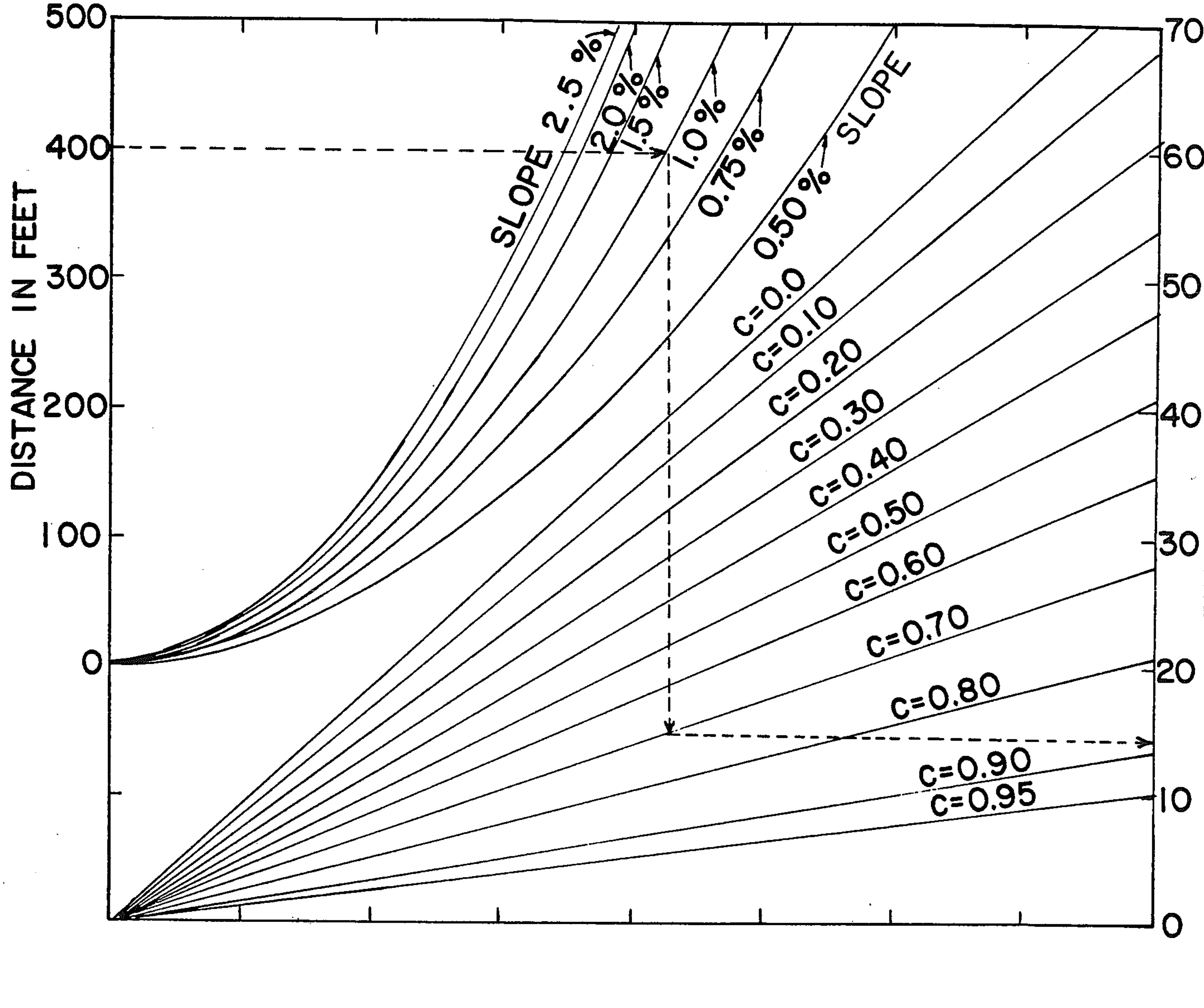


Figure 2.6 Overland flow time (after Denver Regional Council of Governments, 1969).

The Federal Aviation Agency (1970) developed an equation for T_c from airfield drainage data,

$$T_c = 0.03\ (1.1 - C)\ L^{0.5}\ 5^{0.333} \qquad (2.19)$$

in which C is runoff coefficient of the rational method, L maximum length of overland flow in feet and S slope in percent of the longest overland flow path. Evidently this is a surface flow method.

An equation of T_c developed by the Soil Conservation Service (1975) is

$$T_c = 10^{-5} \times 27.78 \sum_i (L\ K\ S^{-0.5})_i \qquad (2.20)$$

in which L is length of overland flow in feet, K an empirical coefficient indicative of land cover, and S slope in percent. T_c is regarded as the ratio of the length and velocity. A watershed is divided into a number of flow segments. The graph, as shown in figure 2.7, provided in TR-55 (Soil Conservation Service, 1975) for the SCS velocity method is used to compute the velocity of flow through the segment. A travel time is obtained for each flow segment. The time of concentration is set equal to the sum of the segment travel times. This is basically an overland flow method but can be extended to include channel flow.

McCuen, Wong and Rawls (1983) calibrated for urban areas the following equations

$$T_c = 0.01462\ L_f^{0.5552}\ I_2^{-0.7164}\ S_{fm}^{-0.207} \qquad (2.21)$$

and

$$T_c = 0.0469\ L_f^{0.445}\ I_2^{-0.7231}\ C_f^{0.5517}\ S_{fm}^{-0.226} \qquad (2.22)$$

where L_f is total length of flow path in feet, I_2 2-year, and T_c-hour rainfall intensity in inches per hour, S_{fm} channel slope in feet per mile, and C_f Espey channelization coefficient (Espey, Morgan and Masch, 1965). These equations include both overland flow and channel flow

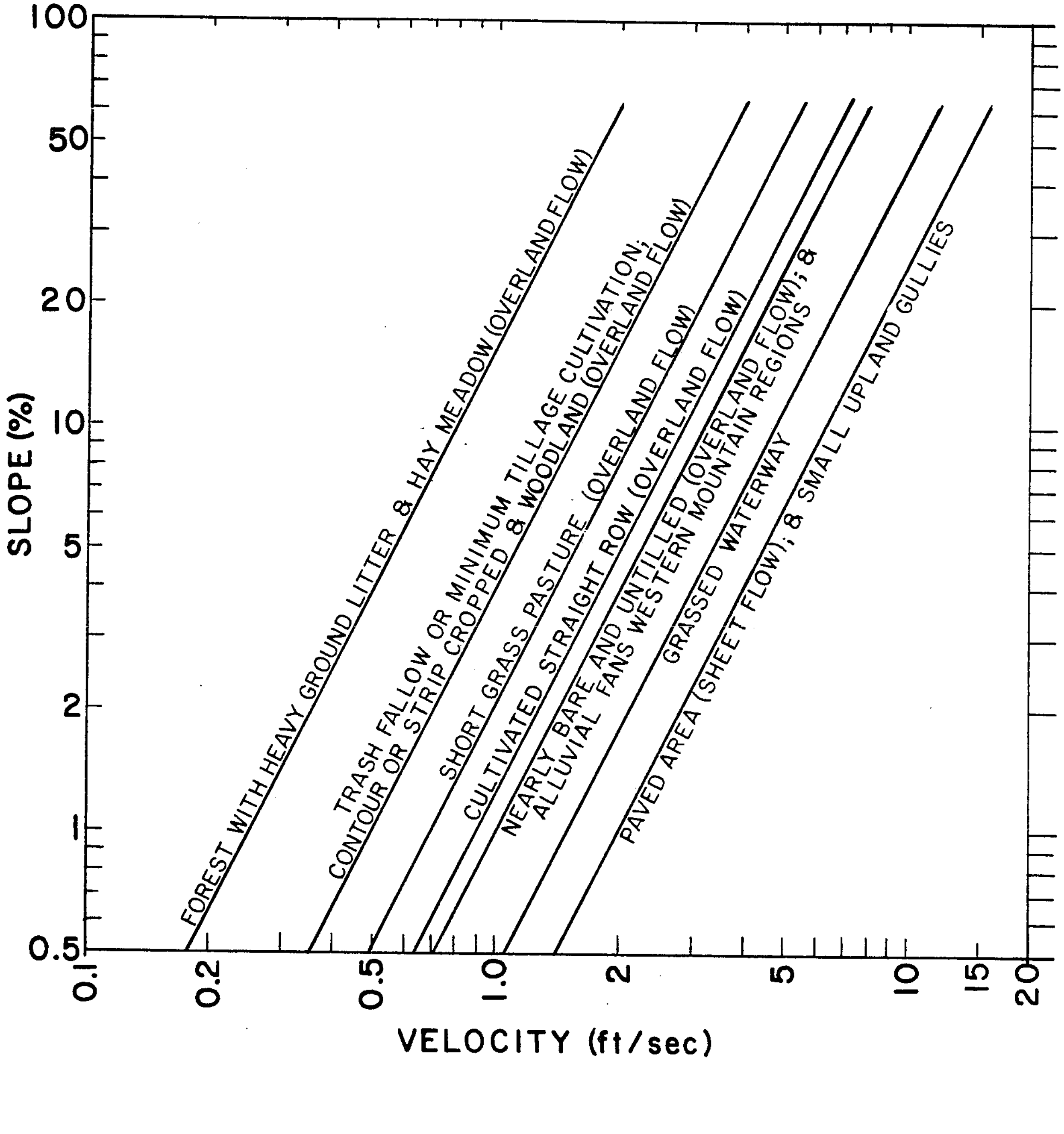


Figure 2.7 Velocities for estimating overland flow time (after Soil Conversation Service, 1972).

regimes. Equation (2.22) includes C_f which is a flow resistance variable. Equation (2.21) should be used except when the channel is undergoing a hydraulic change or is significantly different from the average channel for an area; in such cases an estimate of C_f should be made and equation (2.22) used to estimate T_c.

2.5 CHOOSING BETWEEN EQUATIONS

McCuen, Wong and Rawls (1984) evaluated 11 equations for estimating T_c from 48 urban watersheds located in most regions of the United States. These equations are (2.2), (2.13), (2.14), (2.18), (2.19), (2.20), (2.21), and (2.22). The watersheds had areas less than 4,000 acres, with average impervious area of 22.1 percent and the mean time of concentration 1.49 hours having a range from 0.21 to 6.14 hours. These equations resulted in errors in the estimated value of T_c that was greater than 30 minutes for more than 50 percent of the watersheds. The maximum errors for these equations ranged from 1.48 to 10.29 hours. The standard deviations of the errors ranged from 0.37 to 2.27 hours, with only one of the 11 equations having a value less than one hour. If such errors are translated into errors in design discharges then the design errors can be significant indeed. Given the wide variation in the estimated T_c values and the very large errors, which equation should be used? For watersheds having nonhomogeneous land uses and significant channel flow, a mixed method is certainly preferable. A velocity method is most likely to provide an accurate estimate for such watersheds. Equations (2.2) and (2.20) represent such a method. Similarity in watershed characteristics between the watershed where a design is required and the watersheds used to calibrate an equation is perhaps the most important criterion for selecting an equation. The other considerations are availability of data and physical characteristics of the watershed. The equation(s) selected for computing T_c must reflect pertinent flow and physiographic characteristics.

2.6 LAG TIME

There exists a multitude of definitions of lag time as evident from the foregoing discussion. Horner and Flynt (1936) remarked, "The term, 'lag' . . . has reference only to the difference in phase between salient features of the rainfall and runoff curves, . . . Values, subject to correction, are taken from the time difference between salient features . . . , or between the centers of mass of the rainfall and runoff rate curves." This time difference between salient features appears to have been interpreted in many different ways. We briefly outline some of the more frequently utilized interpretations.

2.6.1 DEFINITION 1

Perhaps the most popular definition of lag time is the time difference between the centroid of effective rainfall and that of direct runoff. If h(t) is the IUH then its first moment defines the lag time,

$$T_\ell = \int_0^\infty t\, h(t)\, dt \qquad (2.23)$$

Further, if a single linear reservoir is used to define the IUH then its storage coefficient k specifies the lag time, that is, $T_\ell = k$. The lag time defined in this way appears to be the most stable measure (Schulz and Lopez, 1974; Espey, Morgan and Masch, 1965). Many attempts have been made to empirically estimate T_ℓ in terms of watershed characteristics. The resulting empirical equations are given in Table 2.5.

Table 2.5 Equations of lag time in hours according to definition 1 (t_1 from figure 2.1).

Author	Equation	Equation Number for Referencing
Clark (1945)	$t_1 = K\ L/\sqrt{S_c}$, $k \varepsilon [0.8,2.2]$	(2.24)
Linsley (1945)	$t_1 = K\ L\ \sqrt{A/S_c}$	(2.25)
Mitchell (1948)	$t_1 = K\ A^{0.6}$, $K = 1.05$	(2.26)
Johnston (1949)	$t_1 = K + 90\ \frac{W}{S}$, $K = 1.5$	(2.27)
Eaton (1954)	$t_1 = K(WA/(Lr))^{1/3}$, $K = 1.2$, $r = \varepsilon[1,2]$	(2.28)
O'Kelly (1955)	$t_1 = K\ S_0^d$	(2.29)
Dooge (1955)	$t_1 = K\ A^{.25}/\sqrt{S_0}$, $K\varepsilon[10,13]$	(2.30)
Hoyt and Langbein (1955)	$t_1 = K\ A^{0.4}$, $K\ [1,3]$	(2.31)
Nash (1960)	$t_1 = K\ A^{0.3}\ S_0^{-0.3}$, $K = 27.6$	(2.32)
	$t_1 = K\ L^{0.3}\ S_c^{-0.33}$, $K = 20$	(2.33)
Carter (1961)	$t_1 = K(L/\sqrt{S_a})^{0.6}$, $K = 1.7$	(2.34)
Morgan and Johnson (1962)	$t_1 = K\ t_2^{0.81}$, $K = 2.8$	(2.35)
Wu (1963)	$t_1 = K\ A^{.94}\ L^{-1.47}\ S_c^{-1.47}$, $K = 780$	(2.36)
Bell (1967)	$t_1 = K\ A^{0.33}$, $K\ \varepsilon\ [0.5,3]$	(2.37)

Table 2.5 (continued).

Author	Equation	Equation Number for Referencing
Kennedy and Watt (1967)	$t_1 = K\ L^{0.66}\ S^{-.33}\ S_b^{1.21},\ K = 6.71$	(2.38)
Cordery (1968)	$t_1 = K\ [\frac{W}{\sqrt{S_0}} + \frac{L_{nm}}{2\sqrt{S_0}}]$	(2.39)
Wu (1969)	$t_1 = K\ A^{0.23}$	(2.40)
Bell and Omkar (1969)	$t_1 = K\ A^{0.33}$	(2.41)
Rao and Delleur (1974)	$t_1 = 0.78\ A^{0.496} L^{0.073} S^{-0.075} (1+I_A)^{-1.289}$	(2.42)
	$t_1 = 0.78 A^{0.542} S^{-0.081} (1+I_A)^{-1.21}$	(2.43)
	$t_1 = 0.803 A^{0.512} (1+I_A)^{-1.433}$	(2.44)
Boyd (1978)	$t_1 = 1.066\ (2\mu - 1)^{0.404}$	(2.45)
	$t_1 = 2.51\ A^{0.38}$	(2.46)

A = Watershed area in square miles

A_m = Area in k_m^2

D_r = Duration of the effective rainfall in hours

I_A = Percentage impervious area

I_d = Depth of the effective rainfall in mm

K = Constant

L = Mainstream length in miles

L_r = Watershed lag ratio

Table 2.5 (continued).

n_m = Manning roughness coefficient

r = Branching factor

S = Mean basin slope in feet/mile

S_a = Mainstream slope calculated by equal area method in feet/mile

S_b = A dimensionless factor representing the ratio of area A_b of lakes, marshes and ponds in the upper 2/3 of the watershed to the watershed area, $S_b = 1 + 20(A_b/A)$

S_c = Mean slope of the mainstream in feet/mile

S_d = Measure of overland slope, a dimensionless ratio

S_0 = Average overland slope in parts/10,000

$(2\mu-1)$ = Total number of interior and exterior links in the channel network

W = Average width of watershed in miles

2.6.2 DEFINITION 2

Another frequently employed definition of lag time is the time difference between the centroid of the effective rainfall and the peak of direct runoff. Several empirical equations have been derived to relate lag time T_p to watershed characteristics. Some of these equations are listed in Table 2.6. The lag time defined in this way is more variable than the one of definition 1 above.

2.6.3 DEFINITION 3

This definition is due to Overton (1970) who defined the lag time as the lapse between the time of occurrence of 50 percent input volume and 50 percent output volume. For the effective rainfall resulting in equilibrium hydrograph, Overton derived this lag time T_ℓ to be the ratio of storage at the equilibrium condition V_e to the intensity of effective rainfall I,

$$T_\ell = V_e/I \qquad (2.55)$$

Using kinematic wave equations, T_ℓ was derived by Overton (1971) for a plane, by Singh (1975) for a converging surface, and by Singh and Agiralioglu (1980) for a diverging surface. For a plane surface T_ℓ can be expressed as

$$T_\ell = \frac{m}{m+1}\left[\frac{LI^{1-m}}{\alpha}\right]^{\frac{1}{m}} \qquad (2.56)$$

where m is exponent and α resistance parameter in kinematic depth-discharge relationship. A nomographic solution is presented in figure 2.8 (Agiralioglu and Singh, 1981).

Table 2.6 Equations of lag time in hours according to definition 2 (t_2 from figure 2.1).

Author	Equation	Equation Number for Referencing
Snyder (1938)	$t_2 = C_t(LL_c)^{0.3}$, $C_t\ \varepsilon[1.8,2.2]$	(2.47)
Linsley (1943)	$t_2 = C_t(LL_c)^{0.3}$, $C_t\ \varepsilon[0.3,1.2]$	(2.48)
Taylor and Schwarz (1952)	$t_2 = C \exp\ (m_1\ D)$, $C = 0.6/\sqrt{S}$, $m_1 = 0.212\ (LL_c)^{-.36}$	(2.49)
Mockus (1957)	$t_2 = K\ L_w^{0.8}\ (M + 1)^{1.67}/S_0^{\ 0.5}$, $K = \frac{1}{1900}$	(2.50)
Hickok, Keppel and Rafferty (1959)	$t_2 = C\ [\frac{A^{0.3}}{S_0\ D^{0.5}}]^{0.61}$	(2.51)
	$t_2 = C\ [\frac{(L_c + W)^{0.5}}{S_0 D^{0.5}}]^{.65}$	(2.52)
Eagleson (1962)	$t_2 = \frac{L_a\ n}{1.5}\ R_a^{\ 2/3}\ S_a^{\ -1/2}$	(2.53)
Soil Conservation Service (1975)	$t_2 = \frac{L_w^{\ 0.8}\ (\frac{1000}{C_N} - 9)^{0.7}}{1900\ S_0^{\ 0.5}}$	(2.54)

C_N = Runoff curve number

D = Drainage density

L = Length of the main channel in miles

L_a = Mean travel distance in feet equal to that portion of the sewer which flows full

L_C = Distance in miles up the main stream to the center of area

L_W = Hydralic length of watershed in feet

M = Soil cover parameter

R_a = Weighted hydraulic radius in feet of the main sewer flowing full

S = Average channel slope

S_0 = Average watershed slope in percent

S_a = Weighted slope of the main sewer in feet/feet

W = Watershed width

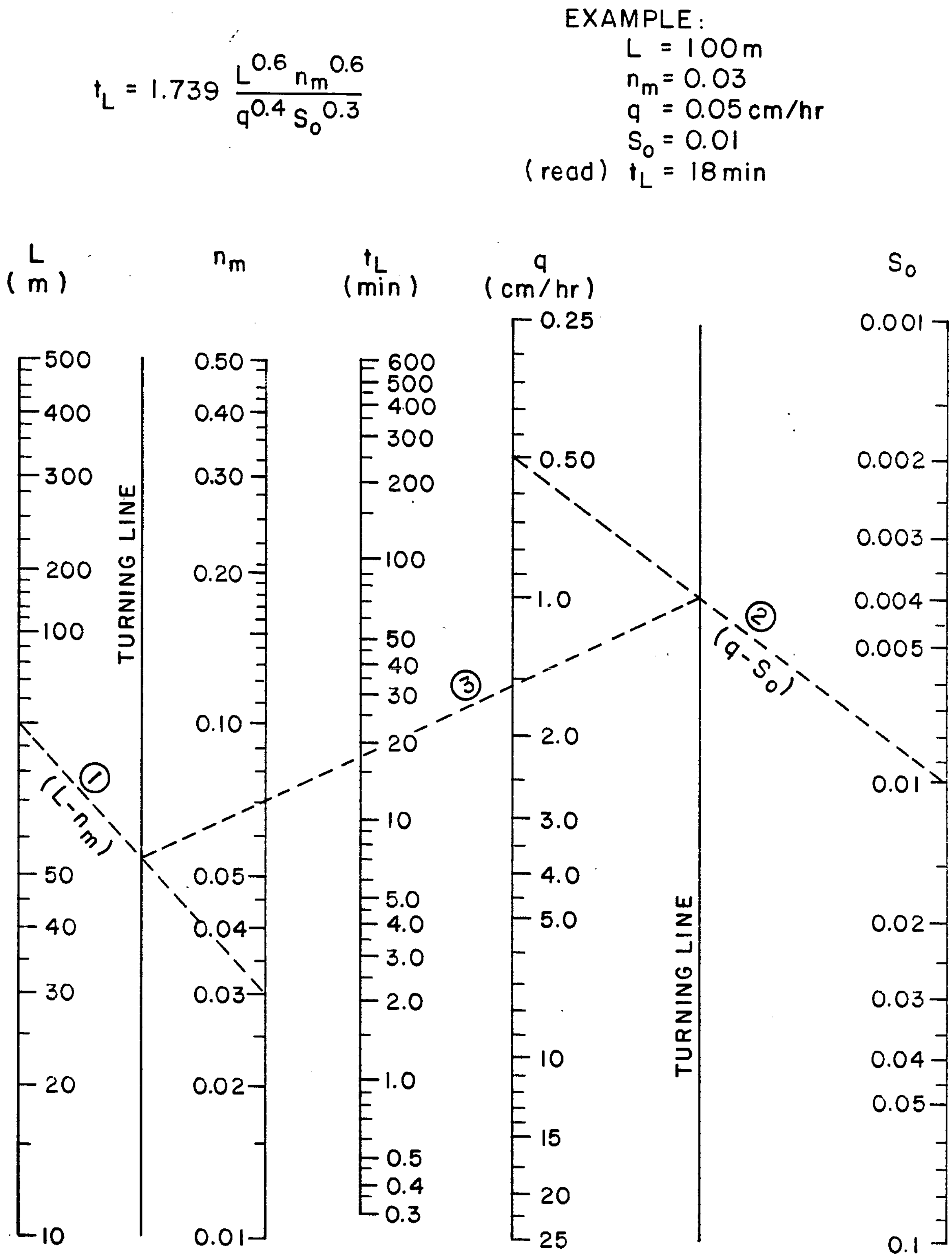


Figure 2.8 Nomograph for lag time for a plane using Manning's roughness coefficient (after Agiralioglu and Singh, 1981).

For a converging surface,

$$T_\ell = [\frac{LI}{2\alpha}]^{1/m} \frac{1}{2\ I(1 - r)} [\frac{\Gamma(1 + \frac{1}{m})\ \Gamma(1 - \frac{1}{2m})}{\Gamma(2 + \frac{1}{2m})}$$

$$- \frac{r^{2(m+1)/m}\ (1 - r^2)^{1 - \frac{1}{2m}}}{(1 + \frac{1}{m})}$$

$$\{1 + \sum_{j=0}^{\infty} \frac{B(2 + \frac{1}{m};\ j + 1)}{B(2 + \frac{1}{2m};\ j + 1)} r^{2(j+1)}\}] \tag{2.57}$$

in which B(.,.) represents the beta function. If r is only partly accounted for,

$$T_\ell = [\frac{LI}{2\alpha}]^{1/m} \frac{\Gamma(1 + \frac{1}{m})\ \Gamma(1 - \frac{1}{2m})}{2\ I(1 - r)\ \Gamma(2 + \frac{1}{2m})} \tag{2.58}$$

where I is the time-invariant intensity of effective rainfall, and α and m are, as before, kinematic wave parameters in the depth-discharge relationship.

