

*The Measurement and Comparison of Costs
for Alternative Water Replacement Projects*

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LOUISIANA STATE UNIVERSITY

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ABSTRACT

THE MEASUREMENT AND COMPARISON OF COSTS
FOR ALTERNATIVE WATER REPLACEMENT PROJECTS

Water replacement projects may be undertaken by water users individually or by collective community action. Cost is a primary consideration in the choice of alternative projects. The cost includes delivery (to user) components of the project water and cost of water, which may be viewed as an interest-free loan sufficient to cover all costs. This may be computed by distributing the cost and interest over the life of the project.

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ABSTRACT

THE MEASUREMENT AND COMPARISON OF COSTS FOR ALTERNATIVE WATER REPLACEMENT PROJECTS

Water replacement projects may be undertaken by water users individually or by collective community action. Cost is a primary determinant in the choice among alternative projects. The cost concept relevant for such comparisons is the present value unit cost of water, which may be viewed as an internal price just sufficient to cover all costs. This may be computed by discounting all costs and water outputs by the cost of capital to the sponsoring agency.

Calculations indicate that the non-profit community project can supply water to corporate users at a lower cost than that at which corporate users can supply their own needs. This is attributed to a) the corporate income tax structure, and b) the lower cost of capital to the community project. The results of detailed calculations for a hypothetical project are presented in tabular form. In addition, a graph is included that shows the extent to which construction costs for the community project must exceed those for the corporate project to equalize the costs of water from the two alternatives.

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INTRODUCTION

Geological studies conducted during the past twenty years indicate that the Baton Rouge, Louisiana, area is confronted with a ground-water supply problem of potentially serious proportions. Successive water samples taken from each of a number of wells in the area have increased in chloride concentrations, and analysis of the data has led to the conclusion that there is a movement of salt water northward toward the pumping centers of the Baton Rouge industrial district in some, if not all, of the ten aquifers currently supplying ground water to the area [Morgan and Winner, 1964; Meyer and Rollo, 1965]. The presence of a fault south of the industrial district may block the movement of salt water to the major pumping centers in one or two of the lowest water bearing formations. There is also a possibility that faults to the north of the industrial district may prevent or inhibit movement of water from the outcrop areas to the center of offtake. In either instance (salt water intrusion or inadequate recharge) existing supplies may have to be replaced, either in whole or in part, from new sources of fresh water.

The water supply problem is particularly crucial to the Baton Rouge area because its industrial base is a petro-chemical complex, which requires large volumes of water of varying purity. In 1965, for example, the total volume of ground water pumped was 102 million gallons per day, of which approximately 72% was for industrial use. If the industrial growth of the Baton Rouge area is to continue at its present pace a large and low cost supply of satisfactory water is required. It would appear to be prudent, therefore, to initiate advanced planning in the event that all, or even part, of the existing water supply must be replaced.

To this end, the Louisiana Water Resources Research Institute, in informal association with the U. S. Geological Survey, is now (1966) engaged in the first phase of a study of the Baton Rouge water supply. The Baton Rouge Ground Water Study is an interdisciplinary project that is investigating the local problem on several fronts:

1. Aquifers underlying the region are being studied to determine the existing geological and hydrological conditions, with the purpose of delineating the physical nature of the problem.

2. A number of practical engineering solutions are being sought, and overall cost estimates are being prepared for each of the possible solutions.

3. Alternative proposals will be evaluated to determine their economic and legal feasibility.

The major water users of the Baton Rouge area own and operate their own wells. It may be assumed, therefore, that in the absence of a single supplier furnishing water at a lower unit cost, the major users will continue to produce their own water from alternate sources if their present supplies are destroyed or downgraded. Thus the economic phase of the Baton Rouge Ground Water Study is concerned with gathering cost data from the major users in an attempt to assist them in estimating the cost of their best alternative replacement supply. These estimated costs will further serve as benchmarks for evaluating the feasibility of any proposed community projects. From a purely economic standpoint, a proposed community project would be considered feasible only if it can provide water at a lower unit cost to the user than the best private alternatives. Moreover, among the possible community projects, that project which can furnish water at the lowest unit cost will be considered optimal from this pragmatic economic standpoint. It should be recognized at the outset that the project which is optimal in the sense of yielding the lowest unit cost may not, because of legal or other considerations, provide the most feasible overall solution. However, cost considerations will play an important part in evaluating alternative proposals.