For a diverging surface,

$$T_\ell = \frac{1}{R(1 - a)} I^{\frac{1-m}{m}} (\frac{1}{2\alpha})^{1/m} \int_{aR}^{R} (\frac{r^2 - a^2 R^2}{r})^{1/m} dr \tag{2.59}$$

Using Binomial theorem,

$$T_\ell = \frac{I^{m^*-1}}{R(1 - a)} (\frac{1}{a\alpha})^{m^*} \sum_{j=0}^{\infty} \binom{m^*}{j} (-1)^j (aR)^{2j} [\frac{R^{m^*-2j+1} - (aR)^{m^*-2j+1}}{m^* - 2j + 1}] \tag{2.60}$$

where $m^* = 1/m$. If a is negligibly small,

$$T_\ell = (\frac{R}{2})^{m^*} (\frac{m}{m + 1}) I^{m^*-1} \tag{2.61}$$

Agiraliogl u and Singh (1981) presented a nomograph based on equation (2.61) as shown in figure 2.9.

$$t_L = 1.147 \frac{R^{0.6} \, n_m^{0.6}}{q^{0.4} \, S_o^{0.3}}$$

EXAMPLE:

$R = 100$ m

$n_m = 0.03$

$q = 0.50$ cm/hr

$S_o = 0.01$

$t_L = 11.7$ min

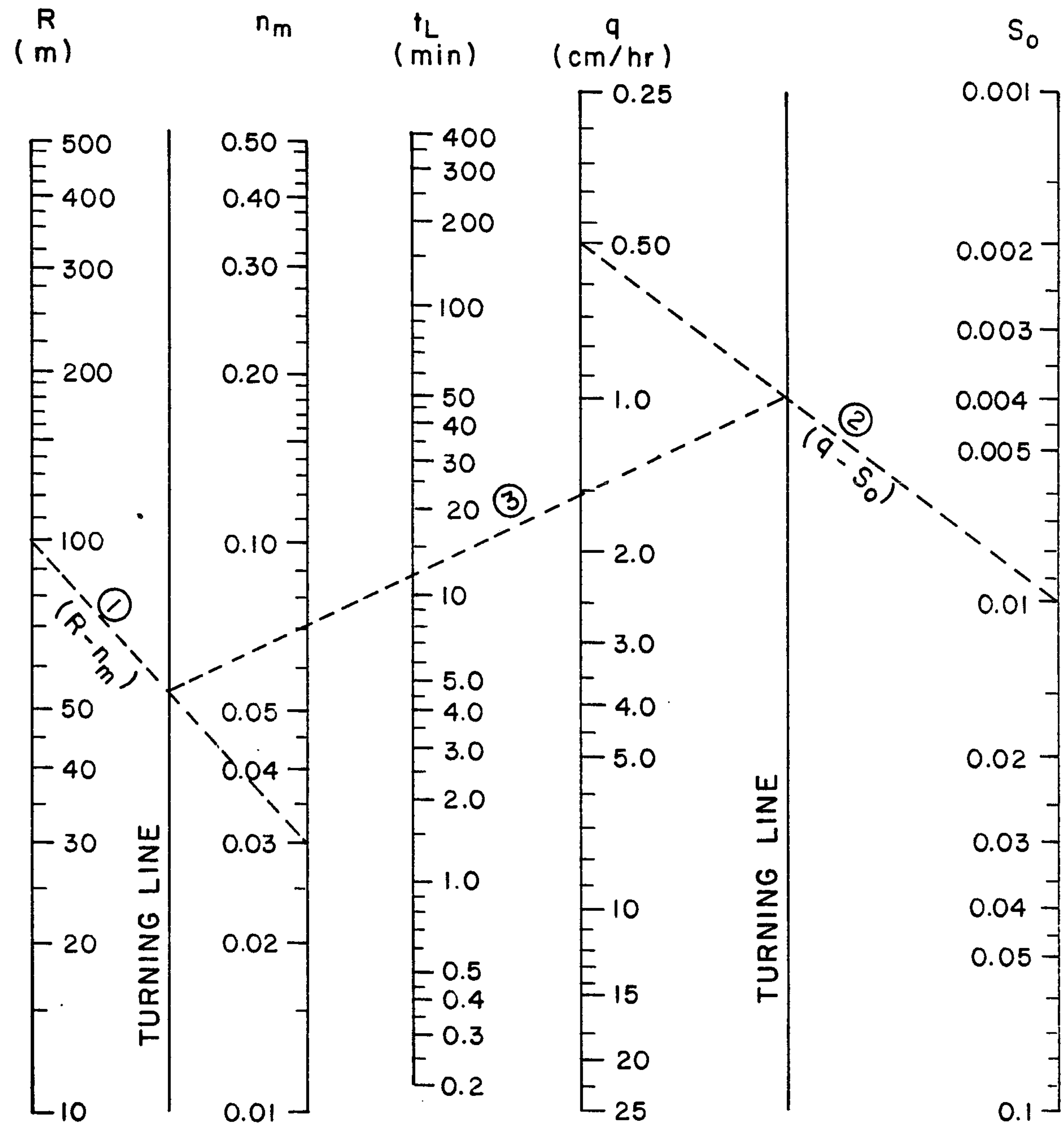


Figure 2.9 Nomograph for lag time for a diverging surface using Manning's roughness coefficient (after Agiralioglu and Singh, 1981).

It is evident that area, channel length, channel slope, and overland slope are the most frequently utilized geomorphologic characteristics for estimation of watershed lag time. Other characteristics, utilized much less frequently, include drainage density, stream order, roughness coefficient, and watershed width.

Soil Conservation Service (1975) presented a nomograph for equation (2.54), as shown in figure 2.10. This is a curve number method applicable to small urban areas less than 2,000 acres. This yields lag time adequately for an area which is completely paved such as a parking lot. It, however, overestimates for composite land use areas where streets, gutters or sewers provide a more efficient flow pattern than lawns, forests or other pervious areas. This method was modified by Soil Conversation Service to incorporate the effect of changes in the main channel and the watershed surface. For example, if the main channel is partly lined or made hydraulically more efficient then its travel time will be reduced as shown in figure 2.11. Likewise, by increasing impervious portion of its area, the watershed lag can be modified as shown in figure 2.12. We illustrate this method by an example.

2.7 LAW OF BASIN LAG

In a study of four nested drainage basins near Sydney in New South Wales, Australia, Boyd (1978) discovered that the basin lag time followed a law similar to the law of basin areas. This can be written as

$$L_w = L_1 R_\ell^{u-1} \tag{2.62}$$

where L_w is lag time of the basin of order w, L_1 lag time of the first order basin, and R_ℓ basin lag ratio. He found R_ℓ to be 1.737 for his

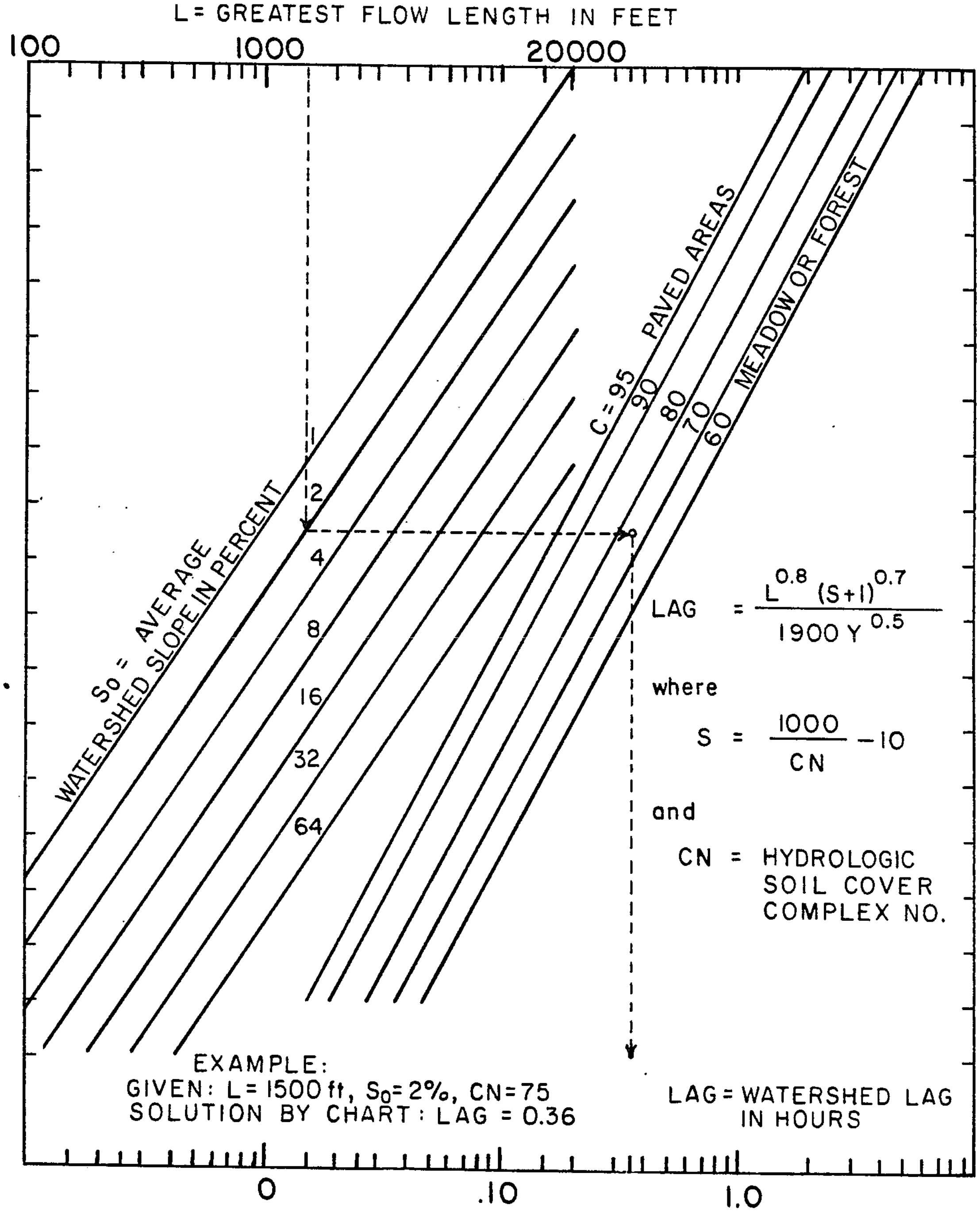


Figure 2.10 Curve number method for estimating lag for homogeneous watersheds under natural conditions up to 2,000 acres (Soil Conservation Service, 1975).

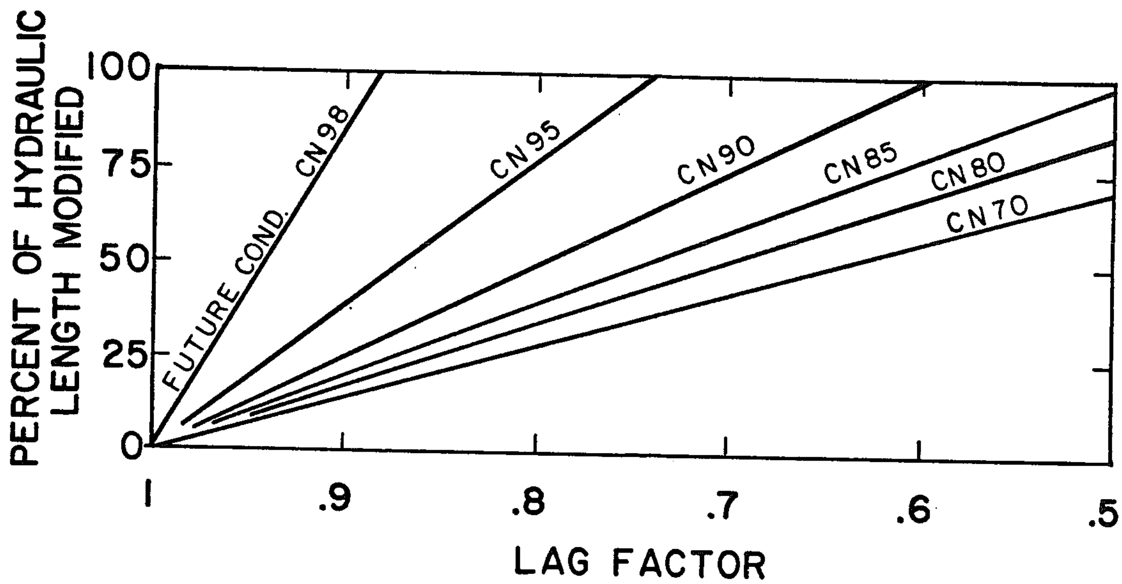


Figure 2.11 Factors for adjusting lag from equation (2.54) or figure 2.10 when the main channel has been hydraulically improved (after Soil Conservation Service, 1975).

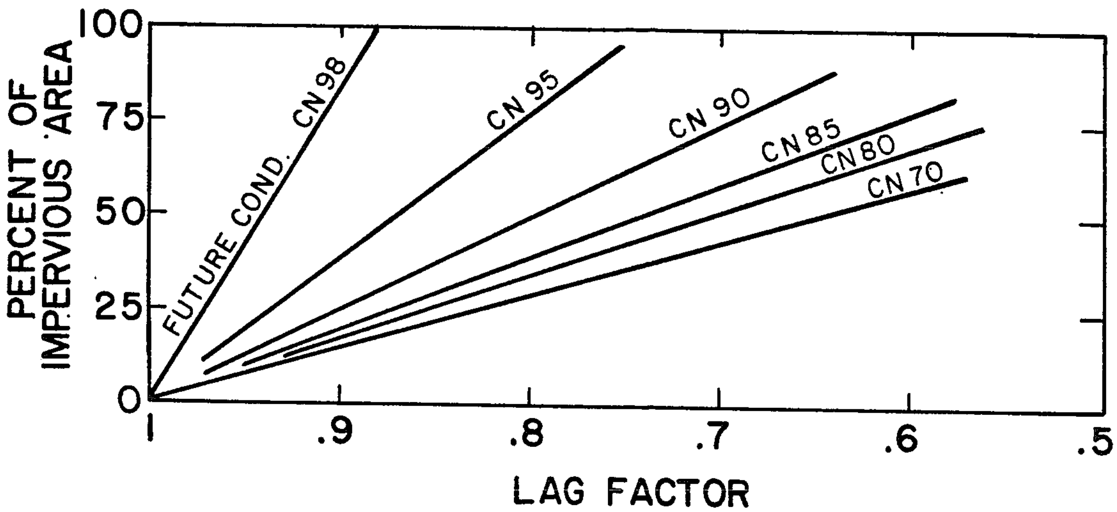


Figure 2.12 Factors for adjusting lag from equation (2.54) or figure 2.10 when impervious areas occur in the watershed (after Soil Conservation Service, 1975).

data. Here L is the same as T_ℓ. Equation (2.62) can be used conveniently in hydrologic synthesis of ungaged basins.

2.8 VARIATION IN LAG TIME

The lag time, defined according to definition one, is more stable and, therefore, more suitable in hydrologic modeling. If the watershed is assumed to be linear and time-invariant, the time parameter is constant and independent of storm properties, whereas most of the other measures depend on storm characteristics, particularly the duration of effective rainfall (Meynink, 1978).

The lag time is frequently found to vary with watershed and rainfall characteristics. The watershed characteristics can be of two types: (1) morphological such as area, main stream length, overland slope, main stream slope, drainage density, etc.; and (2) non-morphological such as vegetation and land use. The morphological characteristics may be considered constant within the time frame of hydrologic design. However, vegetation and land use practices may change within a season and from one season to another. Further, these are influenced by climatic conditions.

Recognizing the variability of lag time, several investigators have attempted to relate it to the characteristics of effective rainfall and discharge (Laurenson, 1962; Askew, 1968, 1970a, 1970b; Meynink, 1978; Overton, 1970, 1971; Rao and Delleur, 1974; Singh, 1975; Singh and Agiralioglu, 1980). Some of the variable lag time models are given in Table 2.7. It may be remarked that of all the studies on lag time, Meynink (1978) conducted perhaps the most comprehensive investigation. The variability of lag time is also supported by kinematic wave theory.

Table 2.7 Equations of variable lag time in hours (as defined in figure 2.1).

Author	Equation	Equation Number for Referencing
Laurenson (1962, 1964)	$t_1 = a\ Q_m^b$, $a = 64$, $b = -0.27$	(2.63)
Askew (1968, 1970a, 1970b)	$t_1\ K\ A^{.57}\ Q_m^{-.23}$ $K = 8.28$	(2.64)
	$t_1 = K\ A^{0.54}\ S_d^{-.16}\ Q_m^{-.23}$, $K = 2.91$	(2.65)
	$t_1 = K\ L^{.8}\ S_d^{-.33}\ Q_m^{-.23}$, $K = 2.91$	(2.66)
Rao and Delleur (1974)	$t_1 = 0.831 A^{0.458} (1+I_A)^{-.166} P_E^{0.267} D_r^{0.371}$	(2.67)
	$t_2 = 0.731 L^{0.943} (1+I_A)^{-4.303} P_E^{-2.114}$ $D_r^{0.238}$	(2.68)
Meynink (1978)	$t_1 = K\ A_m^{.3405}\ Q_p^{-.3619}$, $K = 2.55$	(2.69)
	$t_1 = K\ A_m^{1/3}\ Q_p^{-2/15}\ (\frac{D_r}{I_d})^{1/5}$, $K = 3.12$	(2.70)
Boyd (1978)	$t_1 = 2.12\ A_m^{0.57}\ Q_s^{-0.23}$	(2.71)

A = Watershed area in square miles

A_m = Watershed area in square kilometers

D_r = Duration of the effective rainfall in hours

I_A = Percentage impervious area

I_d = Depth of the effective rainfall in mm

I_e = Depth of effective rainfall in inches

L = Length of the main stream in miles

Q_m = Average discharge rate associcted with the surface runoff hydrograph in cfs

Q_p = Peak discharge in mm/hour

Q_s = Peak discharge in m^3/s

S_d = Measure of overland slope, a dimensionless ratio

2.9 RELATION BETWEEN TIME OF CONCENTRATION AND LAG TIME

The time of concentration is closely related to the time of equilibrium T_e and the lag time T_ℓ. T_e is the time when runoff peak is equal to the maximum intensity of effective rainfall. For uniform effective rainfall intensity lasting for a period equal to or greater than T_c, T_c and T_e will be equal to each other. If the duration is less than T_c then $T_c > T_e$. If the watershed geometry is simple, say a rectangular plane, then for uniform effective rainfall,

$$T_c \geq T_e \geq T_\ell \tag{2.72}$$

For natural watersheds of larger size and complex drainage pattern, the water originating on the most remote part may arrive at the outlet too late to contribute to the peak runoff. Therefore, T_c will generally be greater than T_ℓ. It is physically plausible that on natural watersheds, T_e will be equal to or greater than T_c. Therefore,

$$T_e \geq T_c \geq T_\ell \tag{2.73}$$

It is evident that these time characteristics are not independent of each other for a watershed. The Soil Conservation Service (1975) suggested that

$$T_c = 1.67\ t_2 \tag{2.74}$$

Overton and Meadows (1976) indicated

$$T_c = 1.6\ t_2 \tag{2.75}$$

However, it can be shown by using the Soil Conservation Service (SCS) triangular unit hydrograph with 37.5 percent of the volume under the rising limb that $t_1 = 1.176\ t_2$. By inserting equation (2.74),

$$T_c = 1.417\ t_1 \tag{2.76}$$

McCuen, Wong and Rawls (1984) examined the validity of equations (2.74) and (2.76) by using hyetograph and hydrograph data from 39 urban watersheds. They computed the mean value of T_c and t_1 for the entire sample of watersheds. These were 1.51 hours and 1.12 hours respectively. The ratio of 1.51/1.12 is 1.35 which agrees well with the value of 1.417. Thus, it appears that on the average, computed T_c values agree with T_c values from rainfall-runoff data. However, values on a given watershed show considerable scatter as seen in figure 2.13.

2.10 OTHER TIME CHARACTERISTICS

In addition to lag time, several other measures of the time characteristics of runoff hydrograph have been used in hydrograph synthesis. These are defined in figure 2.1. Some of these have been related to watershed characteristics.

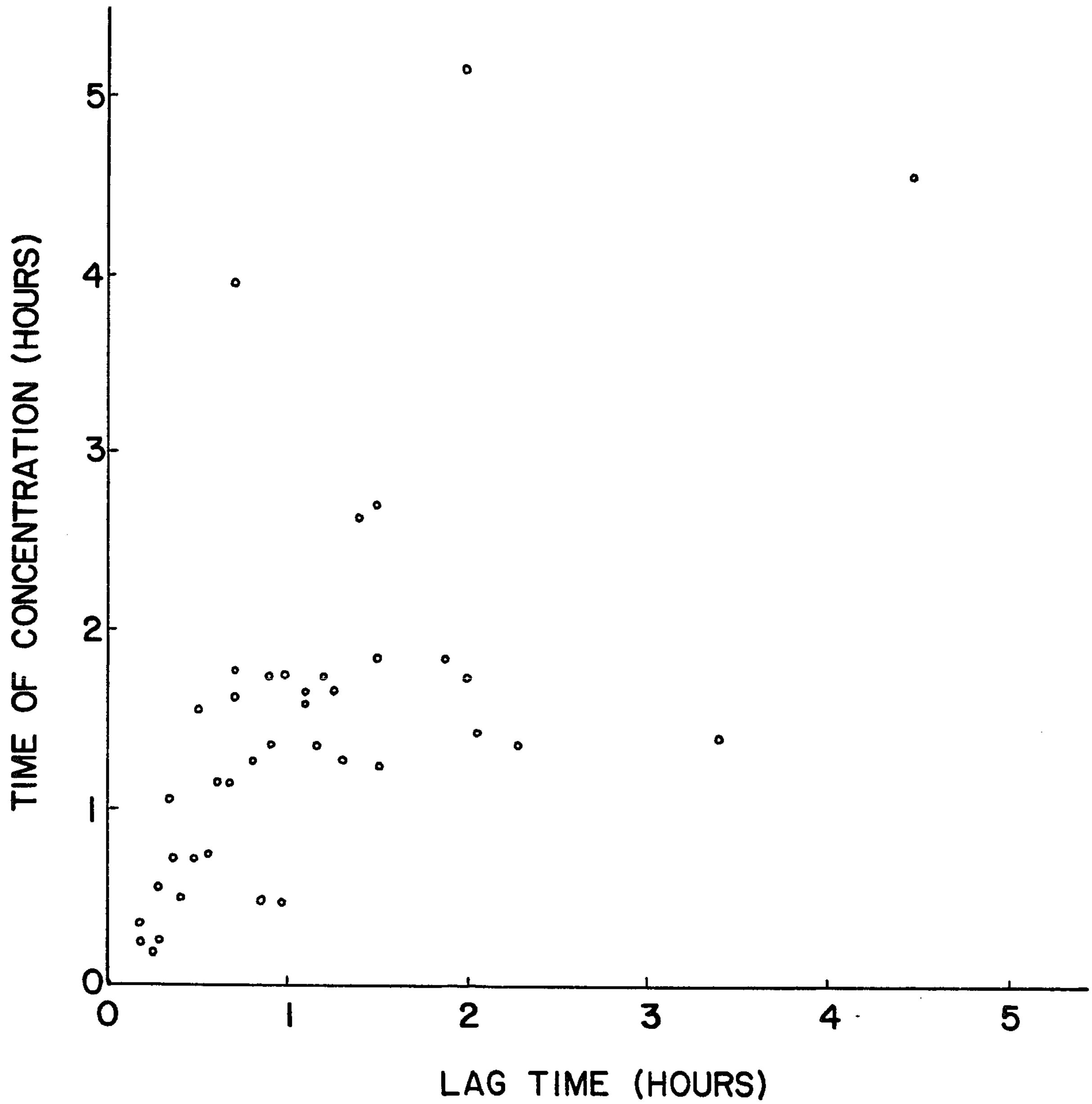


Figure 2.13 Relation between time of concentration and mean lag time estimated from rainfall-runoff data (after McCuen, Wong and Rawls, 1984).

3. MATHEMATICAL MODELS FOR HYDROGRAPH SYNTHESIS

There are various kinds of rainfall-runoff models. Clearly the black-box models are entirely based on analysis of rainfall-runoff data. Consequently, these models are not applicable to those watersheds which lack such data. On the other hand, conceptual models contain parameters which may be determined from either rainfall-runoff data or physically measurable watershed characteristics. Frequently, the model parameters are optimized for some selected rainfall-runoff events over a given watershed, using a suitable optimization algorithm subject to an objective function. The optimized parameter values are then utilized in the model to predict runoff for the rainfall events of interest not used in the optimization. This approach is obviously not applicable to ungaged watersheds. Further, it has other shortcomings. For example, the optimized parameters can best represent the watershed only for the events used in the optimization. No sooner does the optimization set of rainfall-runoff events change than the optimum parameter values change. The extensive amount of data normally required for reliable optimization is often lacking and may prove prohibitive in the widespread model applicability. Equally important, optimization is often expensive.

The other approach attempts to establish relationships between model parameters and physically measurable watershed characteristics. These relationships are then assumed to hold for ungaged watersheds having similar hydrologic characteristics. Thus, the models can also be utilized in evaluating the effect of land use practice on watershed response.

Rainfall-runoff relationships for ungaged watersheds have been developed along two complementary lines: (1) Empirical equations have been developed to relate some individual runoff hydrograph characteristics to watershed characteristics. These characteristics can be partitioned into (a) time characteristics, and (b) discharge characteristics. The time characteristics have been discussed in the preceding chapter. (2) Procedures have been developed to synthesize the entire runoff hydrograph from watershed characteristics. These procedures or models are both linear and nonlinear in character. First we discuss the first line of development.

3.1 DISCHARGE CHARACTERISTICS

Many empirical relationships have been developed for estimating discharge characteristics from rainfall and watershed characteristics (Kinnison and Colby, 1945; Chow, 1962; Thomas and Benson, 1970; Duru, 1976; Chang and Boyer, 1977; Aron and Miller, 1978; Dingman, 1978; Crippen, 1982; Aron, Kibler and Tagliati, 1981; Mosley, 1981; Adejuwon, Jeje and Ogunkoya, 1983; Mimikou, 1983; Harlin, 1984). Of these the peak discharge characteristic has been the most popular; it has been frequently utilized in hydrograph synthesis (Snyder, 1938; Carter, 1961; Wu, 1963; Rao, Assenzo and Harp, 1966; Bell, 1967; Larson and Machmeier, 1968; Bell and Omkar, 1969; Cordery, 1971) as well as frequency analysis (Durant and Blackwell, 1959; Gray, 1970; Alexander, 1972). An excellent discussion of peak discharge formulae is presented by Chow (1962) and Gray (1970). These relationships should be used with caution. They should be used in the regions for which they were developed. A major drawback of these equations arises not so much from their empirical basis, but more so from the lack of knowledge of exact conditions of their applicability.

The simplest of the formulae used in frequency analysis are those based on drainage area, and are of the following forms:

$$Q_m = a\ A^b \qquad (3.1a)$$

$$Q_m = a\ A^{cA^{-b}} \qquad (3.1b)$$

$$Q_m = \frac{aA}{(a_1 + a_a A)^c} + dA \qquad (3.2)$$

in which Q_m is maximum flood flow rate, A is drainage area, c and b are exponents, and a, a_1, a_2 and d are coefficients which depend upon the area, climatic characteristics and frequency. A major drawback of these formulae is that they do not contain a frequency term explicitly. Equations similar to equation (3.1) have also been proposed for estimating hydrograph peak (Black, 1972; Black and Cronn, 1975), as will be clear in the following sections.