The purpose of this paper is to present a practical method for comparing water replacement costs. The only comparison to be made is between replacement of a part of the present water supply by individual private corporations with replacement of the same volume of water by a non-profit community project. The general method outlined will be used by the Baton Rouge study group in

evaluating alternative community project proposals, and in comparing the cost of community project proposals with the cost of private replacement projects.

All too often treatments of the capital budgeting problem focus attention on the problem of discounting net revenue streams, while the problem of evaluating investment projects for which there are no objective revenue streams receives scant attention. It is this second problem that is the concern of this paper.

The general method of measuring the costs of alternative projects is developed in the first section of this paper. Unit costs of the community and corporate projects are then compared in the second section. Finally, the third section provides an example of cost comparisons between corporate and community water replacement projects. The problem of uncertainty is not discussed. In defense of this neglect we can only offer the statement: "The fundamental difficulty of uncertainty cannot really be dodged; and since it cannot be faced, it must simply be ignored." [Solow, 1963]

THE GENERAL METHOD OF MEASURING COSTS

The basic operating premise is that at least a part of the existing water supply must be replaced. It follows, therefore, that while a knowledge of the cost of current supplies is not relevant to the problem at hand, an estimate of the unit costs of alternative replacement projects is pertinent. The estimation and comparison of such unit costs presents several problems inasmuch as costs are incurred at different points in time and must be discounted to be made comparable. What is the appropriate rate of discount? How does the tax structure affect the evaluation of costs for a corporate project and the comparison of costs between corporate and community projects? Should discrete or continuous discounting be used in the computations? Each of these questions is discussed in turn.

The Present Value of Future Costs

Costs that are incurred at different points in time have different values when viewed from the present. To illustrate, let C_0 be a cost arising at the present time and let C_t be a cost arising t years in the future. If C_0 could be postponed for t years, and the consequent saving be invested at the rate of $r\%$ compounded annually, it follows that after t years the original sum C_0 will have a value of $C_0(1+r)^t$. Thus to put the costs C_0 and C_t on an equal footing C_t must be equal to $C_0(1+r)^t$. Alternatively, the present value of a cost incurred in year t is defined as

$$C_0 = PVC_t = \frac{C_t}{(1+r)^t} \quad (1)$$

The present value of a future cost (PVC) may be interpreted as the amount that if set aside now and invested with an annual rate of return of $r\%$ would provide a sum just sufficient to meet the cost when it is incurred.

In general, the present value of a stream of yearly costs, C_t , is given by the formula

$$PVC = \sum_{t=1}^n PVC_t = \sum_{t=1}^n \frac{C_t}{(1+r)^t} \quad (2)$$

Since cost in year t is discounted for the entire year t , as well as for the preceding $t - 1$ years, it is seen that this formula treats costs as being incurred on the last day of the year. Thus annual discounting will understate the present value of future costs; this problem will be dealt with more fully in a subsequent section.

Immediate cash outlays, such as initial costs of construction, should not be discounted. Thus to encompass immediate cash outlays in the general formula it is necessary to sum from zero to n , where costs at time zero, C_0 , represent immediate cash outlays. Formula (2) then becomes

$$PVC = \sum_{t=0}^n \frac{C_t}{(1+r)^t} = \sum_{t=0}^n C_t (1+r)^{-t} \quad (3)$$