In a similar vein, may workers (Getty and McHughs, 1962; Morgan and Johnson, 1962; Rao, Assenzo and Reich, 1965, 1968; Stephens and Mills, 1965; Harp and Hiemstra, 1968, 1966; Colombi, 1978; McCuen and Bandelid, 1982) have proposed empirical formulae for computing peak of the IH of a given duration. Many of them are of the form proposed by Getty and McHughs (1962),

$$h_p = [110{,}860/\{A^{0.45}(LL_{ca}/\sqrt{S})^{0.32}\}]^{0.5} \qquad (3.3)$$

where h_p corresponds to the 2-hr UH peak in $ft^2/sec/mi^2$, A watershed area in mi^2, L length of the main channel in mi, L_{ca} length to the center of gravity of the watershed in mi, and S stream slope in ft/mi.

In a series of papers, Rogers (1980, 1982), and Rogers and Zia (1982) investigated the relation between peak discharge Q_p and volume of direct runoff F for rainfall and runoff events of any duration, spatial distribution or uniformity,

$$\mathrm{Log}(Q_p/V^2) = b - m\ \mathrm{Log}\ V \qquad (3.4)$$

where b is intercept and (m-2) slope of the line in logarithmic domain. This was referred to as the second-order standardized peak discharge distribution and used to interpret the degree of hydrologic nonlinearity of the watershed. If m equals 1 then the watershed is hydrologically linear and less than 1 when it is nonlinear. This relation was tested on 42 watersheds ranging in size from 2.4 km^2 to 701.9 km^2. The watersheds were from Nebraska, Pennsylvania, Maryland and New York. This relation accounted for an average of 86 percent of variation in the dependent variable Mimikou (1983) applied equation (3.4) to 8 watersheds between 200 and 6000 km^2 in northern and western Greece. She supported Roger's finding that the slope has no geographic, climatic or morphologic influence. Furthermore, she found that the coefficient of determination of this relation was linearly related to the degree of hydrologic nonlinearity. The intercept b was found to be significantly related to the logarithm of any of the two watershed morphological indices AS/L and A/L where S and L are the main channel length and slope.

3.2 LINEAR BLACK-BOX MODELS

These models do not account for variations in watershed characteristics or their effect on the unit hydrograph (UH). An empirical curve giving a unique shape of the UH is proposed. Several different models have been proposed as discussed by Benson (1962), Chow (1962), Allison (1967), and Cordery and Pilgrim (1970).

3.2.1 THE BERNARD MODEL

The model by Bernard (1935) appears to be perhaps the first attempt to synthesize the unit hydrograph from watershed characteristics. He studied six watersheds in Ohio varying in area from 500 to 6,000 square miles and in channel slope from 3.5 to 12 feet per mile. He assumes that the peak of the unit hydrograph should be inversely proportional to the time of concentration which, in turn, we assumed to be proportional to a so-called watershed factor u. Using a 24-hour duration, he developed a diagram relating the daily ordinates of various distribution graphs (DG) to the watershed factor u. In the diagram, u was plotted on a logarithmic paper against the ordinates of the 1-day DG at 1 day, 2 days, 3 days, ..., 10 days, after the rainfall. Straight parallel lines were drawn; one to fit all the points representing the ordinates of the 1-day DG on the first day after the rainfall; another to fit the points representing the ordinates on the second day, and so on as shown in figure 3.1.

The watershed factor was expressed as

$$u = \left(\frac{60P}{L}\right)^{4eg} F^{8eg} \frac{S^{1.5eg}}{1000^{2eg}} \tag{3.5}$$

where P is a constant depending upon the shape and area of the watershed, L length of the mainstream, in feet, from the most remote portion of the watershed to the outlet, F a constant depending upon the shape and condition of the main channel, S the slope in feet per 1000 feet of the main channel, e an exponent in the rainfall-intensity equation,

$$I = \frac{KF^n}{D^e} \tag{3.6}$$

in which I is the intensity of rainfall, in inches per hour, for a storm of duration of D in hours, and F, n and e are constants. The term g is

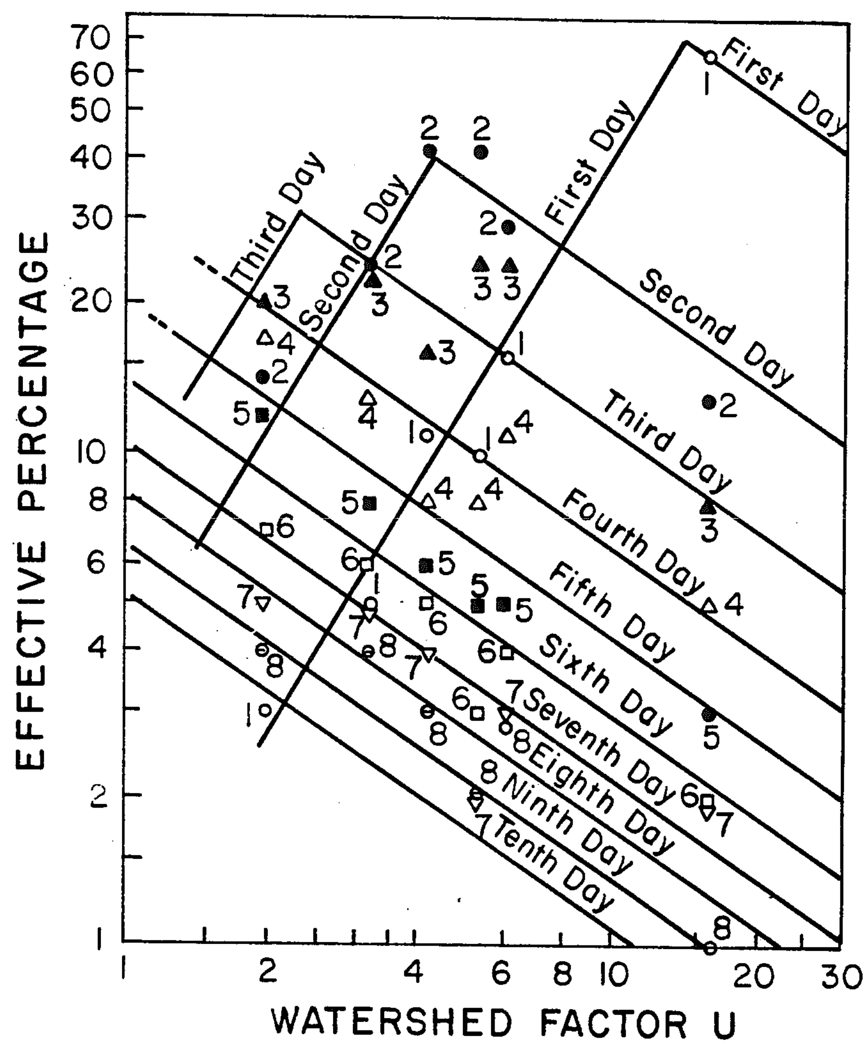


Figure 3.1 Relation of watershed factor u to daily increments of the various distribution graphs (after Bernard, 1935).

$$g = \frac{1}{4 - e} \tag{3.7}$$

If we look at equation (3.5), it is immediately seen that there are only two variables on the right side. Therefore, u can be more simply expressed as a function of S and L, lumping all the constants into one. Jarvis (1935), in a discussion of the Bernard method, suggested to express u simply as

$$u = 20 \frac{S^{0.5}}{L} \tag{3.8}$$

In his closure, Bernard (1935) agreed with the usefulness of this equation. The factor u is supposed to embody the watershed characteristics that govern the UH. It was obtained from a runoff formula derived by Gregory and Arnold (1932). For an ungaged watershed, u can be computed from equation (3.5) or (3.5). By entering the value of u into the diagram, the ordinates of the 24-hour DG can be obtained by reading off the ordinates on the first day, the second day, the third day, and so on. Bernard found that for any particular day within the period of rainfall, the daily value of the DG increased directly as the five-thirds power of u to a maximum daily value, and then receded inversely as the two-thirds power of u. Furthermore, for a particular value of u, the daily value of DG increased directly as the square of the rainfall duration in days to a maximum value, and then it receded inversely as the square of the storm duration in days.

The Bernard model is a one-parameter one since it involves only one measure of watershed characteristics. Therefore, this measure must determine both the scale and the shape of the UH. An implication of this hypothesis is, as Nash (1958) emphasized, that all 1-day UH's having the sample peak will be identical. Further, as the duration changes, a separate diagram of the above type will be required. This diagram may also change with the location. The concept of distribution graph is nevertheless appealing, and has been frequently used in hydrology (Laden, Reilly and Minnotte, 1940; Jetter, 1944).

3.2.2 THE SNYDER MODEL

Snyder (1938) was perhaps the first to have established a set of formulas relating the physical geometry of the watershed to three basic parameters of the unit hydrograph. These formulas were based on a study of 20 watersheds located mainly in the Appalachian Highlands, which varied in size from 10 to 10,000 square miles.

The basic parameter which Snyder defined is t_p, the time of lag to peak in hours taken as the time from the center of mass of the effective rainfall of unit duration to the peak of the UH or simply the watershed lag (see figure 3.2). He derived all other characteristic parameters of the UH in terms of t_p. According to the Snyder model,

$$t_p = C_t \, (LL_c)^{0.3} \tag{3.9}$$

where L is the length of main stream in miles from the outlet to divide, L_c the distance in miles from the outlet to a point on the stream nearest the center of area of the watershed, and C_t a constant varying normally in the range from 1.8 to 2.2, with some indication of lower values for watersheds with steeper slopes. In a study of 27 watersheds in Pennsylvania, Miller, Kerr and Spaeder (1983) found C_t to vary from 1.01 to 4.33. This much of variation was also observed by Bull (1968) for 25 watersheds in the lower Missouri River basin in Kansas and Missouri. Hudlow and Clark (1969) found its range from 0.40 to 2.26 for 13 watersheds (0.5 to 75 mi^2) in central Texas. Linsley (1943) noted its range from 0.3 to 0.7 for northwestern states in the U.S.A. Cordery

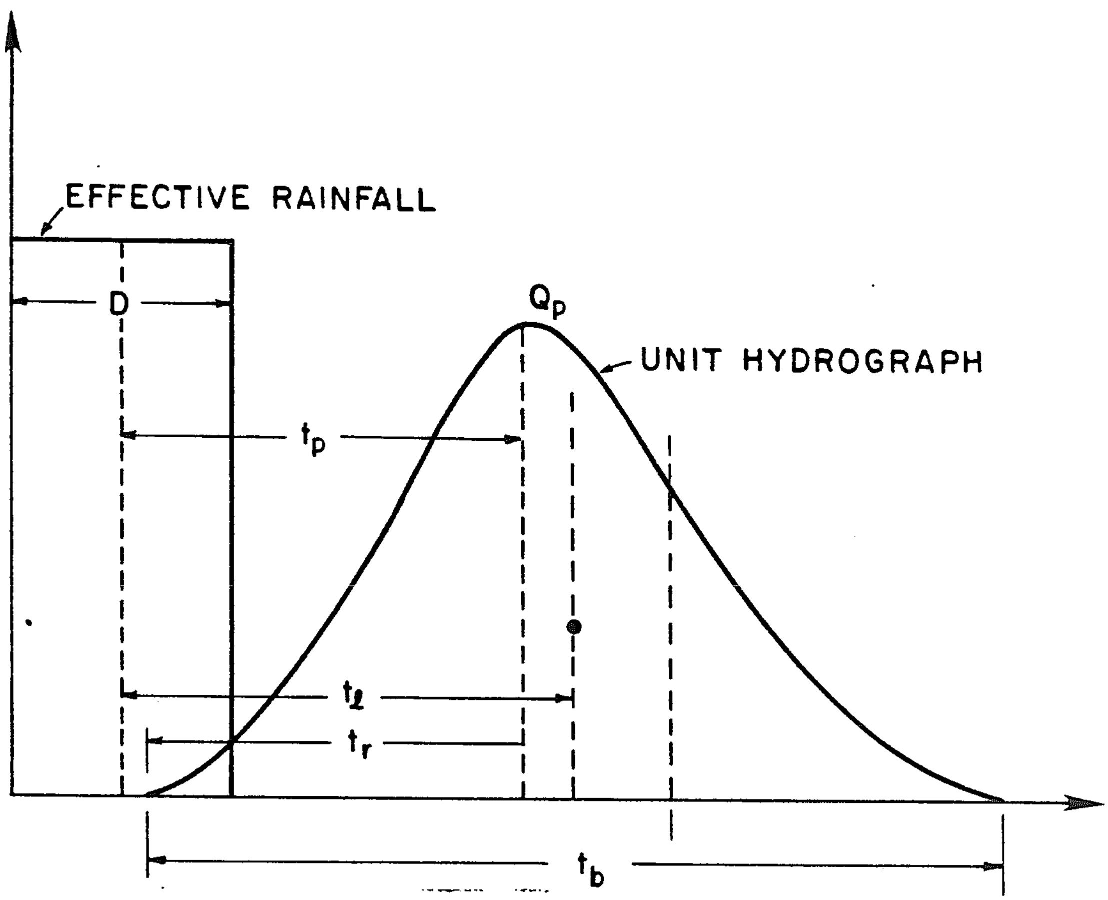


Figure 3.2 Schematic for the Snyder model.

(1968) found C_t to vary from 0.4 to 2.4 for 12 watersheds (0.021 to 248 mi^2) in eastern New South Wales, Australia. The quantity (LL_c) is a measure of the size and shape of the watershed. It is thus seen that watershed slopes were not considered.

The unit duration of the effective rainfall was defined by

$$D = t_p/5.5 \tag{3.10}$$

Due to this effective rainfall, the peak h_p of the resulting UH was given by

$$h_p = C_p/t_p \tag{3.11}$$

where C_p is a constant ranging from 0.56 to 0.69, and h_p has the dimensions (L/T). If h_p is desired in the dimensions of L^3/T then equation (3.11) must be multiplied by A, watershed area, and appropriate conversion factor. Miller, Kerr and Spaeder (1983) found C_p to vary from 0.23 to 0.67 for 27 watersheds in Pennsylvania. Cordery found its range as 0.4 to 1.1 for 12 watersheds in eastern New South Wales, Australia. Hudlow and Clark (1968) found its variability from 0.31 to 1.22 for watersheds in central Texas. For Sacramento and lower San Joaquin Rivers in California, Linsley (1943) noted its variability from 0.35 to 0.59. For example, the time base in days of the UH was defined as

$$t_b = 3 + 3(t_p/24) \tag{3.12}$$

The constants in equation (3.12) are fixed by the procedure of hydrograph separation. Having determined t_p, h_p and t_b, the UH can be constructed such that its volume is equal to one unit. Obviously, many UH's can be sketched which will satisfy this requirement. As an aid to sketching a reasonable UH, the U.S. Army Corps of Engineers (1940) developed a relation between h_p and the width of the UH at values of 50 percent (W_{50}) and 75 percent (W_{75}) of h_p as shown in figure 3.3.

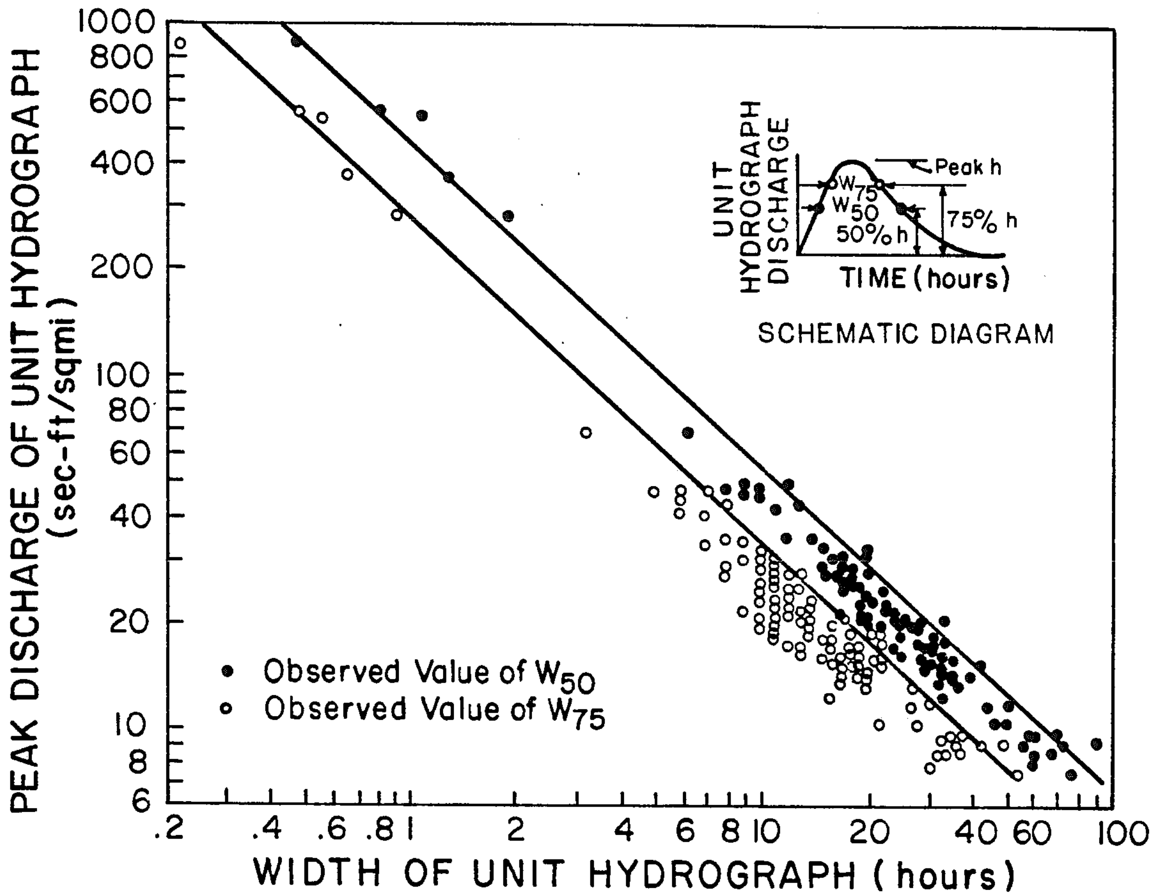


Figure 3.3 Unit hydrograph width at 50 and 75 percent of peak value (after U.S. Army Corps of Engineers, 1940).

These time widths can be evaluated from the following equations

$$W_{50} = \frac{830}{q_p^{1.1}} \tag{3.13}$$

$$W_{75} = \frac{470}{q_p^{1.1}} \tag{3.13}$$

where q_p is the UH peak h_p in ft^2 per sec per mi^2. The Corps of Engineers have suggested, as a guide for shaping the UH, that these widths should be positioned such that one-third of the width is placed to left and two-thirds to the right of the peak. However, Hudlow and Clark (1969) found in their study that an allocation of our-tenths of the width to the left and six-tenths of the width to the right produced optimum results. On the other hand, Viessman, et al. (1977) stated that as a general rule of thumb, the time widths W_{50} and W_{75} should be proportioned each side of peak in a ratio of 1:2 with larger proportion on the right of peak. Equation (3.12) is reasonable for large watersheds but is known to produce exceptionally large t_b for smaller watersheds. As a general rule of thumb, t_b can be taken as 3 to 5 times the time to peak for sketching a UH. Some workers (Hudlow and Clark, 1969) have attempted to fit a mathematical function to the UH.

If the duration of the effective rainfall, say t_R, is different from D defined above, a modified lag time t_p^* can be obtained from

$$t_p^* = t_p + \frac{t_R - D}{4} \tag{3.15}$$

Using this t_p^*, h_p and t_b can be determined from equations (3.11) - (3.12), and then the UH can be constructed.

The Snyder model has been widely used in applied hydrology (Laden, Reilly and Minnotte, 1940; Linsley, 1943; Bull, 1968; Cordery, 1968; Hudlow and Clark, 1969; Miller, Kerr and Spaeder, 1983), and is a one-parameter method. Snyder also developed a relationship between t_p and the ordinates of the distribution graph as shown in figure 3.4. From a hydrographic standpoint, equations (3.9) - (3.11) appear to have a real physical significance, but equations (3.12) - (3.13) and figure 3.4 lack generality.

3.2.3 THE McCARTHY MODEL

In 1938 McCarthy analyzed data from 22 watersheds ranging in area from 74 to 716 square miles located in the Connecticut River basin. He related three parameters of 6-hour UH, including the time of rise, the peak discharge and the base length, to the watershed characteristics including area, overland slope expressed as the average slope of the hypsometric curve and hence in feet/square miles, and stream pattern. This work was not published by McCarthy but its account is available in the U.S. Army Corps of Engineers (1940).

The area is considered by converting the UH's and the watershed characteristics to the corresponding quantities for "model" watersheds of 10 square miles by employing the Froude model law. The overland slope is obtained by constructing a graph of percentage of watershed area greater than a given altitude above the gage versus elevation (hypsometric curve) and dividing the area under this curve by half the square of the watershed area. This quantity, being area divided by length, has the dimension of length and has, therefore, to be multiplied by the length scale in converting to the model watershed.

The stream pattern number can be obtained by an inspection of the watershed map. This is defined as having a value of unity if no stream has a tributary draining more than 25 percent of the total watershed area; a value of two if there are two tributaries of approximately the

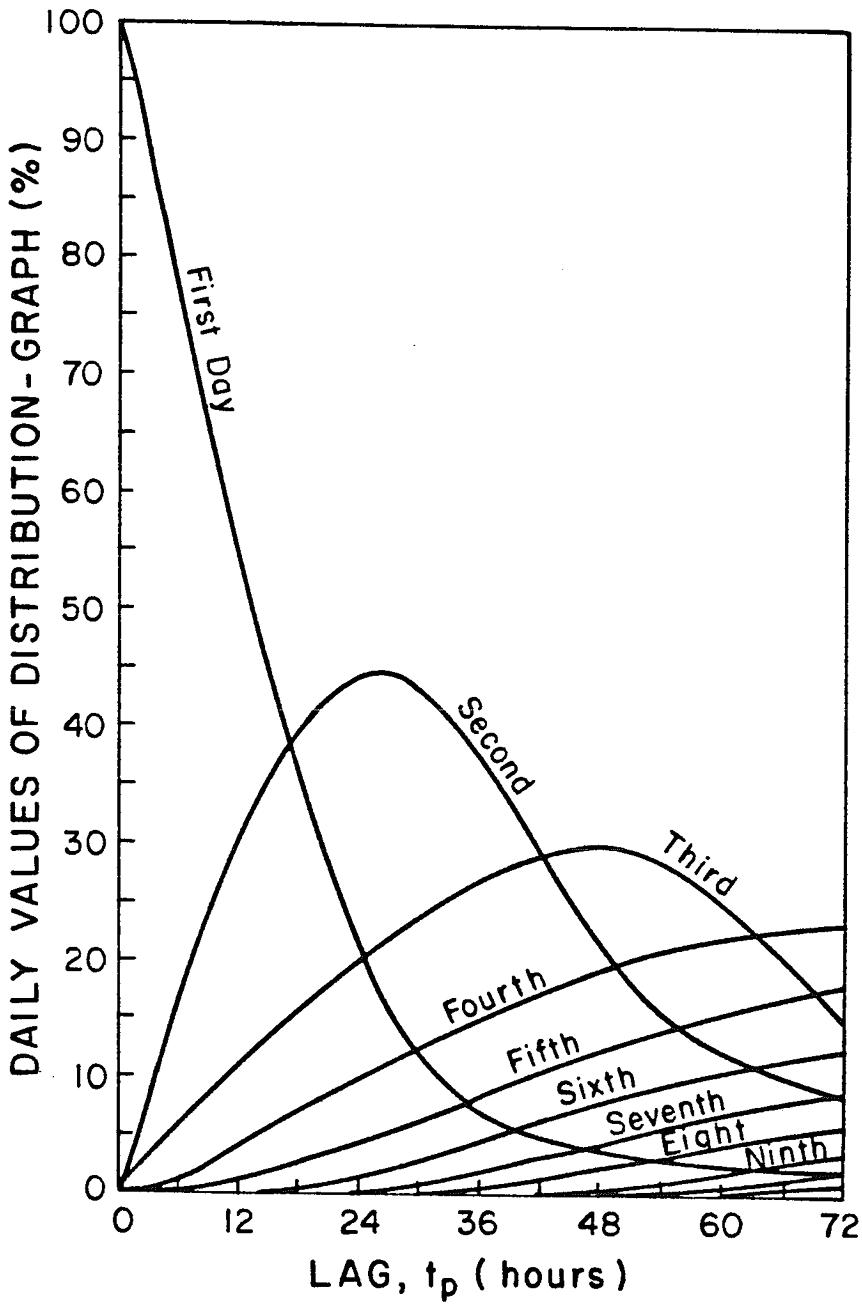


Figure 3.4 Relation between basin lag and distribution graph (after Snyder, 1938).

same size draining at least 50 percent of the total watershed area; and a value of three tributaries drain 75 percent of the watershed area.

McCarthy developed a diagram on a logarithmic paper by plotting the peaks of the UH's against the slope of the respective watersheds for various values of stream pattern number. He essentially constructed a surface whose coordinates are these quantities. Simply put, for each of the three stream pattern numbers a curve was drawn relating peaks to slopes. Except for this McCarthy did not correlate any other parameter of the UH with watershed characteristics. Instead, he separately correlated the lag time and the base time with the UH peak; the lag time was defined as the time from the beginning of the effective rainfall to the peak.

This method has several drawbacks: (1) The slope is tedious to determine. (2) No information is obtained on the UH shape. (3) The diagram developed in the method applies only to one rainfall duration and possibly only to one region. Further, as Nash (1958) pointed out, an implicit assumption in expressing the lag time and base time of the UH as functions of the peak is that all UH's having the same peak are identical. This model is a one-parameter method. The use of the model law implies that small watersheds are similar to large ones. This is hardly a tenable assumption.