Now, the present value unit costs for both a non-profit community project and a profit-making corporation can be calculated by an adaptation of the above method. Let x_p represent the unknown present value unit cost of water from a community project which is assumed to remain constant over the life of the project, and let q_t represent the quantity of water produced in year t . Then total cost for the project in year t will be $x_p q_t$. Similarly, if x_c (also assumed to be constant) represents the unknown present value unit water cost of a company project producing q'_t units of water in year t , then total cost for the company project in year t will be $x_c q'_t$. It follows that for a community project with a life of n years the sum of present value costs will be equal to the sum of quantities produced in each year multiplied by the present value unit cost and discounted by the appropriate factor. Symbolically,

$$\sum_{t=0}^n PVC_t = \sum_{t=0}^n (x_p q_t) (1+i)^{-t} \quad (4)$$

where i is the rate of discount appropriate to the community project. Since x_p is a constant it may be factored out of the summation and upon dividing both sides by the sum of discounted quantities we have

$$x_p = \frac{\sum_{t=0}^n PVC_t}{\sum_{t=0}^n q_t(1+i)^{-t}} \quad (5)$$

If k is the appropriate discount rate for the corporation, the present value unit cost for the corporate project with a life of n years is

$$x_c = \frac{\sum_{t=0}^n PVC_t}{\sum_{t=0}^n q'_t(1+k)^{-t}} \quad (6)$$

It may be helpful to take a somewhat different approach and to view the present value unit cost as an internal price which the company, or community project, charges itself for water. When so interpreted, this internal price multiplied by quantities produced in each year generates a revenue stream which, when discounted, is equal to the present value of all costs. Thus this internal price (present value unit cost) is a price just sufficient to cover all costs associated with the project. Moreover, the rate of discount can be interpreted as the rate of return over cost, or simply the net rate of return, on the investment outlay. This latter interpretation may seem rather strained since the discount rate must be selected first and once selected determines (together with the cost and quantity streams) the present value unit cost of water. However, this interpretation does serve to elucidate the fact that the rate of discount must bear some relationship to the net rate of return that a firm expects to realize on alternative investment projects.

The Cost of Capital as the Relevant Rate of Discount

To say simply that the discount rate used to calculate present value unit cost must reflect the net rate of return on alternative investment projects is

far too vague. We submit that the appropriate rate of discount for either a community or a corporate project is the cost of capital to the sponsoring body. The cost of capital for the non-profit community project is simply the interest rate at which the project is financed.

The cost of capital for a profit-making corporation is not so readily determined. Indeed, the very concept 'cost of capital' as applied to a profit-making corporation raises several theoretical issues that have as yet not been satisfactorily resolved [Modigliani and Miller, 1958; Durand, 1959]. While an attempted resolution or even a complete treatment of these issues is beyond the scope of this paper, a brief discussion of some of the problems is necessary.

As a starting point, it should be noted that a corporation has a number of alternative sources of funds; it may, for example, retain earnings, utilize depreciation allowances, borrow from financial institutions, issue short- or long-term debt, or issue new shares of either common or preferred stock. Each of these sources has a different objective cost. Since a water replacement project will be long-lived only four of these alternative sources of funds need be considered; issuance of new shares, long-term borrowing, issuance of preferred stock, and internal sources (retained earnings and depreciation allowances).

Normally, the cost of equity capital is calculated either as an expected earnings per share/price per share ratio [Dean, 1951] or by some variation of a dividend/price ratio with allowance made for expected dividend growth [Bierman and Smidt, 1960]. The cost of borrowed capital is uniquely dependent on two factors: the effective rate of interest at which the corporation can borrow and the corporate income tax rate. Inasmuch as interest payments are expenses deducted from revenue in arriving at taxable income, the cost of borrowed capital is not the full amount of the effective interest rate, but rather the interest rate reduced by the marginal tax rate multiplied by the interest rate. In short, the cost of debt capital is obtained by multiplying the effective interest rate by one minus the marginal tax rate. The effective rate of interest is readily calculated as the current yield to maturity on the

corporation's long-term debt, adjusted, when necessary, for underwriting costs associated with a new debt issue. The cost of preferred capital is uniquely determined by the current yield on preferred stock.