3.2.4 THE COMMONS MODEL

Commons (1942) hypothesized that the dimensionless hydrographs, the so-called basic hydrographs, derived from all short-period unit hydrographs would be identical. The dimensionless hydrograph represents, in a sense, the shape of the UH. He suggested that, as a first approximation, all short period UH's were of the same basic shape and differed only in scale. Some may be high and of short duration while others low and of long duration. This means that a dimensionless hydrograph is all that is needed to be synthesized for a watershed. The dimensionless hydrograph can be obtained by dividing all the ordinates of the UH by the value of the peak ordinate and multiplying all the abscissae by the peak divided by the volume of the UH. Commons divided the base time into 100 units and the peak discharge into 60 units. The area under the graph would then be 1,196.5 square units (time multiplied by discharge). The choice of these numbers of units was dictated by the units which he used. For example, if the total flood discharge in acre-feet is divided by 1,196.5, it will give the value of one square unit in acre-feet. Dividing the peak flow in cfs by 60 gives the value of one unit of flow in cfs. Since 1 cfs is approximately 1 acre-foot in 12 hours, the value of one square unit multiplied by 12 divided by the value of 1 unit of flow will yield the value of one unit of time in hours. In fact, this rather awkward choice of units can be avoided by dealing with discharge in the dimensions of L/T.

The Commons method is also a one-parameter method. The method assumes that the knowledge of one parameter alone, say the peak of the UH, implies knowledge of the UH. However, the shape of the UH may vary with the value of the chosen parameter.

3.2.5 THE TAYLOR-SCHWARZ MODEL

Taylor and Schwarz (1952) analyzed data from twenty watersheds, ranging in area from 20 to 1600 square miles, located in the northern and middle Atlantic states. In addition to the watershed characteristics employed by Snyder (1938), they introduced the average slope of the main channel. This slope is equal to the slope of a straight line profile channel, having the same length and the same flow time as that of the main stream, that is,

$$S_c = \left[\frac{N}{\sum_{i=1}^{N} 1/S_i^{0.5}}\right]^2 \tag{3.16}$$

where S is the average slope, and S_i, i = 1, 2, ..., N, are the slopes of N equal reaches of the main stream.

In their analysis, Taylor and Schwarz considered several unit hydrographs of different durations. The following equations were presented:

$$t_p = C_1 \exp(m_1 D) \tag{3.17}$$

where

$$m_1 = 0.212 \, (LL_c)^{-0.36} \tag{3.18}$$

$$C_1 = 0.6/S_c^{0.5} \tag{3.19}$$

and

$$h_p = C_2 \exp(m_2 D) \tag{3.20}$$

where

$$m_2 = 0.121 \, S_c^{0.142} - 0.05 \tag{3.21}$$

$$C_2 = 382 \, (LL_c)^{-0.36} \tag{3.22}$$

Here h_p is in ft^2/s/sq.mi. Finally,

$$t_b = 5(t_p + \frac{D}{2}) \tag{3.23}$$

The remainder of the symbols carry the same meaning as defined in the Snyder model.

Taylor and Schwarz presented a nomograph for solving these equations. They also recommended the use of the Corps of Engineer's width graphs for drawing the UH. It is seen that this model is a two-parameter model. As Nash (1958) pointed out, it can be seen from equation (3.20) that they correlated for each watershed the peaks of the UH's of different periods with the periods of the UH's and a function of the form,

$$h(D,P) = h(0,P)\ \exp(m_2 D) \qquad (3.24)$$

where h(D,P) is the peak of the D-hour UH and h(0,P) the peak of the IUH. Clearly, the relation between h(D,P) and h(0,P) is a function of the shape of the IUH measured by m_2. Further, the h(0,P) and m_2 depend upon S and (LL_c).

3.2.6 THE SCS MODEL

The method of hydrograph synthesis employed by Soil Conservation Service (1955, 1971), U.S. Department of Agriculture, uses an average dimensionless hydrograph derived from an analysis of a large number of natural unit hydrographs for watersheds varying widely in size and geographical locations. This dimensionless hydrograph, as shown in figure 3.5, has its ordinates expressed as h/h_p and its abscissae as t/t_p in which h is the discharge at any time t, h_p peak discharge and t_p time from the beginning of rise to the peak. Its ordinates can also be expressed as V_a/V where V_a is the accumulated values of h at time t and V the total volume. This UH has a point of inflection approximately 1.7 times t_p and t_p approximate 0.2 t_b, the base time.

The dimensionless UH (see Table 3.1) has approximately 37.5 per-cent of the total volume in the rising side, which can be represented by one unit of time and one unit of discharge. The SCS method suggests that the dimensionless UH can also be represented by an equivalent triangular hydrograph, as shown in figure 3.6, having the same units of time and discharge, in turn having the same percent of volume on the rising side

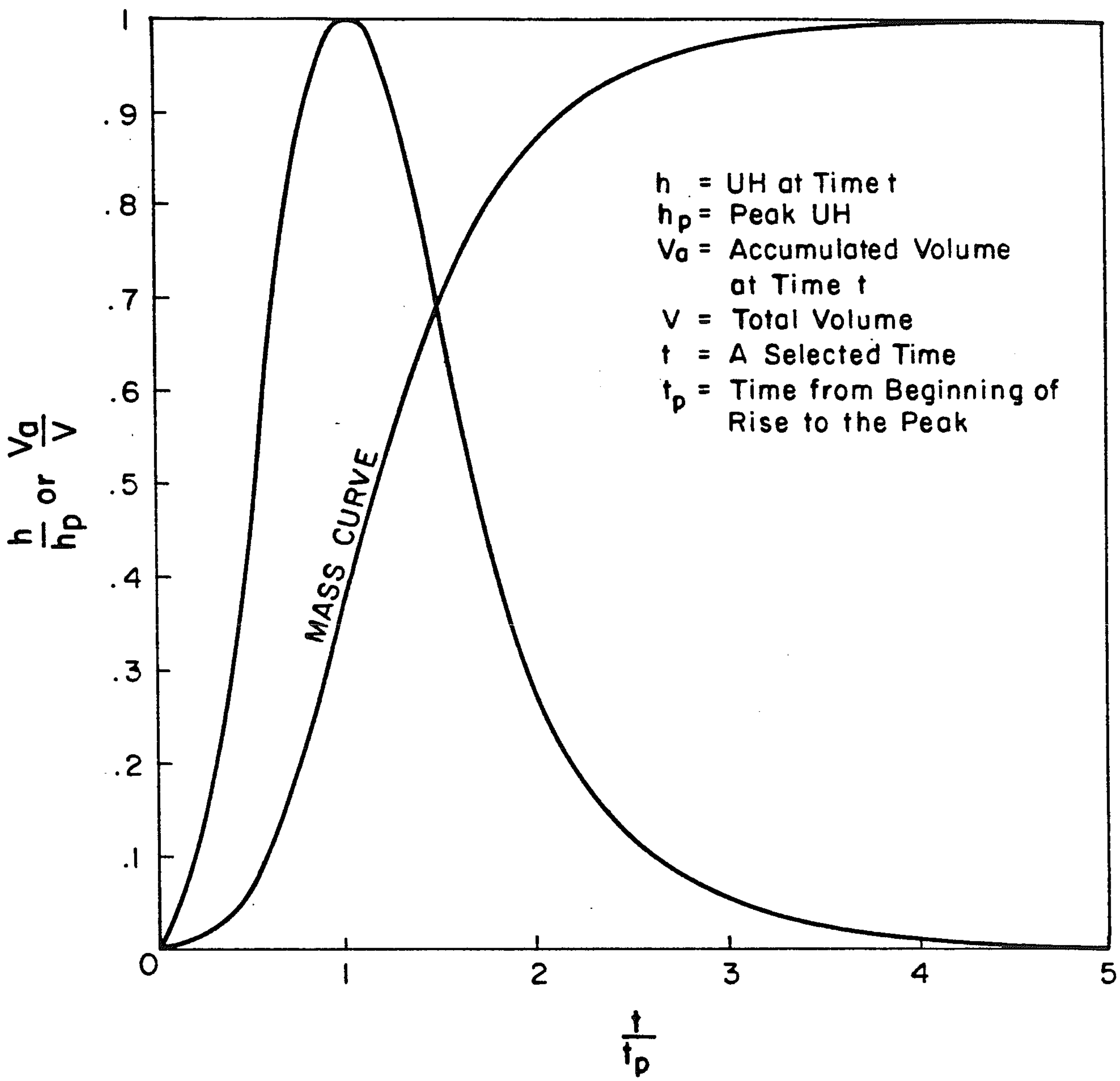


Figure 3.5 Dimensionless unit hydrograph and mass curve (after Soil Conservation Service, 1971).

Table 3.1 Ratios for dimensionless unit hydrograph and mass curve (after Soil Conservation Service, 1971).

Time Ratios (t/T_p)	Discharge Ratios (h/h_p)	Mass Curve Ratios (Va/V)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.012
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.700
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989
3.6	.021	.993
3.8	.015	.995
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

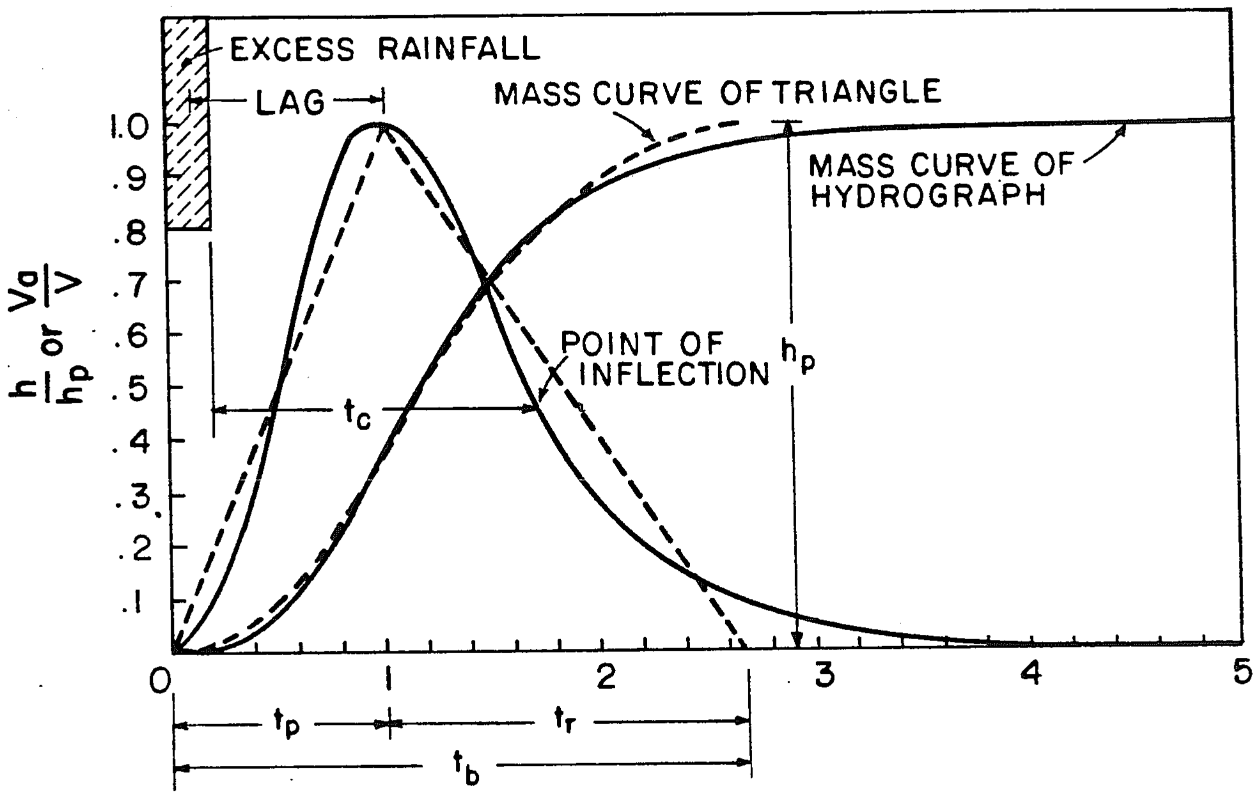


Figure 3.6 Dimensionless curvilinear unit hydrograph and equivalent triangular hydrograph (after Soil Conservation Service, 1971).

of the triangle. This enables defining t_b, the base of the triangle, in relation to t_p. If one unit of time t_p equals 0.375 of volume then

$$t_b = 1/0.375 = 2.67 \text{ units of time}$$

Chu and Lytle (1972) found for 23 watersheds (0.18 to 750 mi^2) in South Dakota that

$$t_b = 0.1021 \left(\frac{L}{S_{0.5}}\right)^{0.968} \qquad (3.25a)$$

in which L is the main channel length in miles and S main channel slope in ft/ft. The correlation coefficient was approximately 0.982. For simplicity one may use

$$t_b = 0.1 \frac{L}{S^{0.25}} \qquad (3.25b)$$

Therefore, the time of recession t_r,

$$t_r = t_b - t_p = 1.67 \text{ units of time} = 1.67\ t_p$$

Consequently, the volume under the triangular unit hydrograph V is

$$V = \frac{h_p}{2}(t_p + t_r) \qquad (3.25)$$

Hence,

$$h_p = \frac{KV}{t_p} \qquad (3.26)$$

where

$$K = \frac{2}{1 + (t_r/t_p)} = 0.75$$

Therefore,

$$h_p = 0.75\ V/t_p \qquad (3.27)$$

If h_p is to be expressed in volumetric units (L^2/T) then the watershed area and appropriate conversion factor can be multiplied with h_p.

From figure 3.6 it is clear that

$$t_p = \frac{D}{2} + t_L \qquad (3.28)$$

where t_L is the lag time between the center of the effective rainfall and t_p. Further,

$$t_L = 0.6\ t_c \tag{3.29}$$

where t_c is the time of concentration. Substituting equations (3.28) - (3.29) into equation (3.27),

$$h_p = \frac{1.5\ V}{D + 1.2\ t_c} \tag{3.30}$$

Thus, the SCS model reduces to a one-parameter model. The parameter t_c can be computed from watershed characteristics as discussed in the preceding chapter.

The Soil Conservation Service defined t_c in two ways: (1) the time of travel from the divide to the outlet, and (2) the time from the end of the effective rainfall to the point of inflection on the UH. The first definition was used to compute t_c in the SCS model. However, the point of inflection, or t_c as mentioned above, is 1.7 time t_p,

$$t_c = 1.7\ t_p \tag{3.31}$$

Using equations (3.28) and (3.29) and (3.31),

$$t_c + D = 1.7\ t_p$$

$$1.2\ t_c + 2D = 2\ t_p$$

Solving these equations,

$$D = .133\ t_c \tag{3.32}$$

Thus, the UH is completely specified by one parameter. The above relationships can be used to construct either the triangular UH or the cirvilinear UH. The SCS model is one of the popular models for synthesizing the UH for small watersheds of less than 500 mi^2 (Hanson and Johnson, 1964; Reich, 1965, 1968; Hiemstra, 1968; Wu, 1969; Wang and Wu, 1971; Chu and Lytle, 1972; Reich and Wolf, 1973; Mostaghimi and Mitchell, 1982; Wheater, Shaw and Rutherford, 1982; McCuen and Bandelid, 1983; Sangal, 1983). Morgan and Johnson (1962) compared the SCS, Snyder, Commons and Mitchell models on 12 watersheds (10.1 to 101 mi^2) in Illinois and found that the errors in prediction of peak discharge ranged up to nearly 200 percent. The four methods were comparable. Recently, Mostaghimi and Mitchell (1982) undertook a comparison on 15 small watersheds (3.4 to 558.1 ha) in central Illinois of four models: the Cypress Creek, Rational, Chow and SCS. They found that the SCS model was the best for predicting peak runoff rates.

3.2.7 HICKOK-KEPPEL-RAFFERTY (HKR) MODEL

The HKR model (Hickok, Keppel, and Rafferty, 1959) is similar to the SCS method, and was developed entirely for small watersheds. The basic data involved the runoff characteristics of 14 watersheds varying in size from 11 to 790 acres which were located in semiarid regions (Arizona, Colorado and New Mexico). An average dimensionless hydrograph was developed by plotting Q/Q_p versus t/t_L. However, t_L was defined as the time difference between the centroid of a limited block of intense rainfall and the resulting peak discharge.

For reasonably homogeneous semiarid range lands up to about 1,000 acres in area,

$$t_L = K_1 \left[\frac{A^{0.3}}{S_a \; D_w^{0.5}}\right]^{0.61} \qquad (3.33)$$

where t_L is lag time in minutes, A watershed area in acres, S_a average land slope in percent, Dw drainage density in feet per acre, and K_1 a constant equal to 106.

For watersheds with widely different physiographic characteristics,

$$t_L = K_2 \left[\frac{(L_{sa} W_{sa})^{0.5}}{S_{sa} D_w^{0.5}}\right] \tag{3.34}$$

where L_{sa} is length from the outlet of watershed to the center of gravity of source area in feet, W_{sa} average width of the source area in feet, W_{sa} average width of the source area in feet, S_{sa} average land slope of the source area in percent, D_w drainage density of the entire watershed in feet per acre, and K_2 a constant equal to 23.

Lag time was found to be a major determinant of the hydrograph shape. Therefore, the ratio of the peak rate of runoff h_p to the total runoff volume V was expressed as a function of lag time,

$$\frac{h_p}{V} = \frac{K_3}{t_L} \tag{3.35}$$

For h_p in ft^3/sec, V in acre feet and t_L in minutes, K_3 equals 545.

Using these equations, generalized dimensionless hydrograph and mass curve were prepared as shown in figure 3.7. Data for these curves are shown in Table 3.2. This model is, again, a one-parameter model, and therefore has the same drawbacks as other one-parameter models.

3.2.8 THE DOUBLE TRIANGLE MODEL

The double triangle method for hydrograph synthesis on small watersheds was developed by Ardis (1972, 1973) and Tennessee Valley Authority (1972) and has been used by Reich and Wolf (1972), Diskin and Lane (1976), and Tay and Singh (1978). This model is essentially an extension of the SCS model using a simple triangular UH. The double triangle UH is obtained by assuming the watershed response to be composed of two parts; each represented by a triangular shape. The first part represents a fast response with a high peak and short time base while the

Table 3.2 Data for dimensionless hydrograph and mass curve (after Hickok, Keffer and Rafferty, 1959)

t/t_L	h/h_p	Q/Q_p	t/t_L	h/h_p	Q/Q_p
0	0	0	1.6	0.545	0.671
0.1	0.025	0.002	1.7	0.482	0.707
0.2	0.087	0.007	1.8	0.424	0.742
0.3	0.160	0.020	1.9	0.372	0.773
0.4	0.243	0.036	2.0	0.323	0.799
0.5	0.346	0.063	2.2	0.241	0.841
0.6	0.451	0.096	2.4	0.179	0.875
0.7	0.576	0.136	2.6	0.136	0.900
0.8	0.738	0.180	2.8	0.102	0.917
0.9	0.887	0.253	3.0	0.078	0.932
1.0	1.000	0.325	3.4	0.049	0.953
1.1	0.924	0.400	3.8	0.030	0.965
1.2	0.839	0.464	4.2	0.020	0.973
1.3	0.756	0.523	4.6	0.012	0.979
1.4	0.678	0.578	5.0	0.008	0.983
1.5	0.604	0.627	7.0	0	0

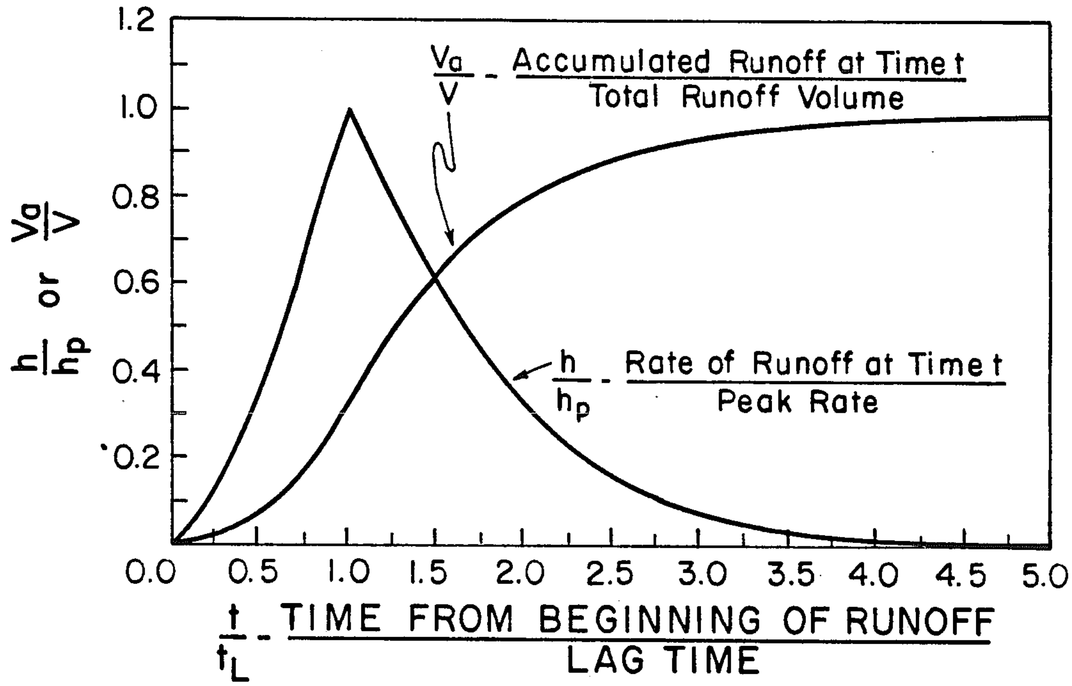


Figure 3.7 General dimensionless hydrograph and mass curve (after Hickok, Keppel and Rafferty, 1959).

second a slow response having a low peak and longer time base. The latter may start either at the same time as the former does, or at the time corresponding to the peak of the fast response. In either case, the superimposition of the ordinates of the two individual responses results in a three line polygon, a double triangle. Evidently, the shape of this polygon depends on the relative proportions of the two responses. Figure 3.8 illustrates the double triangle UH. However, it is tacitly assumed that the delayed response peaks where the initial response ends.

The shape of the double triangle UH can be specified by four parameters. A fifth parameter required for complete specification can be obtained from the condition that UH enclose a unit area. As shown in figure 3.8, the five parameters are: t_p, time to peak; t_r, time of recession; h_p, peak ordinate; h_{pr}, ordinate at the break point (or peak ordinate of delayed response) which can be expressed as $h_{pr} = a\, h_p$; and t_{pr}, time from the peak ordinate to the break point which can be written as $t_{pr} = b\, t_r$. Therefore, the last parameter h_p can be written as

$$h = \frac{2}{t_p + (a + b)\, t_r} \tag{3.36}$$

For the recession limb to be concave,

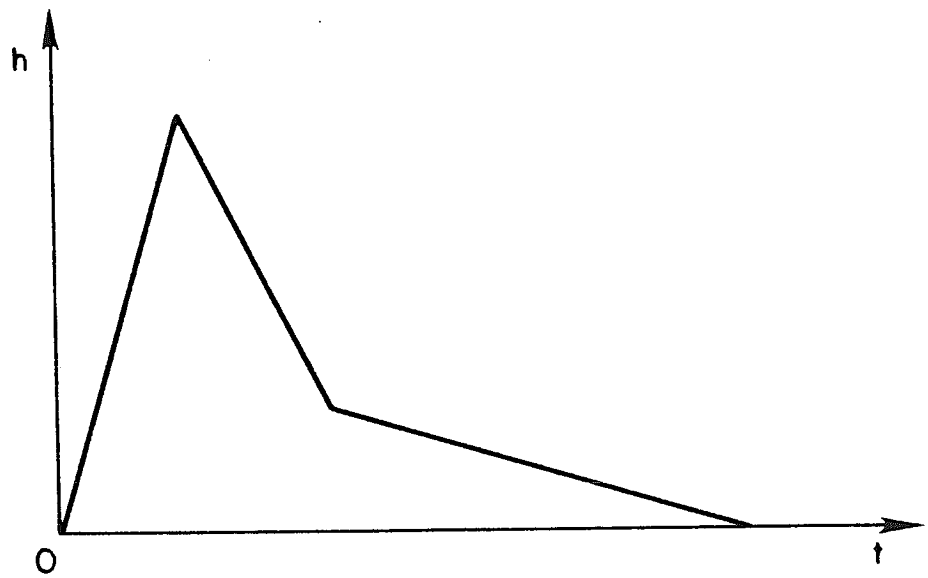
$$0 < a + b < 1.0 \tag{3.37}$$

Any four parameters can be used to define the double triangle UH. For example, if h_p, t_p, t_{pr} and t_r are taken as parameters then

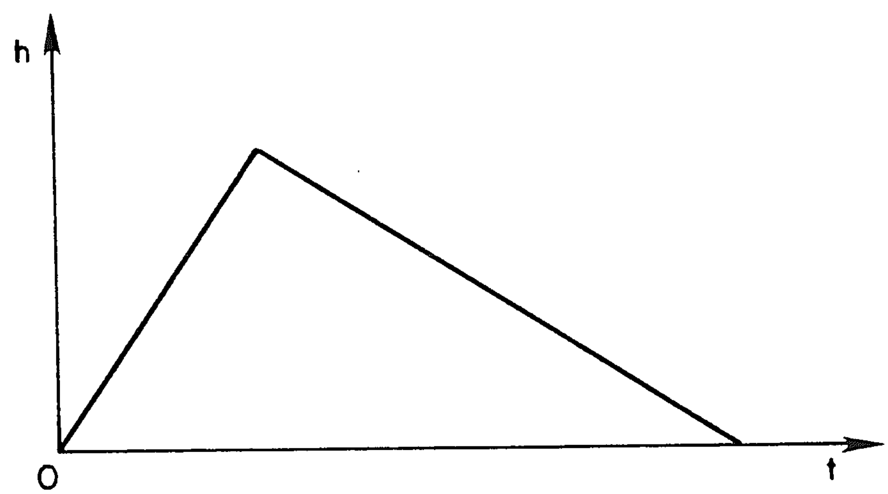
$$h_{pr} = \frac{[2 - h_p(t_p + t_{pr})]}{t_{pr} + t_r} \tag{3.38}$$

Each parameter measures a specific attribute of the UH.

As Ardis (1972) has shown, the double triangle has considerable flexibility to generate the UH of various shapes. This is illustrated



(a)



(b)

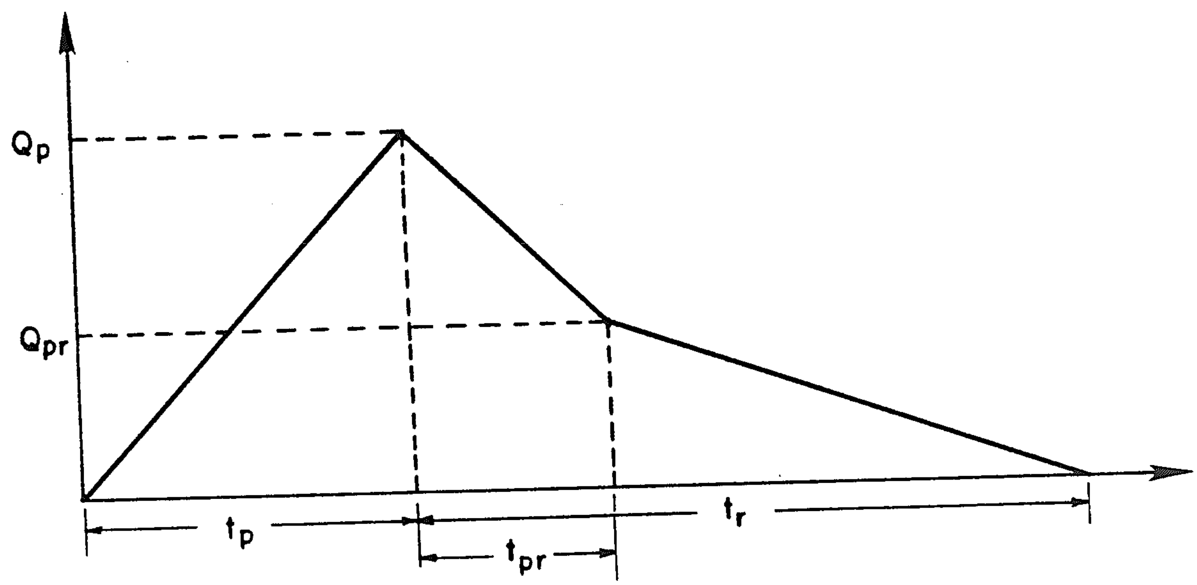


Figure 3.8 The double triangle unit hydrograph.

in figure 3.9 for a constant time base ($t_p + t_{pr} + t_r$). For convenience, let $t_1 - t_p$, $t_2 = t_p + t_{pr}$, and $t_3 = t_2 + t_r$. Therefore, the basic four parameters defining the double triangle UH can be h_p, t_1, t_2 and t_3. Ardis selected the following watershed characteristics which were related to these parameters using multiple linear regression analysis, with the choice of the duration of the UH as 1 hour. Eleven watersheds (7.04 to 16.8 mi^2) lying in the Tennessee River basin were used.

(1) Drainage area A in mi^2.

(2) Watershed shape, a dimensionless measure defined as

$$S_p = L_c^2/A \tag{3.39}$$

where S_p is shape factor measuring watershed elongation, L_c length of the mainstream channel measured to the watershed boundary.

(3) Mainstream sinuosity, S_u, a measure of sinuosity and defined as

$$S_u = (L_c/L_1) - 1 \tag{3.40}$$

where L_1 is the length of the mainstream channel measured in 1-mile chords.

(4) Drainage density D_w, a measure of the extent of channelization in a watershed and hence the runoff-carrying capacity, defined as

$$D_w = 1.571 \ \Sigma N/\Sigma L_i \tag{3.41}$$

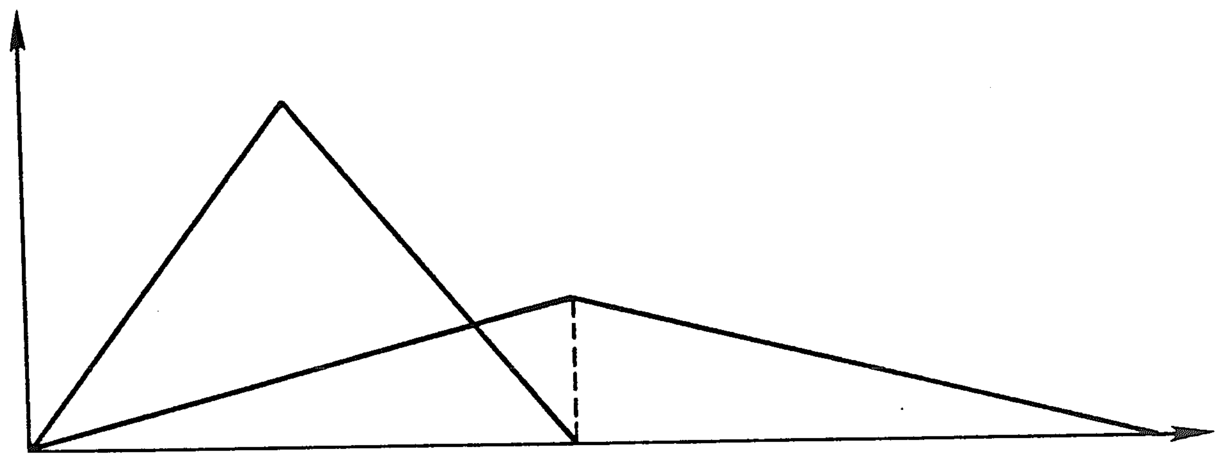
where Σ N denotes the total number of intersections all streams and extended channels make with the grid system, and ΣL_i the total length of all grid lines within the watershed in miles.

(5) A measure of soils from low-flow data, S_0 defined as

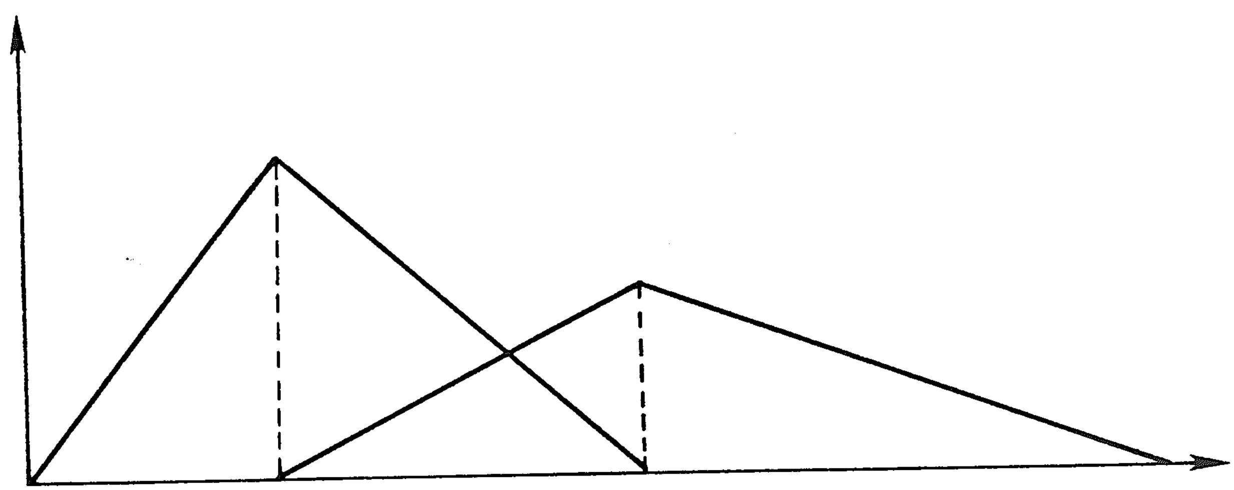
$$S_0 = Q_{70} \tag{3.42}$$

where Q_{70} is the discharge equalled or exceeded 70 percent of the time.

(6) Percent of watershed in forest, W_f.



(a)



(b)

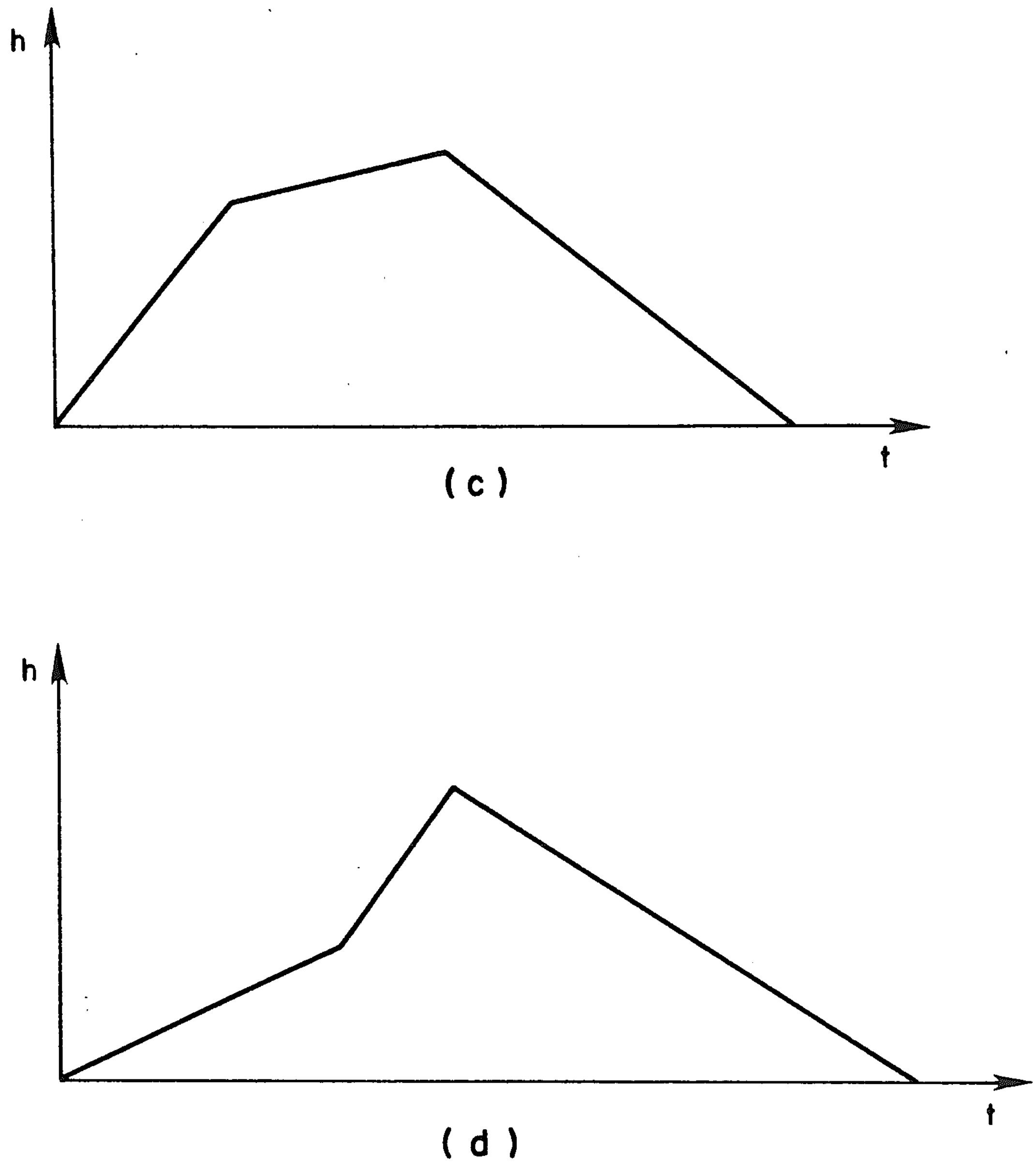


Figure 3.9 Various shapes of the UH that can be represented by the double triangle.

(7) Slope and time measures. The slope of a watershed or channel is a characteristic related to the corresponding velocity of flow. The corresponding time of travel for a specific reach is then its length divided by its slope. Therefore,

$$S_c = (L_c/T_M)^2 \tag{3.43}$$

where T_M is time factor and S_c weighted mean channel slope, in feet per mile.

(8) Total direct runoff, V.

(9) Duration of the effective rainfall in multiples of D, equal to 1 hour in this case.

Table 3.3 gives the correlations between watershed characteristics and the UH parameters. The double triangle model was found to be a significant improvement over the SCS model (Ardis, 1973; Diskin and Lane, 1976).

3.3.1 THE CLARK MODEL

This model involves determination of the time area concentration (TAC) diagram and the storage coefficient k. Clark (1943) suggested that k can be approximated by the relation

$$k = CL/S_c^{0.5} \tag{3.44}$$

where L is channel length in miles, S_c mean channel slope, and C a constant varying from about 0.8 to 2.2.

Linsley (1945), in a discussion of Clark's paper, suggested a modification of equation (3.44) by including a square root of area term,

$$k = bL\ (A/S_c)^{0.5} \tag{3.45}$$

Table 3.3 Parameter estimation equations (after Tennessee Valley Authority, 1973).

$Y = WF1 * V^{WF2} * D^{WF3}$, Y = Dependent Variable, V = Amount of Direct Runoff (inches), and D = Duration of Effective Rainfall (hrs).

Individual Equations		Leading Coefficient	Powers on Watershed Characteristics for WF1, WF2, and WF3*								r_a	r_1
			Area	S_c	S_p	W_f	D_w	S_u	Soils	$(L_c/S_c)^{0.5}$		
h_p												.858
	WF1	650			.280	-.530	-3.09	-.850		-.711		
	WF2	.357				.238			-.142			
	WF3	-0.490	.234			.0189	.341					
t_1											.831	
	WF1	.0000167			-1.24	.582	4.79	.816	-.350	.681		
	WF2	0										
	WF3	.0115			.383	.344		-.401		.141		
t_2												.792
	WF1	.00188	.396	-.314	-.259	.290	3.71	1.00	-.106			
	WF2	0										
	WF3	.136										
t_3												.850
	WF1	.285	.679		-.558	2.02	.843	-.0757				
	WF2	0										
	WF3	0										
PHI = .0340 + .0903 * LOSS -.0445 * WS - .00538 * D - .0172 * SOILS											.819	

*For Example: WF2 equation in QMAX/AREA model is $WF2 = .364 * W^{.261} * SOILS^{-.145}$
WF1, WF2 and WF3 are watershed factors depending upon Y being evaluated.
PH = ϕ-index (inches/hours).
WS = Weights of S from SCS curve number method.
LOSS = Difference between accumulated rainfall and V.

The average slope S_c was computed as

$$S_c = [\frac{\Sigma L_i S_i^{1/2}}{L}]^2 \tag{3.48}$$

where S_i is the slope of the stream segment L_i.

The derivation of this equation is not given by Johnston but it is intended to represent the uniform slope that would result in the same total time of travel as the actual stream if length, roughness, channel cross-section and other pertinent factors other than slope were unaltered (Laurenson, 1962). An independent derivation on this premise leads to an expression different from equation (3.48) as shown by Laurenson (1962). Let it be assumed that the effects on velocity of roughness and hydraulic radius are the same for all reaches of the stream. This assumption is questionable but has been made previously (Taylor and Schwarz, 1952). The velocity of flow v_i through a reach i can be expressed as

$$v_i = B\ S_i^{1/2} \tag{3.49}$$

where B is a constant. The time of flow t_i then is

$$t_i = L_i/v_i = L_i/[B\ S_i^{1/2}] \tag{3.50}$$

The total time of flow down the main stream, t_c, is

$$t_c = \frac{1}{B} \sum_{i=1}^{M} L_i/S_i^{1/2} \tag{3.51}$$

where M is number of reaches. The mean velocity of flow v_m can be expressed as

$$v_m = \sum_{i=1}^{M} L_i/t_c = \frac{B\ \Sigma\ L_i}{\Sigma\ L_i/S_i^{1/2}} \tag{3.52}$$

Since

$$v_m = B\ S_c^{1/2} \tag{3.53}$$

where b is a coefficient which was found to vary from 0.04 to 0.075 for 11 rivers in California and Virginia. This better represents the effect of storage in tributaries on flood flows.

The time base of the TAC diagram t_c was considered to equal the time interval between the end of rain and the point of contraflexure on the hydrograph recession limb. This time base was measured from the recorded floods, and not related to watershed characteristics. It can, however, be determined by using any of the methods discussed in the preceding chapter.

3.3.2 THE JOHNSTON MODEL

Johnston (1949) correlated the parameters t_c and k of the Clark model for 19 watersheds, varying from 25 to 1,624 square miles in the Scotie and Sandusky River basins. He found, using correlation analysis on 15 of his 19 watersheds, that

$$t_c = \frac{4.7}{r^2} \left(\frac{L}{S_c}\right)^{1/2} \qquad (3.46)$$

where t_c is time base of the TAC diagram in hours, L length of the main stream in miles, S_c average slope of the main stream in feet per mile, and r a branching factor based on the stream pattern.

The factor r is found by dividing the area under a graph showing the total area tributary to the main stream above any point by the area under a similar curve constructed, assuming the watershed to be single branched and of uniform width. Mathematically,

$$r = \frac{\int_0^L \int_y^L w \, dx \, dy}{0.5 \, A \, L} \qquad (3.47)$$

where w is the effective width of the watershed.

we can express

$$S = \left[\frac{\Sigma\, L_i}{\Sigma\, L_i / S_i^{1/2}}\right] \quad (3.54)$$

Equation (3.54) is different from equation (3.48) and should be preferred.

Johnston noted that equation (3.46) could be modified without appreciable loss of accuracy to

$$t_c = 5\ (L/S_c)^{1/2} \quad (3.55)$$

From a correlation analysis he found that

$$k = 1.5 + 90\ (W/R)$$

where W is the average width of the watershed in miles, and is equal to A/L, and R an overland slope factor computed by the intersection grid method (that is, placing a square grid over the contour map and counting the number of intersections of contour lines and grid lines). Mathematically

$$R = NE/g \quad (3.56)$$

where N is number of intersections of contour lines with grid lines, E contour interval in feet and g total length of grid lines within the watershed in miles.

3.3.3 THE EDSON MODEL

Edson (1951) proposed that if isochrones were drawn to represent the time required for each element of the effective rainfall to reach the watershed outlet, the cumulation of area A with time t would result in an approximate parabola,

$$A \propto t^x, \quad x > 1 \quad (3.58)$$

so that the discharge of direct runoff Q might become

$$Q \propto t^2, \quad x > 1 \quad (3.59)$$

However, when runoff accumulates simultaneously over all parts of a watershed, the time of travel required for each component is so affected by the presence of other components that the hypothetical isochrones remain no longer valid. It is reasonable to regard the consequent delay in discharge as a result of valley storage. In other words, a portion of the total runoff must be stored in the valley upstream of the watershed outlet to produce discharge. The valley of the watershed acts as a reservoir; the discharge from its storage decreases exponentially with time,

$$Q \propto \exp(-yt), \quad y > 0 \tag{3.60}$$

where y is recession constant > 0. Thus, the reservoir action of the valley storage exerts a damping effect on the flow given by equation (3.60) which is valid for an indefinite period of time. Further, the valley storage must exist for even the smallest Q implying thereby that equation (3.60) is valid from $t = 0$.

The combined effect of translation and storage effects can be expressed by coupling equation (3.59) - (3.60),

$$Q \propto t^x \exp(-yt)$$

or

$$Q = B\, t^x \exp(-yt) \tag{3.61}$$

where B is a proportionality constant. It may be noted that equation (3.60) is dominant during the recession whereas equation (3.59) during the rise.

If V is the total volume of discharge then

$$V = \int_0^\infty Q\, dt \tag{3.62}$$

where Q is specified by equation (3.61). To facilitate integration of equation (3.62), let $m = x + 1$ and $w = yt$. Then

$$V = \int_0^\infty B(w/y)^{m-1} \exp(-w)\ dw/y \qquad (3.63)$$

Recalling that

$$\Gamma(m) = \int_0^\infty w^{m-1} \exp(-w)\ dw$$

equation (3.62) becomes

$$V = By^{-m}\ \Gamma(m)$$

Therefore,

$$Q = \frac{V\ y^m}{\Gamma(m)}\ t^{m-1} \exp(-yt) \qquad (3.64)$$

Edson argued by taking $dQ/dt = 0$ that $m > 2$ and that $Q = 0$ when $t = 0$ and $t = \infty$. Furthermore, the peak discharge and its time would be given as

$$Q_p = Vy[(m-1)e]^{m-1}/\Gamma(m)$$

$$t_p = (m-1)/y$$

Similarly, by taking $d^2Q/dt^2 = 0 = [(m-1) - yt]^2 - (m-1)$ whence $t = [(m-1) \pm (m-1)^{0.5}]/y$. It then follows that

$$t_r = [(m-1) - (m-1)^{0.5}]/y$$

$$t_f = [(m-1) + (m-1)^{0.5}]/y$$

where t_r is the time of maximum dQ/dt, and t_f time of minimum dQ/dt.

Equation (3.64) can be taken to represent the IUH or UH with $V = 1$. Therefore

$$h(t,0) = \frac{y^m}{\Gamma(m)}\ t^{m-1} \exp(-yt) \qquad (3.65)$$

which is identical to the Nash model. Equation (3.65) has two parameters m and y which can be estimated from watershed characteristics. Edson, except for alluding to it, did not attempt to determine them in this manner. Instead, he computed from flow records, and found m to vary from 3.0 to 7.2 and y from 0.55 to 1.28/day.

3.3.4 THE EATON MODEL

Eaton (1954) also used the Clark method and correlated its parameters t_c and k for 7 Tasmanian river in Australia with watershed areas ranging from 48 to 322 square miles. His analysis yielded

$$t_c = 1.35 \left(\frac{AL}{r}\right)^{0.37} \tag{3.66}$$

and

$$k = 1.2 \left(\frac{WA}{Lr}\right)^{1/3} \tag{3.67}$$

where the symbols are as defined previously. The branching factor r varies between 1 and 2. Use of slope factor would be preferable to r, but a lack of contour maps for these watersheds prevented Eaton from using it.

3.3.5 THE O'KELLY MODEL

This model (O'Kelly, 1955), like the Clark model, involves determination of the TAC diagram and the storage coefficient, k. His basic data included 10 flood-free watersheds in Ireland, varying in areal extent from 43 to 366 square miles and in ground slope from 20 to 330 feet per mile. O'Kelly proposed to represent, without loss of accuracy, the TAC diagram by an isosceles triangle. Therefore, his method is a two-parameter one containing the base of the isosceles triangle t_c and k as parameters.

O'Kelly modified the values of t_c and k to correspond with a watershed of 100 squares miles in area, using a hydrologic time-scale factor based on one-fourth root of the area. The modified values of t_c and k were then plotted against the overland slope S_0 defined as the median value of the maximum slope occurring at the intersections of a grid of square mesh imposed on the watershed map. Thus, he concluded that

$$t_c = a\ S_0^b \tag{3.68}$$

and

$$k = c\ S_0^d \tag{3.69}$$

where a, b, c and d are empirically derived constants. It may be noted that if b = d, $t_c/k = a/c$. This implies that the shape of the IUH would be fixed as suggested by Commons (1942). However, O'Kelly used slightly different values of b and d, and thus obtained a basic shape which varied slightly with S_0 and, therefore, with k.

Nash (1958) used O'Kelly's data and found that the evidence for varying the basic shape of the IUH with S_0 was inadequate; practically equally good results could be obtained by using Commons' basic shape and varying a single parameter with S_0. In either case, since t_c and k are both correlated with S_0, a functional relationship between t_c and k is implied. Therefore, the IUH can be completely characterized by a single parameter S_0.

In a discussion of O'Kelly's work, Dooge (1955) proposed to express t_c in hours as

$$t_c = a\ \frac{A^{1/2}}{S_0^{\ 1/2}} \tag{3.70}$$

where A is watershed area in square miles, S slope in parameter 10,000, and a an empirical constant. For the values of t_c obtained by O'Kelly, a varied from 10 for a slope of 10 in 10,000 to 14 for a slope of 500 in 10,000. Similarly, he expressed k in hours as

$$k = b\ \frac{A^{1/2}}{S_0^{\ 1/2}} \tag{3.71}$$

where b is an empirical constant. For the values of k obtained by O'Kelly, b varied from 13 for a slope of 10 in 10,000 to 10 for a slope of 500 in 10,000.

Based on a least squares analysis of O'Kelly's data, Dooge (1956) derived

$$t_c = 2.58 \frac{A^{0.41}}{S_0^{0.17}} \tag{3.72}$$

$$k = 100.5 \frac{A^{.28}}{S_0^{0.7}} \tag{3.73}$$

The tedium of evaluating average ground slope might be avoided by finding the average channel slope and using the approximate formula

$$S_0 = 4\ (S_c)^{0.8} \tag{3.74}$$

where S_0 is overland slope in degrees and S_c channel slope in degrees. If the slopes are expressed in feet per mile, equation (3.74) becomes

$$S_0 = 10\ (S_c)^{0.8} \tag{3.75}$$

3.3.6 THE NASH MODEL

Nash (1962) applied his model to British catchments. This model has two parameters n and k. Nash showed that these parameters were related to the first and second moments of the IUH about the origin as

$$m_1 = nk \tag{3.76}$$

$$m_2 = 1/n \tag{3.77}$$

These moments were then correlated empirically with watershed characteristics as

$$m_1 = 27.6\ A^{0.3}\ S_0^{-0.3} \tag{3.78}$$

and

$$m_2 = 0.41\ L^{-0.1} \tag{3.79}$$

where S_0 is overland slope in parts per 10,000, calculated as the mean of a grid sample of slopes.

3.3.7 THE GRAY MODEL

Gray (1961, 1962), using the Edson model, developed a procedure for derivation of the UH from physically measurable watershed characteristics. He analyzed topographic and hydrologic characteristics of 42 small watersheds, varying in size from 0.23 to 33,000 square miles, located in the States of Illinois, Iowa, Missouri, Nebraska, Ohio and Wisconsin.

Gray derived the UH's for these small watersheds and reduced them to a dimensionless form by expressing the ordinates in percent flow based on a time increment equal to one-quarter of the period of rise t_r (% flow/0.25 t_r) and the abscissa as the ratio of any time t divided by the period of rise (t/t_r). Although each ordinate is expressed as % flow/0.25 t_r, the connotation simply infers that it is the percentage of the total volume of flow based on a time increment duration of 0.25 t_r.