The cost of capital from internal sources requires more detailed treatment. It might seem at first glance that since internally generated funds require no explicit cash outlay their cost is zero. This, however, is not the case. The alternative to retaining earnings and using these funds to finance an investment project is the payment of dividends. If the financial well-being of shareholders can be taken to be the overriding concern of corporate directorships, then the decision of whether to retain earnings or to pay dividends should hinge on the effect that each of these alternatives has on the net worth of the shareholder. Specifically, an investment project financed by internal funds should be undertaken only if it increases the market value of corporate shares by an amount at least as great as the dividend foregone. Since the market evaluates shares on the basis of expected earnings, the increase in the market value of shares will be greater than the investment outlay only if the expected net rate of return on the investment outlay is greater than the ratio of expected earnings to the price of shares, i.e. the cost of equity capital. Thus the cost of equity capital is also the appropriate cost of capital to be applied to internal sources. Moreover, although the argument has been couched in terms of retained earnings, the same argument applies to depreciation allowances. If the use of such funds does not provide a rate of return at least as great as the cost of capital then such funds should not be retained [Cohen and Robbins, 1966].

To restate the argument symbolically: let E represent expected annual earnings; M , the total market value of the corporation's shares; then r , the cost of capital will be equal to E/M . In a perfectly functioning capital market, the cost of equity capital may be taken as a constant with respect to a change in expected earnings, because an increase in expected earnings will lead to an increase in the price of shares, as the market discounts the expected increase in earnings. Thus we may write

$$\Delta M = \frac{\Delta E}{r} \quad (7)$$

Let ΔI represent the increase in investment outlay (foregone dividend) and let $\Delta E'$ represent the resulting increase in expected annual earnings on the investment outlay which, for purposes of exposition, we assume to be a perpetual flow, then

$$\Delta I = \frac{\Delta E'}{i} \quad (8)$$

where i is the expected rate of return on the investment outlay. Now, it can be seen that if knowledge of the investment project is communicated to the market and the market evaluates earnings expectations in the same way as corporate management ($\Delta E = \Delta E'$), $\Delta M > \Delta I$, only if $i > r$. In words, the change in the market value of the corporation's outstanding shares is greater than the investment outlay, or foregone dividend, only if the rate of return on investment is greater than the cost of capital.

One is now tempted to conclude that the cost of capital will depend upon the method of finance. If an investment outlay is financed by equity capital or from internal sources one rate would seem applicable, if by long-term debt or preferred stock still other rates seem applicable. Again, this is an unwarranted conclusion. The method of financing an investment project will have an effect, even if only a minor one, on the capital structure. Internal or equity financing will lower the debt/equity ratio and conversely debt financing will raise it. Under the assumption that each corporation has an optimal capital structure that it seeks to achieve and maintain, investment outlays must be financed in such a way as to leave the capital structure unaffected. In other words, it is argued that the decision to alter the capital structure is a separate decision. This leads to the conclusion that there is but one cost of capital which must be a weighted average of the cost of equity and senior capital [Donaldson and Pfahl, 1963]. Moreover, if one assumes that the corporation has achieved its optimum capital structure, or what it conceives to be its optimum capital structure, the appropriate weights are the current

percentages of the corporation's capital provided from debt and equity sources. Thus, although a particular project may be financed by the issue of debt, for example, maintenance of the desired capital structure will require that other projects be financed by equity capital.

In the preceding paragraphs it was shown that the cost of capital that is the applicable rate of discount for a long-term investment project is a weighted average of the costs of equity and senior capital. It should be recognized that this conclusion rests on several arbitrary assumptions. First, it was assumed that the overriding objective of the corporation is the maximization of the net worth of its shareholders. The second assumption was that shareholders are indifferent between an equal increase in dividends and in the market value of shares; an assumption which because of differential tax rates on income and capital gains is not likely to be realized [Gordon, 1962]. Third, it was assumed that there exists an optimum capital structure for any corporation which the corporation achieves and seeks to maintain. Even if these assumptions are accepted, the cost of equity capital depends upon expectations with respect to earnings. Thus a cost of capital can only be calculated by making some arbitrary assumption with regard to earnings or dividend expectations. Consequently, there emerges no uniquely determined single rate that may be taken to represent the 'true' cost of capital.

One way out of this dilemma is to make the necessary assumptions regarding expected earnings, calculate the expected earnings/price ratio, determine the cost of debt capital, apply the appropriate weights, and call the resulting figure the cost of capital. The other alternative is to compute a range of rates, representing a confidence interval, so to speak, for the 'true' cost of capital. We propose to combine both approaches, presenting first a point estimate of the cost of capital based upon the trend of earnings and earnings/price ratios in the recent past; and, second, testing the sensitivity of present value unit cost of water to a range of discount rates. The 'cut-off point' that the corporation employs in deciding whether or not to undertake an investment project will serve as an upper bound for this range, since this is the rate below which,

risk considered, the corporation feels that a given investment would be detrimental. The cut-off rate of return may be regarded as the corporation's own implicit estimate of its cost of capital, perhaps inflated by a risk premium. The current yield to maturity on the corporation's long-term debt reduced by the corporate income tax rate, i.e. the cost of its debt capital may be taken as the lower bound.

The use of a range of rates, particularly as broad a range as this one, to represent the cost of capital has a distinct disadvantage, both in comparing alternative projects within the same corporation and for comparing a corporation and a community project. First, for two alternative company projects, one involving high construction costs but low operating and maintenance costs, and the other involving a smaller initial expense but higher costs in subsequent years it is quite possible that discounting by the highest rate will yield the lowest present value unit cost for the second project, whereas discounting by the lowest rate may reverse the ranking. In such cases, the economist may take some comfort in the fact that ultimately the decision between the two projects must be made by corporate management. Moreover, in deciding between the two projects corporate management not only provides the economist with what it considers to be the best alternative, but also narrows the range within which it regards its appropriate cost of capital to lie. In the absence of a decision by management, the economist is justified in placing more confidence in rates of discount lying closer to the upper than to the lower bound.

If the single rate representing the cost of capital for the community project results in a present value unit cost that lies outside of the limits described previously, the answer is unambiguous. In one case the community project is more expensive than the corporate project; in the other, it is less expensive. If the present value unit cost of the community project lies between these limits, a problem does indeed exist. We can offer no a priori rule for resolving this problem, but suggest that if such a problem arises, it can be resolved only by a careful examination of the circumstances peculiar to that particular case. The analysis contained in the following sections suggests, however, that this problem is not likely to be encountered in practice.

Discrete or Continuous Discounting?

In introducing the concept of present value cost, it was pointed out that the formula employed implicitly treats costs as being incurred at the end of each year. Moreover, the formula for present value unit cost treats the entire production of water as taking place on the last day of the year. In reality, the pumping of water will continue at a more or less even pace throughout the entire year and costs must be met on a monthly or even weekly basis. Therefore, the practice of discounting costs and quantity for the entire year, although understating the present value of costs, overstates the 'true' present value unit cost.

This problem is not serious, however, and the objection can be met by shortening the time period over which costs and quantities are discounted from years to months or even months to weeks. In general the present value (C_0) of a cost incurred in year t (C_t) discounted at an annual rate r is

$$C_0 = C_t (1 + r)^{-t} \quad (9)$$

If C_t is discounted α times per year at a rate $\frac{r}{\alpha}$ (9) becomes

$$C_0 = C_t \left(1 + \frac{r}{\alpha}\right)^{-\alpha t} \quad (10)$$

and as is well known

$$\alpha \lim_{\rightarrow} \infty C_0 = C_t e^{-rt} \quad (11)$$

In the limiting case, therefore, C_t is discounted continuously; a procedure that is appropriate to the view that costs are spread over the entire year [Grant and Ireson, 1960].

Continuous discounting is certainly preferable to annual discounting. As an alternative to continuous discounting, however, costs may be discounted monthly or even weekly. This latter alternative is appropriate to the view that costs must be met only at certain points in time (e.g. monthly or weekly payrolls). Moreover, the convergence of discrete to continuous discounting is quite rapid, so that the difference between present value unit costs

resulting from weekly or even monthly discounting as an alternative to continuous discounting is quite small.

Present Value Unit Cost: Community Project

The composition of the cost stream and hence the present value of costs in each case must be considered. Before a comparison can be made between the present value unit costs of a community and a corporate project, the total cost (C_t) associated with a water project in any year t may be divided into three categories: construction cost (K_t), cost of operation and administration (A_t), and maintenance cost (M_t). In addition there may be some cost offsets such as salvage value (S_t). Thus total cost in any year t is obtained by summing the three types of cost and deducting cost offsets. Symbolically,

$$C_t = K_t + A_t + M_t - S_t \tag{12}$$

Normally construction costs are heavily concentrated toward the beginning of the project, whereas cost offsets occur toward the end of the project's life.