The dimensionless graph represents a modified form of the UH and can be expressed as

$$Q(t/t_r) = \frac{V^*(\alpha^*)^{\beta}}{\Gamma(\beta)} \exp(-\alpha^* t/t_r)\ (t/t_r)^{\beta-1} \qquad (3.80)$$

where V* is the volume in percent, t_r the period of rise, $Q(t/t_r)$ the % flow/0.25 t_r an any value of t/t_r, and α and β are parameters. $\alpha^* = \alpha/t_r$ and is dimensionless. α has the dimensions of reciprocal of time and β is dimensionless. α is a scale parameter and β a shape parameter.

The time increment of 0.25 t_r was chosen for the following reasons: (1) The period of rise is an important time characteristic. (2) The use of 0.25 t_r enables definition of the rising limb at four points. (3) The shape of the hydrograph is retained by using this size of increment.

Equation (3.80) must be modified to represent the UH as

$$Q(t/t_r) = \frac{25(\gamma^*)^{\eta}}{\Gamma(\eta)} \exp(-\gamma^* t/t_r)\ (t/t_r)^{\eta-1} \qquad (3.81)$$

where γ^* and η are parameters. γ^* is dimensionless and is a scale parameter. $Q(t/t_r)$ represents the % flow/0.25 t_r or the percent of the total volume of flow which occurs during a time increment of 0.25 t_r at a specific value of t/t_r. Referring to figure 3.10,

$$\%\ \text{flow}/0.25\ t = \frac{Q_1}{Q_1 + Q_2 + \ldots + Q_N} \times 100$$

where N is the number of increments in t. Therefore,

$$Q_1 = (\%\ \text{flow}/0.25)\ t_r) \times \Sigma\ Q_i/100$$

which can be expressed

$$Q_1 = \frac{\%\ \text{flow}/0.25\ t_r}{100} \frac{AP}{0.25\ t_r} \qquad (3.82)$$

where A is area and P the depth of effective rainfall.

Equation (3.81) has two parameters γ^* and η which can be evaluated from the following relationships:

$$\frac{t_r}{\gamma^*} = a\left[\frac{L}{S_c^{0.5}}\right]^b \qquad (3.83)$$

where L is the length of the mainstream in miles and S_c the slope of the main channel in percent. The values of the coefficient a and exponent b were respectively found to be 11.4 and .531 for Ohio watersheds, 7.4 and 0.498 for Nebraska and western Iowa watersheds, and 9.27 and .562 for Illinois, Missouri, Wisconsin and central Iowa watersheds.

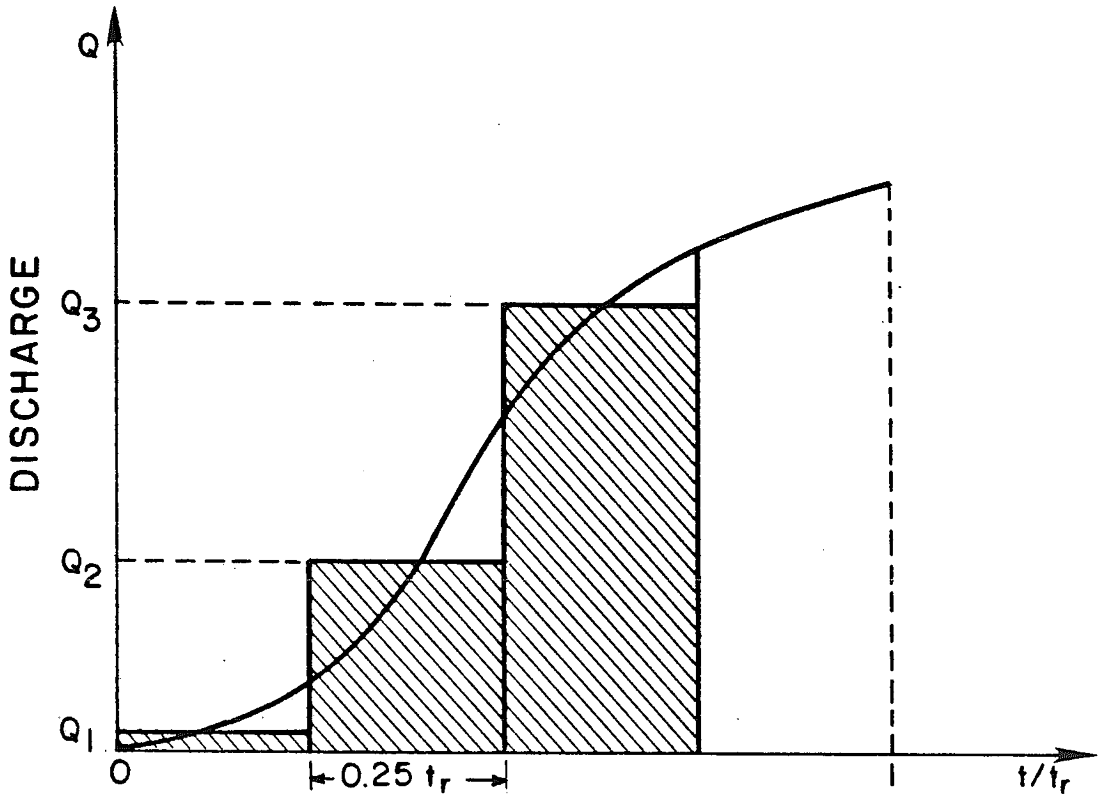


Figure 3.10 A dimensionless plot showing percentage of flow occurring during 0.25 t_r for the Gray model.

$$\eta = \gamma^* + 1 \tag{3.84}$$

$$\frac{t_r}{\gamma^*} = 1/[(0.262/t_r) + 0.0139] \tag{3.85}$$

in which t_r/γ^* and t_r are minutes. Gray prepared a table giving % flow/0.25 t_r versus t/t_r for different values of η as shown in Table 3.4.

3.3.8 THE CORDERY MODEL

Cordery (1968, 1971) used the Clark model and estimated its parameters t_c and k using data from a number of watersheds in eastern New South Wales, Australia. His equations are

$$t_c = 56\ [\frac{(LL_{ca})^{0.8}\ n}{S_c}]^{0.8} \tag{3.86a}$$

and

$$k = 11\ [\frac{W}{S_0^{0.5}} + \frac{Ln}{2\ S_c^{0.5}}]^{0.8} \tag{3.86b}$$

with

$$W = A/L$$

where A is watershed area in square miles, L length of main stream measured to watershed divide in miles, L_{ca} length in miles measured along path water would follow from watershed center of area to the outlet, S_c slope of main stream in feet per mile, S_0 overland slope in feet per mile, and n Manning's roughness coefficient estimated for the highest order stream in the watershed.

3.3.9 THE LEINHARD MODEL

In a series of papers (Leinhard, 1964; Leinhard and Meyer, 1967; Leinhard and Davis, 1971; Leinhard, 1972) Leinhard, using the Boltzman

Table 3.4 Dimensionless graph coordinates for different values of the parameter, η (after Gray, 1961)

	% Flow/0.25 t_r								
t/t_r	η=2.0	η=2.5	η=3.0	η=3.5	η=4.0	η=4.5	η=5.0	η=5.5	η=6.0
0.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.125	1.2	0.8	0.5	0.3	0.2	0.1	0.1	0.1	0.1
0.375	6.6	6.3	5.8	5.2	4.7	4.2	3.7	3.2	2.8
0.625	11.2	12.0	12.6	13.0	13.3	13.6	13.6	13.6	13.5
0.875	13.3	14.9	16.4	17.7	18.9	20.0	21.0	22.0	22.9
1.000	13.5	15.3	16.8	18.2	19.5	20.8	21.9	23.0	24.1
1.125	13.4	15.0	16.5	17.8	19.0	20.1	21.1	22.1	23.1
1.375	12.1	13.3	14.2	14.9	15.6	16.1	16.6	16.9	17.2
1.625	10.3	10.7	11.1	11.2	11.2	11.1	10.9	10.7	10.5
1.875	8.3	8.2	8.0	7.7	7.3	6.8	6.4	6.0	5.5
2.125	6.5	6.0	5.5	5.0	4.4	3.9	3.4	3.0	2.6
2.375	4.9	4.3	3.6	3.1	2.5	2.1	1.7	1.4	1.1
2.625	3.6	2.9	2.3	1.8	1.4	1.0	0.8	0.6	0.5
2.875	2.6	2.0	1.4	1.0	0.8	0.6	0.4	0.3	0.2
3.125	1.9	1.3	0.9	0.6	0.4	0.3	0.2	0.1	
3.375	1.3	0.9	0.5	0.3	0.2	0.1	0.1		
3.625	1.0	0.6	0.3	0.2	0.1				
3.875	0.6	0.3	0.2	0.1					
4.125	0.4	0.2	0.1	0.1					
4.375	0.3	0.1	0.1						
4.625	0.2	0.1							
4.875	0.1	0.1							
5.125	0.1								
5.375	0.1								
Sum	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

statistics, derived a generalized gamma distribution for representation of the IUH. His analysis can be briefly summarized as follows.

Let (N + M) raindrops constitute an instantaneous burst of rainfall. Of these raindrops, M disappear into the ground or evaporates into the air; that is, these represent abstractions. The remaining N raindrops constitute the effective rainfall, and eventually find their way to the watershed outlet. The direct runoff hydrograph can be interpreted in terms of raindrops N_i in each of a series of time increments of duration Δt. The result will be a histogram as shown in figure 3.11.

It is assumed that each raindrop is distinguishable from and independent of the other raindrops. Each raindrop will have the same apriori probability of reaching the gaging station during the i th time interval. Whether or not it does will depend upon where it lands and what obstacles it encounters during its journey. Let us consider the slug of water composed of N_i raindrops that reach the watershed outlet during the time t_{i-1} to t_i. There can be g_i ways in which these raindrops can find their way to the outlet. It is reasonable to assume that there are, on the average, a constant number of ways by which a raindrop must leave each unit of watershed area. Furthermore, the time required for a raindrop to reach the outlet after travelling a distance L_i will be approximately proportional to L_i raised to some power.

The following requirements can now be imposed on N_i's:

1. The total number of occurrences of the event is fixed,

$$\sum_{i=1}^{\infty} N_i = N$$

The N_i's and N are assumed to be large numbers.

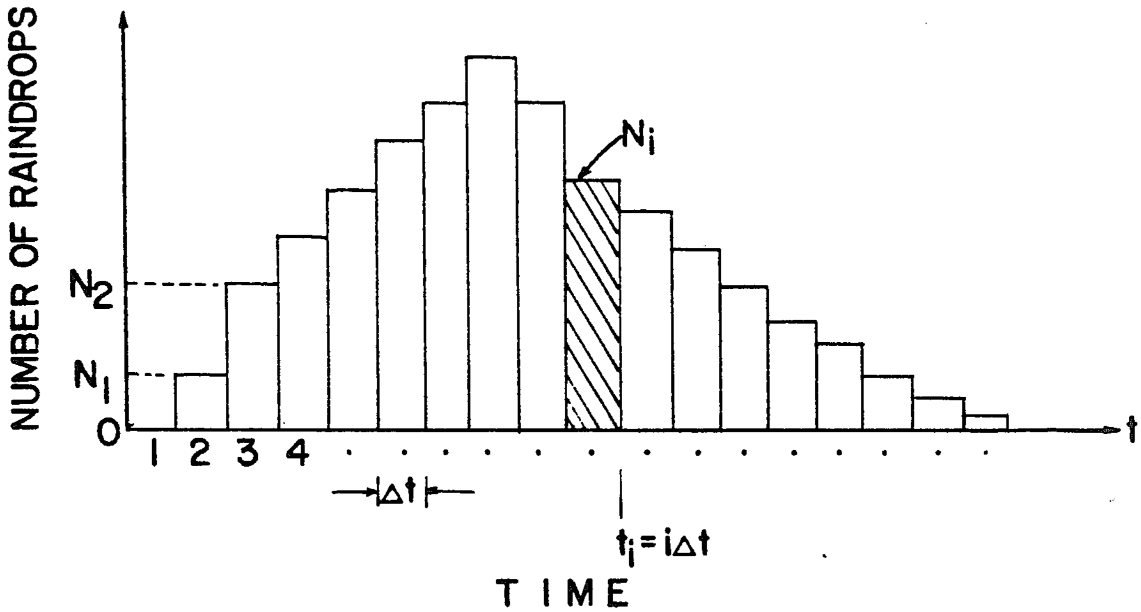


Figure 3.11 The runoff hydrograph interpreted as a raindrop histogram.

2. For each choice of β the following sum is a positive constant,

$$\sum_{i=1}^{\infty} (N_i/N)\ t_i^{\beta} = k, \quad \beta > 0, \quad k > 0$$

3. The number of distinguishable ways g_i in which the N_i raindrops can reach the outlet in the interval $[t_{i-1}, t_i)$ is proportional to a specified power of t_i. Thus,

$$g_i = c\ t_i^{\alpha-1}, \quad \alpha > 0$$

where c is some constant. The problem thus reduces to the derivation of the distribution of the number of events which will consist of determining the most probable distribution satisfying these requirements. Let us designate by W the number of ways in which N distinguishable occurrences of the event can take place, if N_i of these occurrences must take place in $[t_{i-1}, t_i)$ and if the number of ways the event can occur $[t_{i-1}, t_i)$ is given by g_i. Then, using Boltzman statistics (Davidson, 1962) it follows that

$$W = N! \prod_{i=1}^{\infty} \frac{g_i^{N_i}}{N_i!} \tag{3.87}$$

The particular set of numbers $(N_1, N_2, \ldots, N_i, \ldots)$ for which W is maximum will be the most probable distribution. Since the numbers involved are very large, the most probable distribution will be certain for all practical purposes. Let $\tilde{N}_i$ denote those values of N_i which give maximum of W. Then, subject to the three requirements indicated above, it can be shown that

$$\frac{\tilde{N}_i}{N} = \frac{\Delta t[\beta(\beta k/\alpha)^{-\alpha/\beta}]}{\Gamma(\alpha/\beta)}\ t_i^{\alpha-1} \exp[-\frac{\alpha}{\beta} \frac{t_i^{\beta}}{k}] \tag{3.88}$$

We may suppose that $\tilde{N}_i/N$'s represent the discrete probability distribution associated with the random variable T, where T is the time at which the first occurrence of the event under consideration takes place. Therefore,

$$P(t_{i-1} \leq T < t_i) = \tilde{N}_i/N, \quad i = 1, 2, \ldots$$

Approximating this discrete distribution with a continuous probability density function f(t),

$$\tilde{N}_i/N = \int_{t_{i-1}}^{t_i} f(t)dt$$

Using the mean value theorem and letting $\Delta t \to 0$, we obtain from equation (3.88),

$$f(t) = [\frac{\beta}{\Gamma(\alpha/\beta)} (\frac{\alpha}{\beta k})^{\alpha/\beta}] \, t^{\alpha-1} \exp[-\frac{\alpha}{\beta}\frac{t^\beta}{k}], \quad t \geq 0 \qquad (3.89)$$

It may be of interest to note that several distributions can be obtained as special cases of equation (3.89). If $d = \beta$, we obtain the Weibull distribution,

$$f(t) = \frac{\alpha}{k} t^{\alpha-1} \exp[-t^\alpha/k] \qquad (3.90)$$

If $\beta = 1$, it leads to the familiar gamma distribution. If $\alpha = \beta = 2$ then we obtain the Rayleigh distribution,

$$f(t) = \frac{2}{k} t \exp[-t^2/k] \qquad (3.91)$$

This is a special case of the Weibull distribution.

If $\alpha = 3$, $\beta = 2$ then we obtain the Maxwell molecular speed distribution,

$$f(t) = [\frac{(54/\pi)^{0.5}}{k^{1.5}}] \, t^2 \exp[-\frac{3}{2}\frac{t^2}{k}] \qquad (3.92)$$

If $\alpha = 1$ and $\beta = 2$ then we obtain the Maxwell molecular velocity distribution,

$$f(t) = (\frac{2}{k\pi})^{0.5} \exp[-\frac{t^2}{2k}] \qquad (3.93)$$

If $\alpha = \beta = 1$, the resulting distribution is exponential,

$$f(t) = \frac{1}{k} \exp[-t/k] \tag{3.94}$$

The function f(t) can be interpreted as the IUH. Equation (3.89) is a generalized 3-parameter gamma distribution from which several distributions can be obtained as special cases (Leinhard and Meyer, 1967). Leinhard (1964) applied his model to two natural watersheds in Illinois. He observed that for representation of the IUH, β can be taken as 2, and α as 2 and 3 respectively for long slender and fan shaped watersheds. The parameter k establishes the scale of the distribution and will be the second moment of the IUH about its origin. Betson and Green (1968) used an empirical expression similar to equation (3.89) for analysis of streamflow of the Upper Bear Creek in the Tennessee Valley region.

3.3.10 THE LINEAR WATERSHED BOUNDED NETWORK (LWBN) MODEL

Boyd (1978, 1982) developed the WBN model for synthesis of the IUH employing geomorphologic and hydrologic properties of the watershed. The conceptual framework embodied in this model parallels the one developed independently by Singh and McCann (1980). The model divides a watershed into sub-areas bounded by watershed lines using large-scale topographic maps. Following Leopold, Wolman and Miller (1964) two types of sub-areas having different collection and drainage of water are distinguished: (1) ordered basin, and (2) interbasin area. An ordered basin is a complete sub-watershed and does not allow for flow across its boundaries. Its operation is independent of other sub-watersheds. These basins have been shown to be geomorphologically similar to the complete watersheds (Strahler, 1964). An interbasin area is a sub-area with a stream flowing through it. The direct runoff at its outlet consists of both runoff from upstream areas and that generated within it. Each of ordered basin and interbasin area is represented by a lumped storage element. These elements are connected in the topology as the watershed stream network. Thus, the watershed is represented by a branched network of storage elements. For linear elements, this can be represented by an arrangement of parallel cascades wherein the number of cascades is equal to the number of links (interior as well as exterior) plus the number of sub-areas. To be more precise, if there are N sources then (2N - 1) parallel cascades of storage elements will result. The number of elements in a cascade is specified by the position of the link or subarea. This suggests that a knowledge of stream network implies a knowledge of the arrangement of cascades. Figure 3.12 illustrates the model structure (a) showing a second order hypothetical watershed, (b) outlining a branched network model structure, and (c) giving an equivalent model structure. The linear storage element can be described by

$$\frac{dS}{dt} = I - Q \tag{3.95}$$

$$S = kQ \tag{3.96}$$

in which S is volume of water stored in the area, I inflow to the area, Q outflow from the area, k storage coefficient or time parameter, and t time. Note that the LWBN model considers two storage effects separately which are (1) those acting during transformation of the effective rainfall to direct runoff via overland flow and flow in small stream channels, and (2) those acting during transmission of upstream direct runoff through a main stream segment. For a single storage element k is precisely equal to the lag time. Accordingly, k and c are used to correspond to the above two types of storage effects separately.

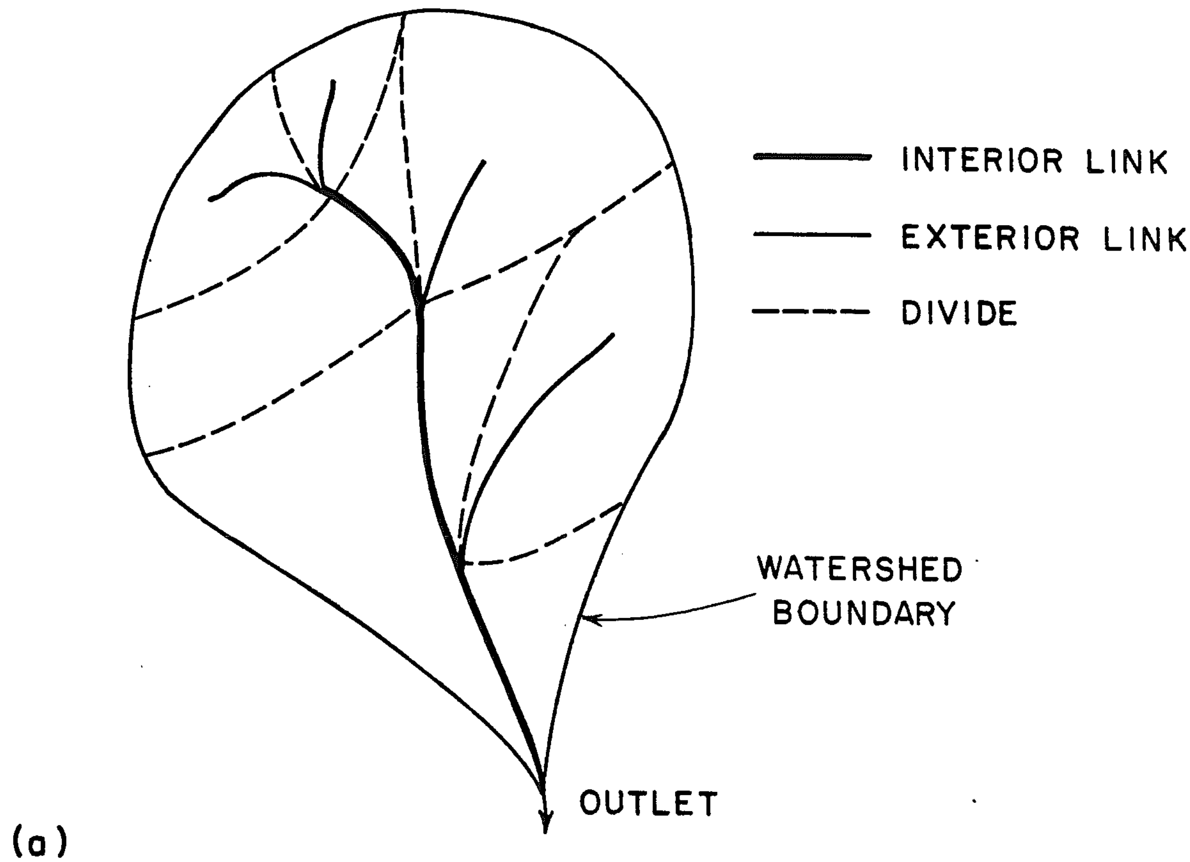


Figure 3.12a A hypothetical watershed and its stream network.

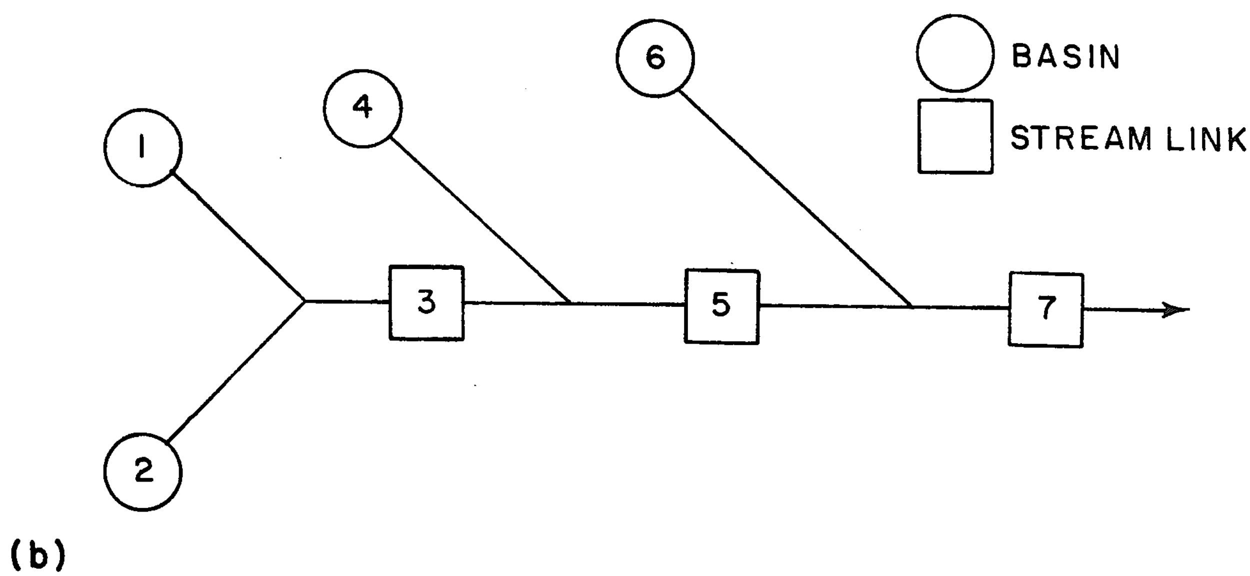


Figure 3.12b A branched network model structure for watershed in (a) above.