The present value of all costs of the community project is obtained by discounting cost in each year by the cost of capital and summing over the life of the project. Present value unit cost is then obtained by dividing through by the sum of discounted quantities. Thus discounting (12) by the cost of capital, summing to obtain the present value of costs and substituting the result into (5) gives [Hirshleifer et al., 1960]

$$x_p = \frac{\sum_{t=0}^n (K_t + A_t + M_t - S_t) (1+i)^{-t}}{\sum_{t=0}^n q_t (1+i)^{-t}} \tag{13}$$

Present Value Unit Cost: Corporate Project

Computation of the present value cost for the corporate project is, however, not so straightforward. A cost incurred by a profit-making corporation reduces revenue by an amount equal to the cost, but, and this is the important point,

it also reduces the corporate income tax liability. Thus the relevant cost figure for the corporation is not the full amount of the cost, but only that part of the cost that reduces after tax revenue, which we shall refer to as the net cash outlay resulting from the cost. For operation cost (A'_t) and maintenance cost (M'_t) the reduction in after tax income in year t will be $A'_t + M'_t - T_t(A'_t + M'_t)$ which is equal to $(1 - T_t)(A'_t + M'_t)$, where T_t represents the marginal corporate income tax rate in year t . By the same reasoning, a cost offset (S'_t) that increases income by an equivalent amount also increases the corporation's tax liability, so that the increase in after tax income, or the reduction in net cash outlay, is only $(1 - T_t)S'_t$.

Construction costs (K'_t) are treated differently. In that such costs must be capitalized and then depreciated over the life of the asset, it is appropriate to include the full amounts of construction costs in the year, or years, in which they are incurred. In subsequent years an allowance for depreciation (D_t) appears as a cost offset. Since the allowance for depreciation allows the company to write down its taxable income by an equivalent amount, the resulting reduction in the tax liability is $(T_t)(D_t)$. Combining the results of this and the preceding paragraph, the net cash outflow in year t is then

$$C'_t = K'_t + (1 - T_t)(A'_t + M'_t - S'_t) - (T_t)(D_t) \quad (14)$$

Present value unit cost of water for the company project is now obtained in the same manner as for the community project; discounting (14) by the cost of capital and summing to obtain present value cost and then substituting the result into (6) to obtain

$$x_c = \frac{\sum_{t=0}^n [K'_t + (1 - T_t)(A'_t + M'_t - S'_t) - (T_t)(D_t)] [(1+k)^{-t}]}{\sum_{t=0}^n q'_t (1+k)^{-t}} \quad (15)$$

A COMPARISON OF CORPORATE AND COMMUNITY PROJECTS

For the purpose of comparing company and community projects, we will make two assumptions: first, the management of the community project is neither more nor less efficient than the managements of corporate projects; and second, that there are constant returns to outlay in the production of water. The first assumption requires no comment. The second is necessitated by the fact that the community project, supplying water to a group of water users will be larger than any single corporate project. Under the assumption of constant returns to outlay, corporate projects may be viewed in one of two ways. First, they may be thought of as one large corporate project composed of a number of individual projects providing, in total, the same quantity of water as the community project and hence having the same costs. Alternatively, the corporate project may be viewed as the project of only one water user, but being identical to the community project reduced by an appropriate scale factor. If the second view is adopted then, although each cost for the corporate project will be smaller, the output will be reduced proportionately. Thus in either case, the primes in formula (15) can be eliminated in comparing the corporate and community projects.