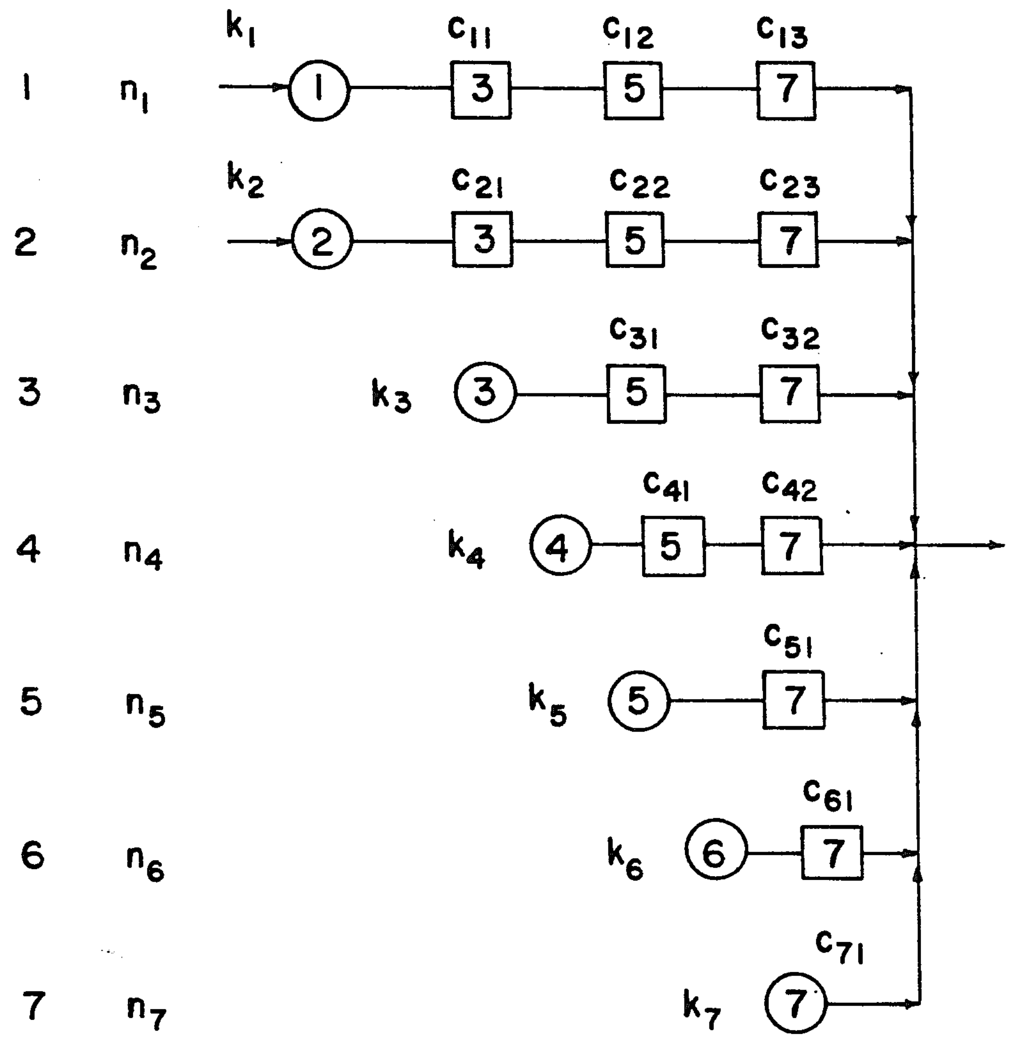


Figure 3.12c An equivalent model structure.

The IUH of the LWBN model can be expressed as

$$h(t) = \frac{1}{A} \sum_{i=1}^{2N-1} \frac{1}{(1 + k_i D) \prod_{j=1}^{n_i} (1 + c_{ij} D)} \delta(t)\ a_i \tag{3.97}$$

where a_i is the area drained by the i-th cascade and n_i the number of stream links in the cascade. Equation (3.97) is a special case of the Dooge model with pure translation deleted. Its solutions have been presented previously. In practice the model equations can easily be solved numerically for each storage element by expressing equation (3.95) - (3.96) in finite-difference form with Δt as the routing period,

$$Q_{t+\Delta t} = \frac{Q_t (2k - \Delta t) + (I_t + I_{t+\Delta t})\ \Delta t}{2k + \Delta t} \tag{3.98}$$

The LWBN model has several attractive features. The model allows for areal variations of rainfall, losses and watershed conditions. It can be used to quantify the effects of land use changes, and constructing dams and reservoirs. The model structure is uniquely defined by watershed stream network. In other words, components of the model can be identified with specific parts of the watershed. The model has a large number of parameters. These include the number n of storage elements, the n values of k values for these elements, and n_i values of C for the n_i interbasin stream segments. However, most of these parameters are deduced from geomorphologic properties as will be clear from the ensuing discussion. A drawback of this model is that there may be hundreds of links in the network representation requiring larger computation than actually needed. If some of the links, exterior or interior, are lumped or suppressed then an objective way of accomplishing this is lacking.

3.3.10.1 Parameter Estimation

The parameters to be estimated are k_i; C_{ij}, $j = 1, 2, \ldots, n_i$; and n_i, $i = 1, 2, \ldots, (2N-1)$. n_i is specified by the stream network. For each storage element i representing the area A, the parameter k is estimated as

$$k = a\ A^b \tag{3.99}$$

where a and b are constants. For 10 watersheds (drainage area 0.39 to 251 km^2) Boyd, Pilgrim and Cordery (1979a, 1979b) reported a = 2.51 and b = 0.38 with A in km^2 and k in hours. Similarly, for each element C is computed as

$$C = a_1\ A^{b_1} \tag{3.100}$$

where a_1 and b_1 are constants. For the same 10 watersheds $a_1 = 1.5$ and $b_1 = 0.38$, with C in hours and A in km^2.

The constants, especially a and a_1, in equations (3.99) - (3.100) may vary from one region to another. It is therefore desirable to discuss the method by which to estimate them. This can be accomplished by computing the watershed lag time which can be obtained from inflow-outflow records, i.e., the time difference between the centroid of the effective rainfall hyetograph and that of the direct runoff hydrograph. The lag time t_L of the LWBN model is

$$t_L = \frac{1}{A} \sum_{i=1}^{2N-1} a_i\ g_i \tag{3.101}$$

where A is total watershed area, a_i are corresponding to the cascade i, and g_i is the total lag along the cascade i, and equals

$$g_i = k_i + \sum_j C_{ij} \tag{3.102}$$

Equation (3.101) can be written alternatively as

$$t_L = \frac{1}{A} [\sum_{i=1}^{2N-1} a_i k_i + \sum_j C_{ij} \sum_{m=1}^{j-1} a_m] \qquad (3.103)$$

3.3.10.2 Operation of the Model

For any effective rainfall pattern, inflows are routed through each storage element using equations (3.95) - (3.96). Computed outflows at the confluence of basins are added together. For interbasin areas, the runoff from upstream is routed through the main channel segment, and to this outflow is added the runoff resulting from the effective rainfall of the interbasin. The computations are carried out from the uppermost element moving downstream to the watershed outlet.

3.4 NONLINEAR MODELS

3.4.1 NONLINEAR RESERVOIR

The nonlinear storage-discharge relation,

$$S = k\, Q^m \qquad (3.106)$$

has been used in a number of nonlinear models (Laurenson, 1965; Laurenson, Mein and McMahon, 1975; Boyd, Pilgrim and Cordery, 1979a, 1979b). Alternatively,

$$S = k\, Q \qquad (3.107)$$

$$k = k_1\, Q^n \qquad (3.108)$$

Therefore, m = N = 1.

Equation (3.106) has two parameters m and k. Laurenson, Mein and McMahon suggested a value of 0.75 for m, unless an evidence supported otherwise. They argued that k can be conveniently expressed as a product of two factors as

$$k = k_1 \; k_2 \qquad (3.109)$$

where k_1 is a constant for a watershed, and k_2 proportional to the delay time of the individual model storage. Thus, k_1 and k_2 depend upon the size and physical characteristics of the watershed.

Both k_1 and k_2 are dimensional quantities. Laurenson, Mein and McMahon (1975) made k_2 dimensionless by expressing it as

$$k_r = (d_i - d_j)/d_c \qquad (3.110)$$

where k_r is dimensionless k_2, d_c mean travel distance for the watershed, and $(d_i - d_j)$ travel distance of the ith subwatershed having area a_i in a watershead composed of N subwatersheds marked by equal increments of travel distance (i denoting upstream and j downstream). Therefore,

$$d_c = \sum_{i=1}^{N} (a_i d_i)/A \qquad (3.111)$$

where A is the area of the watershed.

With k_2 replaced by k_r, k_1 can be supplanted by k_c which can be determined for ungaged watersheds as

$$k_c = 2.12 \; A^{0.57} \qquad (3.112)$$

where A is in km^2. For gaged watersheds an optimization is suggested. Thus

$$k = k_c k_r \qquad (3.113)$$

3.4.2 UNIFORMLY NONLINEAR CASCADE

The uniformly nonlinear model has three parameters: N, k and m. Based on a study of 39 small agricultural watersheds, Singh (1976, 1977, 1979a, 1979b) suggested m to be 1.4, N to be 3 and k to be estimated from watershed characteristics. He correlated k to watershed area A, stream length, L_c, watershed width W, channel slope S_c and watershed shape S_h and developed three alternative relationships for k:

$$k = A^{-.3087} L_c^{-.2161} S_c^{-.0833} S_h^{.3038} \tag{3.114}$$

$$k = A^{-.0283} W^{-.2431} \tag{3.115}$$

and

$$k = A^{-.2289} L_c^{-.264} S_c^{.1008} \tag{3.116}$$

where A is in h_a, L_c in m, W in m and S in percent. The shape factor was computed as proposed by Chorley, Malm and Pagorzelski (1957).

3.4.3 THE LAURENSON MODEL

This model was developed by Laurenson (1962). It divides the watershed into 10 sub-areas by drawing isochrones at equal time interval. These sub-areas can be delineated differently if one chooses to do so. A node is placed at the center of each sub-area where the inflow is assumed to be concentrated. Between each pair of sub-areas is placed a nonlinear storage element (reservoir) defined by equation (3.107) with k also being a function of Q. In this way a watershed is represented by 10 nonlinear storage elements connected is series. Each element receives the effective rainfall occurring over its corresponding sub-area of the watershed as well as the direct runoff from all upstream elements. The model is exhibited in figure 3.13.

The total watershed travel time is divided evenly between ten isochrones. The travel time between each pair of nodes is one-tenth of the total travel time. This is true for all storage elements except the lowest which has one-twentieth of the total travel time. Storage routing is performed between each pair of adjacent nodes with storage allocated to each element based on the storage properties between the nodes. Since the upstream runoff is routed between nodes, allocation of one-tenth of the total travel time to upper nine elements and one-twentieth to the lowest element is reasonable. For the effective rain-

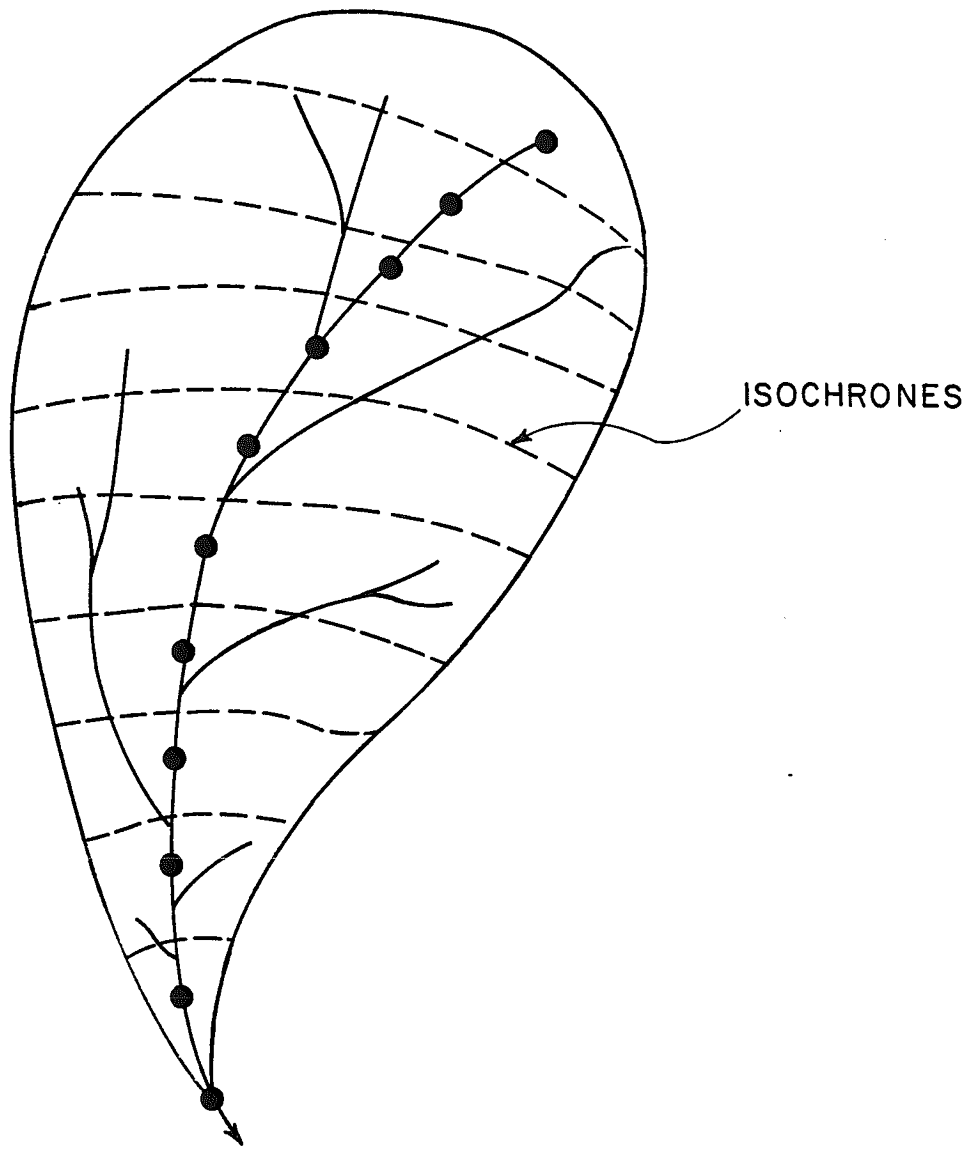


Figure 3.13a A hypothetical watershed with isochrones drawn at equal interval.

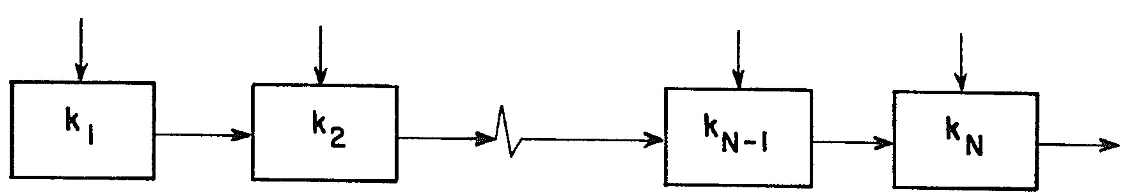


Figure 3.13b The Laurenson model for the watershed in (a) above.

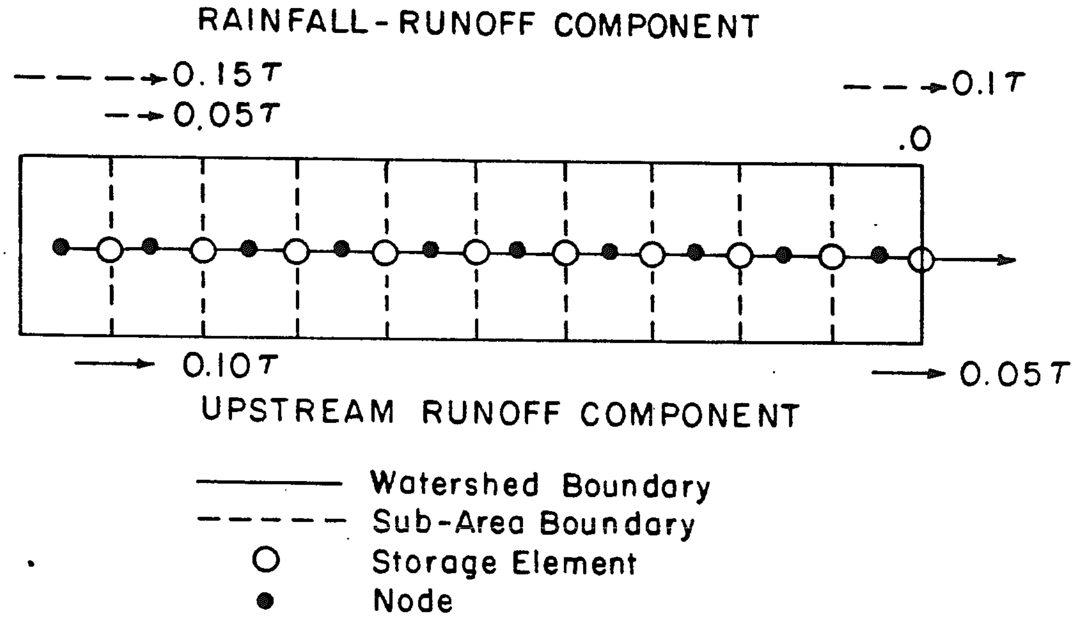


Figure 3.13c Allocation of travel time (or storage) for ten series elements in the Laurenson model.

fall occurring on each sub-area, the travel time ranges from one-twentieth from the downstream isochrone to one-fifteenth from the upstream isochrone; this also yields an average of one-tenth of travel time for the upper nine elements. For the lowest sub-area it ranges from 0 to one-tenth of the travel time giving an average of one-twentieth of the travel time.

The time of travel is analogous to k in equation (3.107) and is related to the physical geometry of the watershed by the assumption that is proportional to the distance and inversely proportional to the square root of slope. This can be estimated for any point in the watershed from its topographic map. The centroid of the TA diagram is the mean time of travel τ_m for the watershed and can be expressed as

$$\tau_m = \int_A \tau \, da / \int_A da = \frac{1}{A} \int_A \tau \, da \tag{3.117}$$

where τ is dimensionless time of travel, and A watershed area. Because of the analogy between time of travel and storage delay time, τ_m is equivalent in dimensional terms to k. Thus, using equation (3.108) for any value of Q, the dimensionless τ can be made dimensional for any point in the watershed as

$$k_* = \tau \, k_1 \, Q^n / \tau_m \tag{3.118}$$

Note that this does not correspond to the mean delay time. If $\Delta\tau$ is the interval of time of travel then each storage element has the time of travel as

$$k_i = \Delta\tau \, k_1 \, Q^n / \tau_m \tag{3.119}$$

The effective rainfall occurring on the topmost sub-area is first routed through the topmost storage element. The direct runoff from this element is combined with the effective rainfall to the second sub-area and the combined flow routed through the second element. This operation is repeated all the way up to the watershed outlet. Since each element is nonlinear and its effect depends on the magnitude of its inflow, it is not permissible to route the inflow to each sub-area individually to the outlet, and then combining the ten outflows there. Note that the watershed storage is distributed rather than concentrated because each inflow is routed through a series of concentrated storages thus providing for both translation and attenuation effects.

This model may be considered to have 3 parameters: number of storage elements N, the coefficient k, and exponent m of the storage discharge relation of equation (3.108). N is fixed at 10. Based on a regression of lag time versus mean discharge using a number of recorded rainfall and runoff hydrographs of the South Creek watershed in New South Wales, Australia, Laurenson obtained

$$t_L = 64\ Q_m^{-0.27} \tag{3.120}$$

where t_L is lag time in hours and Q_m mean discharge in cfs. He adopted m as -0.27. He computed k by assuming that the coefficient 64 was applicable to the points on the watershed corresponding to the centroid of the TA diagram. For the South Creek watershed, the relative time to the centroid was 0.66. The value of k for the entire watershed was 64.0/0.66 = 97.0. For each of the 10 equal storage elements, the coefficient k was 97.0/10 = 9.7.

The Laurenson model considers the internal properties of the watershed. Specifically, it accounts for (a) areal variation of inflow, (b) areal variation of watershed characteristics, (c) nonlinear storage effects, and (d) effective rainfall subjected, depending upon the point of its occurrence, to different storage amounts. Thus, the model is physically realistic. However, there are some weaknesses of the model which should be recognized. Since the method of constructing isochrones is uncertain, the representation of a watershed into sub-areas is less than objective. Storage properties between each pair of nodes cannot be measured so the storage allocation is difficult. The assumption of the watershed lag applying to the centroid of the TA diagram holds only for a linear model and for spatially uniform effective rainfall. Equation (3.120) refers to average values in time and magnitude, but is applied to instantaneous values in equation (3.117). The time parameter k is assumed proportional to travel times which, in turn, are assumed proportion to $L/S_0^{0.5}$ with L being length of travel and S_0 slope. Boyd (1976) has shown that this is a poor parameter for measuring travel time. These limitations introduce uncertainty into model results and the parameters estimated as above. When sub-areas are defined by isochrones, flows in more than one well-defined channel may be lumped together for determination of lag time; this superimposition of flows is not valid because of the nonlinearity.

3.4.4 THE PORTER MODEL

This model is a variation of the Laurenson model. It divides the watershed into a number of sub-areas defined by internal watershed boundaries or stream network so that each sub-area contains only one well-defined channel. Streams of lower order may be lumped, but major tributaries are not. Each sub-area is represented by a nonlinear storage element defined by equation (3.104). Sub-areas are determined by higher order streams such that the maximum number of storage elements will be about five. The resulting arrangement of elements is in parallel and series as shown in figure 3.14. It is a more versatile arrangement and may be of value in modeling large watersheds.

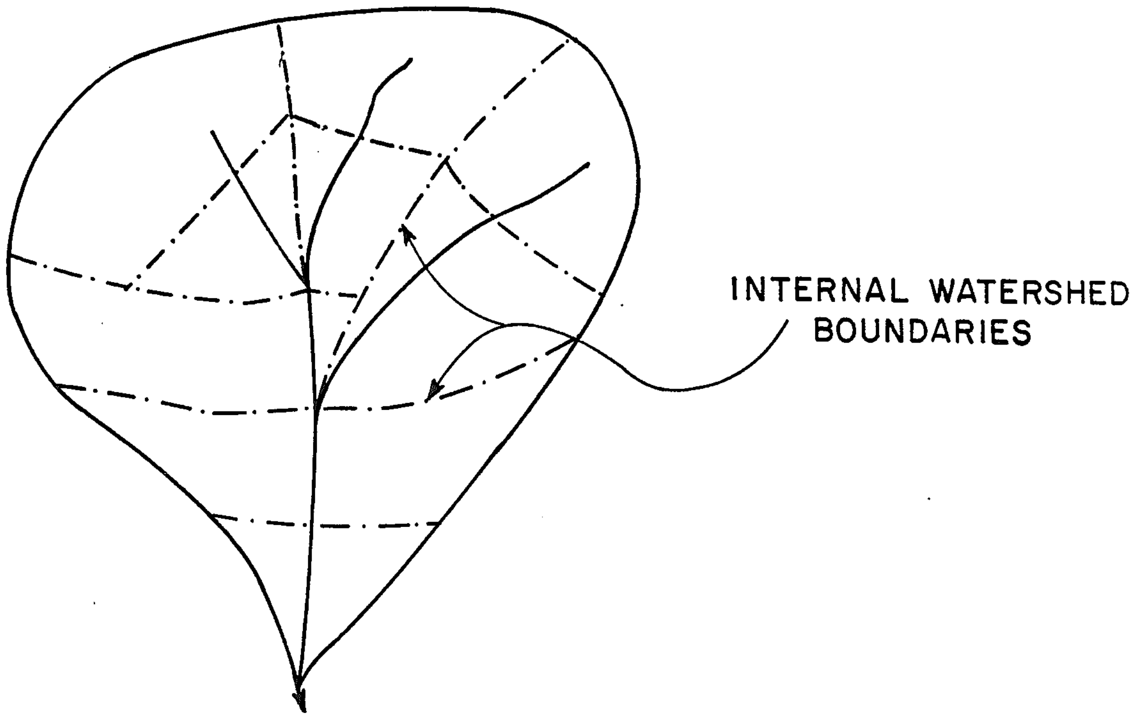


Figure 3.14a A hypothetical water with sub-areas demarcated by internal watershed boundaries.

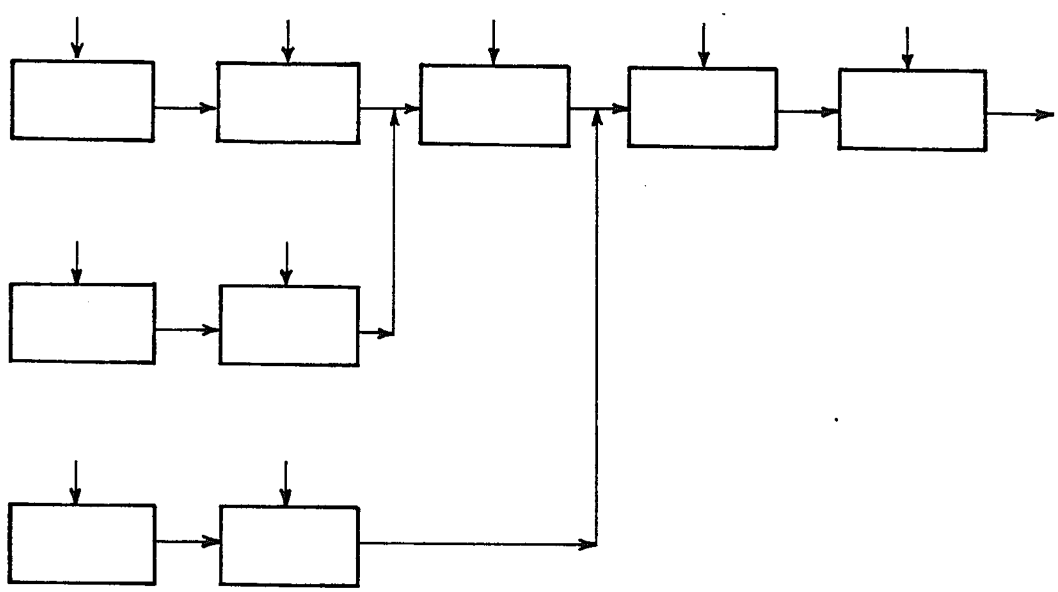


Figure 3.14b The Porter model for the watershed in (a) above.