If we make the further initial assumption that the cost of capital is the same for the community as for the corporate project ($i = k$), then substituting i for k in formula (15) and forming the ratio

$$\frac{x_c}{x_p} = \frac{\frac{\sum_{t=0}^n [K_t + (1-T_t)(A_t + M_t - S_t) - (T_t)(D_t)] [(1+i)^{-t}]}{\sum_{t=0}^n q_t (1+i)^{-t}}}{\frac{\sum_{t=0}^n (K_t + A_t + M_t - S_t) (1+i)^{-t}}{\sum_{t=0}^n q_t (1+i)^{-t}}} \quad (16)$$

$\sum_{t=0}^n D_t (1+i)^{-t}$ relative to K_0 , thus reducing the numerator of (19) relative to the denominator, which will narrow the difference between the present value unit costs of the corporate and community projects. Finally, the use of accelerated depreciation by concentrating depreciation in the early years of the project will raise the numerator relative to the denominator, therefore widening the difference between present value unit costs. In the extreme case, if the company were allowed to write off the entire investment at the time it occurred, and if $S_t = 0$ for all t

$$\sum_{t=0}^n D_t (1+i)^{-t} = K_0 \quad (23)$$

and x_c (the present value unit cost of the corporate project) would equal $(1 - T_t)x_p$. It should be noted that the existence of an investment tax credit, allowing the company to take a tax offset equal to a certain fraction of the investment in the first year also widens the gap between present value unit costs. The failure to include the currently applicable investment tax credit in the calculation of the present value unit cost of water for the corporate project, thus biases the results in favor of the community project. This bias is not serious, however, and the results are not materially affected [See Appendix].

The most important conclusion stems from Equation (22): Although the present value unit cost of the corporate project is always less than that of the community project, it is greater than the complement of the tax rate times the present value unit cost of the community project. The importance of this conclusion is that when a corporation purchases water from an outside source of supply it incurs a cost equal to the purchase price (x_p), but since it also decreases its tax liability by an equal amount, the net cash outlay is $(1 - T_t)x_p$. But, this has been shown to be less than the cost at which the corporation can supply water.

The basic conclusion is, then, that when the assumptions of this section are satisfied, the corporation, or group of corporations, can meet their water

needs at a lower cost by purchasing water from the non-profit community project than by providing their own supplies. Again, this result depends upon the assumptions of constant returns to outlay and on equality of the cost of capital between the corporate and community projects.

To the extent that economies of scale are present in the production of water the conclusion that the community project can supply water at a lower unit cost is strengthened. If, on the other hand, diseconomies of scale are important, then the preceding analysis has overstated the advantage of the community project. However, it is shown (Figure 1, p.25) that diseconomies of scale must indeed be substantial before the basic conclusion is overturned.

The community project's cost of capital will almost inevitably be lower than the cost of capital to the corporate project. If the community project is an arm of a state or municipal government, this will mean that the community project will be enabled to issue tax exempt bonds, and hence in most circumstances be able to borrow at a lower rate than the corporation. In short, the federal and state corporate income taxes combined with a lower cost of capital for a community project provide a cost advantage to the community project, which can be offset only by strong diseconomies of scale in the production of water.

PRESENT VALUE UNIT COST COMPARISONS: AN EXAMPLE

A general method was developed for calculating and comparing costs for alternative water replacement projects. It now remains to provide an application of this method to illustrate the significance of the conclusions.

We again start with the assumption of constant returns to outlay. The corporate project may then be viewed as an agglomeration of company projects having total cost and water output streams identical to those of the community project. It is further assumed that either project must replace a portion of the groundwater supply (45,000 gallons per minute or 23,652 million gallons per year) at a given point in time (time zero); and that the demand for water will remain constant at this level over the life of the project. Construction costs for the project, or group of projects, are treated as being incurred at time zero. The total of such costs is \$10 million. Facilities are sold for a net salvage return of \$100,000 at the end of year 25 (the project's terminal date). Operating costs are assumed to be directly related to output, and, at 1.7¢ per thousand gallons, are \$402,000 annually. Maintenance costs are treated as a constant annual percentage (2%) of construction costs and thus total \$200,000 a year.