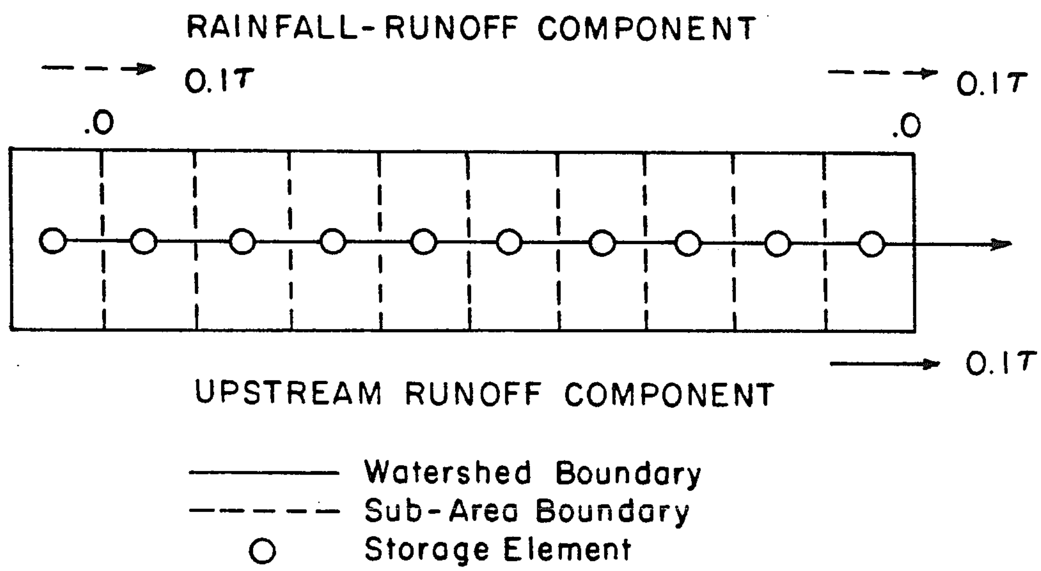


Figure 3.14c Allocation of travel time (or storage) for ten series elements in the Porter model.

The storage elements represent specific sub-areas rather than storage routing between sub-areas. The storage is allocated to each element by assuming that it is proportional to travel time. Since travel time is not known, it is assumed to be a function of the properties of the stream segment in each sub-area, that is, it is proportional to the ratio $L/S_c^{0.5}$ which was shown by Boyd (1976) to be a poor indicator. The storage allocation is essentially the same as in the Laurenson model. The storage routing, however, occurs in each sub-area.

Porter criticized the Laurenson model for halving the lag time for the lowest sub-area on the grounds that the runoff from all upstream sub-areas passes through sub-area and is hence subject to modification by its storage. In addition, it transforms its share of rainfall to runoff. Porter used the full lag time for the lowest sub-area but only one-half for the upper most sub-area. His argument was that this sub-area has no upstream runoff. For example, assume that there are 10 storage elements in series. The travel time for the effective rainfall ranges from zero for the lower isochrone to one-tenth for the upper, given an average travel time of one-twentieth for all sub-areas. For runoff routing the travel time is one-tenth for the lower nine elements but zero for the uppermost element. The storage allocation appears inconsistent with the effective rainfall component from nine of the elements. Like the Laurenson model this model does not consider separately the effects of storage on the transformation of the effective rainfall to direct runoff in a sub-area and the storage effects on upstream runoff which passes through the main stream segment of the sub-area. Both sources of runoff are subject to the same storage routing.

Another modification made by Porter involves applying a translation time between each pair of storage elements. The observed watershed lag is considered to be comprised of two components, a storage delay time k for each element and a translation time T between the elements,

$$t_L = k + T \tag{3.121}$$

t_L is nonlinearity related to Q. For simplicity T is assumed independent of Q. Thus, all nonlinearity of the watershed is attributed to storage elements. To further simplify, a constant value of T_* may be applied between successive storage elements. [This may be thought of as being a time for movement of water from one sub-area to the next.] T_* is the same for each sub-area even though its size varies from one element to the next.

The translation time T for the watershed is obtained by integrating over the watersheds area the time of translation incurred to the outlet as

$$T = \frac{1}{A} \int_A t \ da \tag{3.122}$$

where A is watershed area, t translation time incurred between the point in question and the watershed outlet, and a area-variable of integration. If there are N sub-areas in series with areas $a_1, a_2, \ldots, a_N$, then

$$\int_A t \ da \cong T_* \sum_{j=1}^{N-1} (N - j) \ a_j \tag{3.123}$$

in which the number of sub-areas is in downstream direction. Note that the direct runoff from the lowermost element (No. N) incurs no translation, that from the next sub-area upstream (No. N-1) incurs one translation T_*, and so on. Equation (3.122) can be expressed differently,

$$\frac{1}{A} \int_A da \cong \frac{T_*}{A} \sum_{j=1}^{N-1} \alpha_j \tag{3.124}$$

where α_i is the sub-area drained by the jth sub-area. Therefore,

$$T \cong \frac{T_*}{A} \sum_{j=1}^{N-1} \alpha_j \tag{3.125}$$

If T is known then k can be determined from equation (3.121) with t_L obtained from observations.

From a physical standpoint, introduction of travel time may be difficult to justify. The elements in the model are storage elements. The storage allocation is based on the travel time between the upper and lower boundaries of the sub-area. Hence, there is no room to place this additional storage between the sub-areas. Delay time and translation are considered separately to independently account for the effects of hydrograph attenuation and translation; since they both result from storage routing, this consideration is inconsistent.

3.4.5 THE MONASH MODEL

This model is also a variation of the Laurenson model, and has been presented by Mein, Laurenson and McMahon (1974). The watershed is divided into several subwatersheds such that subwatersheds are based on the major tributaries, gaging stations and the proposed dam site fall on subwatershed boundaries, and all subwatersheds are of the same order of area. Nodes are placed so that there is a node on the main stream in each subwatershed at the point nearest its centroid, at each confluence where subwatershed flow combine, at each gaging station, and at the proposed dam site. If large storage, natural or artificial, exist in the watershed then nodes should be placed at each subwatershed entry to the storage, and at a point immediately downstream of the storage. This allows the flows to be routed through the reservoir or storage using, of course, appropriate storage-discharge relation.

Storage effects are represented by a nonlinear storage element. A storage element is placed between each pair of adjacent nodes. Thus, storage routing occurs between the nodes. However, if there is a reservoir then it is represented by a single storage element even if it has more than one upstream node. The resulting model structure therefore consists of a network of nonlinear storage elements having a shape similar to the watershed stream network. Storage is allocated to each element based on the properties of the stream segment between the nodes. Three relations are suggested:

$$k = L/S_c^{0.5} \tag{3.126}$$

$$k = L \tag{3.127}$$

$$k = [L^2/S_c^{0.5}]^{0.71} \tag{3.128}$$

The allocation of storage elements is basically the same as in the Laurenson model, assuming that storage effects are proportional to travel time between each pair of nodes.

4. DEVELOPMENT OF PROPOSED MODELS

Rodriguez-Iturbe and Valdes (1979) developed an approach for derivation of the IUH by explicitly incorporating the characteristics of drainage basin composition (Horton, 1945; Strahler, 1964; Smart, 1972). This approach coupled the empirical laws of geomorphology with the principles of linear hydrologic systems. Rodriguez-Iturbe and his associates have since extended this approach by explicitly incorporating climatic characteristics, and have studied several aspects including hydrologic similarity (Rodriguez-Iturbe, Devoto and Valdes, 1979; Valdes, Fiallo and Rodriguez-Iturbe, 1979; Rodriguez-Iturbe, 1982; Rodriguez-Iturbe, Gonzalez-Sanabria and Bras, 1982; Rodriguez-Iturbe, Gonzalez-Sanabria and Caamano, 1982). Motivated by the work of Rodriguez-Iturbe and his associates, Gupta, Waymire and Wang (1980) examined this approach, reformulated its, simplified, and made it more general and even more elegant. Its assumptions, limitations and potential for application to synthesis of direct runoff from ungaged watersheds became more apparent. Others (Wang, Gupta and Waymire, 1981; Hebson and Wood, 1982; Kirshen and Bras, 1983; Singh, 1983; Hill, Singh and Aminian, 1984) have also applied this approach to hydrologic analysis.

This geomorphologic approach, as formulated, is probabilistic in character. However, we develop here a deterministic interpretation which is much simpler, easier for hydrologists to understand, equally versatile, and retains the essential import of the original approach. This interpretation can be explained as follows: A watershed is of a fixed order W. It contains channel c_1, c_2, ..., c_W, where c_i denotes the i th order stream. The watershed network is ordered according to

Horton–Strahler ordering scheme (Strahler, 1964, Smart, 1972). When it rains, runoff is generated. The time taken by a body of water to travel to the watershed outlet depends upon the position where the travel is initiated and the path it follows. In a watershed there can be an infinite number of positions where raindrops will land and initiate their travel in association with physiographic characteristics. Likewise, there can be an infinite number of paths of travel. These paths are, however, carved by topographic slope configuration and channel network.

A watershed surface is comprised of overland regions and channels. The surface area occupied by the overland regions is many times greater than that occupied by the channels. It can therefore be assumed without undue loss of accuracy that the raindrops fall and start their journey on the overland regions, and travel through the channel network for reaching their destination at the watershed outlet. Let the overland regions be assigned the same order as the channels they directly contribute to. For notational convenience let an overland region of order i be denoted by r_i. To recount, rain water, depending upon the position of its landing, starts its travel in the overland region and then goes through a string of channels. Logically its travel follows the following rules:

Rule 1. The only possible route from the overland region r_i is to the channel c_i, $1 \leq i \leq W$.

Rule 2. The only possible route from the channel c_i is to the channel c_j for some $j > i$, $1 \leq i \leq W$.

Rule 3. The only possible route from the channel c_W is the watershed outlet.

These rules define a collection of paths which water can take from the start of its journey to the watershed outlet. The number of these paths N equals $N = 2^{W-1}$. To illustrate, consider a third order watershed as shown in figure 4.1. Then paths R_i, i = 1, 2, ..., N, can be demarcated as follows:

Path R_1: $r_1 \rightarrow c_1 \rightarrow c_2 \rightarrow c_3$

Path R_2: $r_1 \rightarrow c_1 \rightarrow c_3$

Path R_3: $r_2 \rightarrow c_2 \rightarrow c_3$

Path R_4: $r_3 \rightarrow c_3$

It should be noted that every 3rd order watershed will have less than or equal to four paths.

These paths specify the spatial evolution of rainwater through a geomorphic network of channels and overland regions. It is clear that the time of travel that water spends in following a given path depends upon its composition. Obviously, different paths may have different travel times. A path is composed of one overland region and one or more channels. This means that the travel time of a path can be specified by computing travel times of its overland region and channel(s) and then summing them up. The transformation of rainfall to runoff from the overland region is determined by the region's such characteristics as area, roughness, vegetal cover, slope, length of flow, shape, etc. The effect of most of these factors can be lumped through the area. The transmission of overland flow through channels downstream depends upon the channel characteristics such as length, slope, width, roughness, etc. However, the effects of all these can be combined through the length.

With these considerations it is surmised that a given watershed can be represented by a parallel arrangement of cascades. Each cascade represents a particular path. For the watershed in figure 4.1 this arrangement is shown in figure 4.2. Each cascade is composed of elements (one representing overland region and the other channels) which can be linear or nonlinear. We assume them to be linear here. Note that a cascade represents all those portions of the watersheds which drain following the same path. For example, for the above watershed R_1 (or its corresponding cascade) consists of 10 overland regions, R_2 consists of 1 overland region, R_3 consists of 2 overland regions, and R_4 consists of 1 overland region. The area drained by R_1 is the sum of its 10 overland regions, and likewise for the remaining R_i's.

An overland region can be represented by a linear reservoir of lag time k, and a channel by a linear reservoir of lag time C. Since a cascade corresponds to a specific path, the lag time of an element is an average of the lag times of the portions of the actual watershed represented by this element. For example, the lag time of the first order overland region is the average of the lag times of all first order overland regions in the watershed. Therefore, k is an average value. This same is true of the channels. This averaging reduces the number of parameters needed to define the cascade arrangement. The total number M of parameters to be determined is 2W half of which being for overland regions and half for channels. Referring to figure 4.2, the unknown parameters are $k_1 = k_2$, k_3, k_4; $C_{11} = C_{21}$, $C_{12} = C_{31}$, $C_{13} = C_{22} = C_{32} = C_{41}$. As will be seen these parameters can be determined following the procedure employed in the LWNB model.

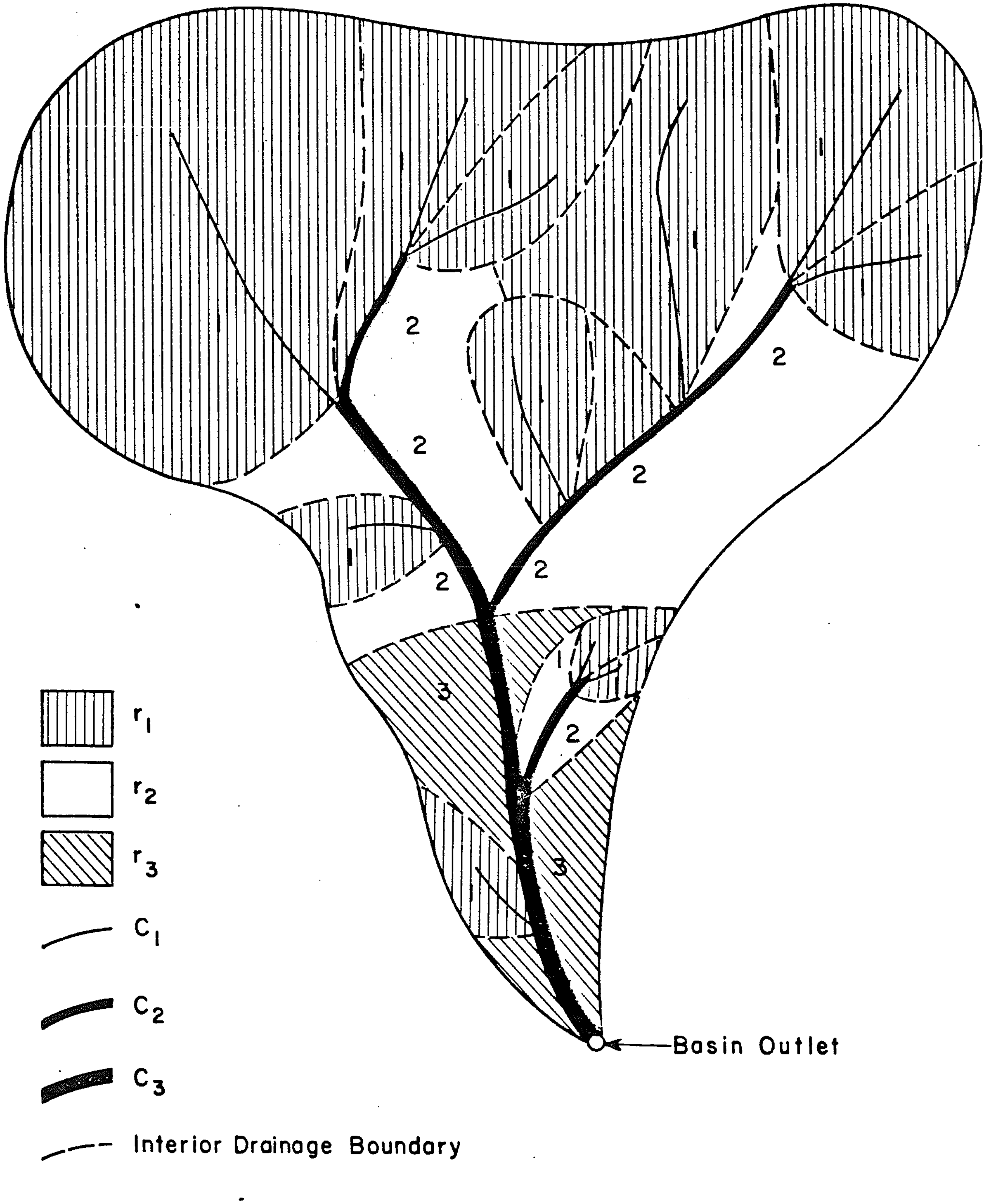


Figure 4.1 A hypothetical third-order watershed with Strahler ordering system.

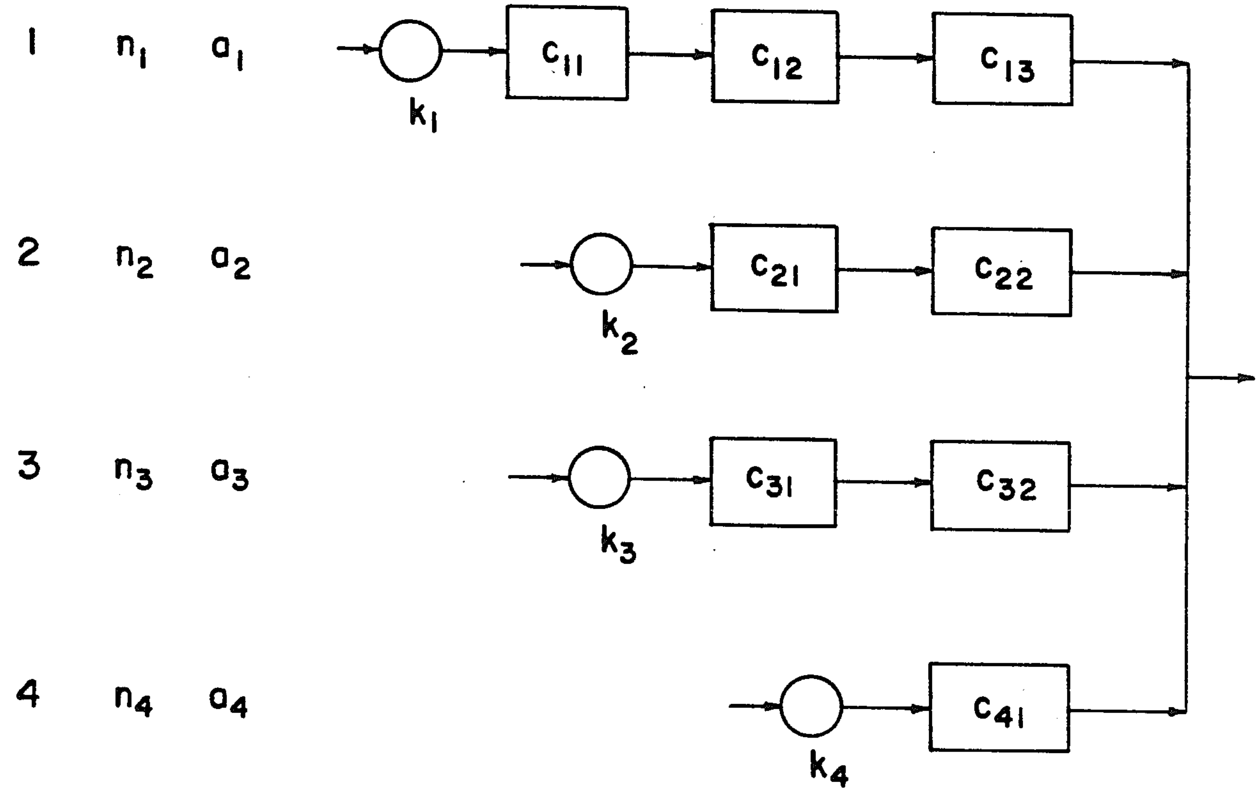


Figure 4.2 An equivalent representation of the linear geomorphic model.

4.1 MATHEMATICAL SOLUTIONS

Two linear elements are involved: (1) linear reservoir, and (2) linear channel. For a linear reservoir, the governing equations are:

$$\frac{dS}{dt} = I - Q \tag{4.1}$$

$$S = kQ \tag{4.2}$$

in which S is storage, I is effective rainfall hyetograph ordinate, Q is direct runoff and k is lag time. Because the reservoir is linear, it may suffice to determine its IUH denoted by h(t),

$$h(t) = \frac{1}{(1 + kD)} \delta(t) = \frac{1}{k} \exp(-t/k) \tag{4.3}$$

Likewise, the governing equations of a linear channel are:

$$\frac{dS}{dt} = I - Q \qquad (4.1)$$

$$S = C[xI + (1 - x)Q] \tag{4.4}$$

in which I is inflow hydrograph of direct runoff received from the overland region, x is weighting factor ranging from 0 to 0.5 and C is lag or travel time. Its IUH can be expressed as

$$h(t) = \frac{(1 - C x D)}{C(1 - x)\ [1 + (D/C(1 - x)]} \delta(t) - \frac{x}{1 - x} \delta(t)$$

$$+ \frac{1}{C(1 - x)^2} \exp([-t/(C(1 - x))] \tag{4.5}$$

For purposes of simplicity x was considered as zero. Therefore,

$$h(t) = \frac{1}{(1 + CD)} \delta(t) = \frac{1}{C} \exp(-t/C) \tag{4.6}$$

Equations (4.3) and (4.6) are identical except for the symbolic difference of the lag time.

Now consider a given path or the cascade representing it. This cascade consists of a number of unequal elements. For purposes of simplicity, let the elements be decnoted by k_i, $i = 1, 2, . . . , N$. Then its IUH is

$$h(t) = \frac{1}{\prod_{i=1}^{N} (1 + k_i D)} \delta(t) = \sum_{j=1}^{M} \frac{k_j^{N-2} \exp(-t/k_j)}{\prod_{\substack{i=1 \\ i \neq j}}^{N} (k_j - k_i)}, \quad N \geq 2 \qquad (4.7)$$

in which N specifies the number of elements in the cascade. Equation (4.7) can be generalized for the case consisting of M number of parallel cascades. Let N denote the number of elements in the i-th cascade draining the area A_i. Let k_{ij} denote the lag time of the j-th element of the i-th cascade. Its IUH can be expressed as

$$h(t) = \sum_{i=1}^{M} \frac{A_i}{A} \frac{1}{\prod_{j=1}^{N_i} (1 + k_{ij} D)} \delta(t) \qquad (4.8)$$

$$= \sum_{i=1}^{M} \frac{A_i}{A} \left[\sum_{j=1}^{N_i} \frac{k_{ij}^{N_i - 2} \exp(-t/k_{ij})}{\prod_{\substack{r=1 \\ r \neq j}}^{N_i} (k_{ir} - k_{ij})} \right], \quad N_i \geq 2 \qquad (4.9)$$

in which $\sum_{i=1}^{M} A_i = A$ - area of the watershed. Equation (4.9) is the general expression for an arrangement of parallel cascades each having unequal linear elements.

If, however, the elements are equal, equation (4.9) needs modification. For example, if a cascade has all equal elements, $k_1 = k_2 = . . . = k_N = k$, then its IUH is

$$h(t) = \frac{1}{k\Gamma(N)} \left(\frac{t}{k}\right)^{N-1} \exp(-t/k) \qquad (4.10)$$

In most cases, some elements will be equal and some unequal. Then equations (4.7) and (4.10) can be combined appropriately. Because the elements are linear, their positions can be re-arranged. That is, in a given cascade, all equal elements can be connected on one side and all the remaining unequal elements on other side.

4.2 CONTRAST WITH PROBABILISTIC APPROACH

The probabilistic approach leading to the IUH is analogous to its deterministic counterpart. Both modes are comparable and equally flexible in virtually all respects. The hydrology of an element in a path (or cascade) is assumed in terms of its IUH in the deterministic mode and in terms of probability density function (pdf) in the stochastic mode. For example, assuming an element to be a linear reservoir with lag time k yields a negative exponential IUH. This is equivalent to saying that the pdf of the lag time of element is negative exponential. The notion of probability arises from the observation that this element represents an average behavior of an ensemble of portions of the watershed. Likewise, assuming an element to behave like the rational equation is equivalent to stating that its IUH is uniform over a finite period. Stochastically, this implies that the pdf of its lag time is a uniform distribution.

4.3 A COMPARISON WITH THE LWBN MODEL

The geomorphic and LWBN models share several common features. Both treat the watershed as a quasi-distributed system. The degree of lumping is considerably greater with the proposed model than will the LWBN model. Both contain parameters that can be estimated from geomorphic attributes in essentially the same fashion. Both models are linear and time invariant. Although the concepts in representing a watershed by a network of planes and channels are different for the two models, their operation is identical. The proposed model prescribes a unique network representation of a watershed while the LWBN model does not. Of course, depending upon the watershed size the representations can be made less detailed. However, the degree of lumping can be achieved more objectively with the proposed model than with the LWBN model.

4.4 QUANTIFICATION OF THE EFFECT OF LAND USE CHANGES

Land use changes affect virtually the entire spectrum of streamflow characteristics. The effect on response time characteristics is discussed in Chapter 3. Here, we discuss the effect on hydrograph characteristics with particular reference to IUH characteristics. To this end we consider the case of a single linear reservoir. The peak of its IUH, h_p, is

$$h_p = \frac{1}{k} \tag{4.11}$$

It is clear that as land use changes from forest to agricultural to urban, the value of k declines meaning thereby an increase in h_p. For example, if k is reduced by 10 percent then h_p increases by more than 11 percent. If k is reduced by 20 percent then h_p increases by 25 percent. k will be large for forest watersheds but a lot less for urban watersheds. This then says that as watersheds are urbanized the potential for flooding increases. This simple analysis can be extended to the case where the IUH is represented by equation (4.9). This, of course, requires computer simulation. A paucity of data did not allow execution of this aspect of study.

5. CONCLUDING REMARKS AND SUGGESTIONS

The following conclusions are drawn from this study:

1. The existing models of watershed response time characteristics are adequate for determining time of concentration, lag time, etc.
2. The existing models of hydrograph synthesis are less than adequate for ungaged basins and require modifications.
3. These models are not entirely satisfactory for quantifying the effect of land use changes on streamflow characteristics.
4. Sufficient hydrologic data showing the effect of land use changes on streamflow are not easily accessible.
5. The proposed geomorphic model is suitable for ungaged basins.
6. The proposed model can be employed to quantify the effect of land use changes. However, it remains to be validated to that effect.

The following suggestions are advanced for further work:

1. A data bank of land use changes, hydrologic characteristics and drainage basin characteristics need to be developed. This should be made available to whoever may wish to use. This will provide a sound basis for testing and reliability analysis of hydrologic models.
2. How much geomorphic detail is needed by the geomorphic model? This aspect is important and requires further investigation.
3. What is the effect of spatial variability of rainfall on geomorphic model results? This is required for deciding if the watershed can be treated as one unit or a collection of subunits.
4. What is the relation between geomorphic parameters and land use changes? This is crucial for assessing the impact of these changes on stremflow.

5. A practical procedure is needed for computing the amount of runoff due to a given rainfall event.

6. A procedure for quantifying the antecedent moisture condition is vitally needed.

7. The model for computing the amount of runoff should be coupled with the geomorphic mdoel. This is a realistic way to synthesize streamflow.

8. A simple model is needed for computing baseflow. This should be integrated with 7. above.

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