The Non-Profit Community Project

Calculation of the present value unit water cost of the community project is now straightforward, by either the discrete or continuous methods. Using the data from the preceding paragraph, formula (13) becomes

$$x_p = \frac{10,000,000 + \sum_{t=1}^{25} (402,000 + 200,000) (1+i)^{-t} - 100,000 (1+i)^{-25}}{\sum_{t=1}^{25} 23,652 (1+i)^{-t}} \quad (24)$$

which reduces to

$$x_p = \frac{10,000,000 + 602,000 \frac{(1+i)^{25} - 1}{i(1+i)^{25}} - 100,000 (1+i)^{-25}}{23,652 \frac{(1+i)^{25} - 1}{i(1+i)^{25}}} \quad (25)$$

where x_p represents the cost per million gallons.

Table 1 provides unit costs for rates of discount (costs of capital) of 4, 6, 8, and 10%, obtained by both annual and continuous discounting. If, as we have argued, it is appropriate to view costs as being spread uniformly over the year, discrete annual discounting overstates unit costs by amounts ranging from 31¢ to 52¢ per million gallons. This discrepancy, however, is not large--less than 1% of the unit cost at each rate of discount.

TABLE 1. Community Project: Water Costs at Selected Rates of Discount

Discount Rate	Present Value Unit Water Cost Per Million Gallons		Average Annual Cost
	Annual Discounting	Continuous Discounting	Continuous Discounting
.04	\$52.42	\$52.11	\$1,232,506
.06	58.45	58.03	1,372,526
.08	65.00	64.52	1,526,027
.10	71.99	71.47	1,690,408

The sensitivity of unit cost to the rate of discount is substantially more important than its sensitivity to the method of discounting. An increase in the cost of capital (rate of discount) from 4 to 6% raises unit costs by \$5.92 per million gallons (if continuous discounting is employed), while an increase in the cost of capital from 4 to 10% raises the cost per million gallons by \$19.36. The average annual cost rises by \$457,902 as the cost of capital is increased from 4 to 10%.

The Corporate Project

The unit costs of the corporate project differ from those of the community project as a result of corporate income tax and depreciation. It is assumed

that each corporation participating in the corporate project has an annual income in excess of \$25,000 so that the marginal federal income tax rate of 48% and the Louisiana state corporation income tax of 4% may be applied to the entire project. Inasmuch as each level of government allows the deduction of the tax of the other in arriving at taxable income, the tax rate (T) is calculated as follows

$$T = (96\%) (48\%) + (52\%) (4\%) = 48.16\%$$

The tax rates are assumed to remain constant over the life of the project. Again, no allowance is made for an investment tax credit.

Calculation of depreciation is complicated by the existence of more than one allowable method. If it could be assumed that each participating company employed the 'straight line' method of depreciation, then annual depreciation would be equal to 4% of construction costs after deducting salvage value. This assumption, however, lacks realism. A more realistic first approximation is to assume that each company uses 'double declining' depreciation as long as the resulting allowance is greater than that which could be obtained by straight line depreciation applied to the undepreciated balance net of salvage. Thus for each year depreciation is 8% of the undepreciated balance at the beginning of the year, until that year is reached when a greater allowance can be obtained by the straight line procedure. A third alternative would have been to use the 'sum of the years-digits' method of depreciation. If this method had been employed, the results would not have been materially different [See Appendix].

Table 2 presents computations of unit costs for the corporate project. The figures were obtained by substituting the data of this section into formula (15). As in the case of the community project, it is seen that unit costs are slightly overstated by the method of discounting annually, and that they are highly sensitive to variations in the discount rate. The average annual cost of water ranges from \$732,029 when costs and outputs are discounted at 4% to \$1,154,927 when discounted at 10%.

TABLE 2. Corporate Project: Water Costs at Selected Rates of Discount

Discount Rate	Present Value Unit Water Cost Per Million Gallons		Average Annual Cost Continuous Discounting
	Annual Discounting	Continuous Discounting	
.04	\$31.27	\$30.95	\$ 732,029
.06	36.86	36.41	861,169
.08	42.97	42.43	1,003,554
.10	49.55	48.93	1,157,292

A Comparison of Costs

Comparison of Tables 1 and 2 reveals that the corporate project can apparently produce water at a lower unit cost than the community project. In fact, even if the cost of capital were 10% for the corporate project and only 4% for the community project, the unit cost of water would appear to be less for the company project. As shown in the first section of this paper, this conclusion is a result of the tax structure.

The relevant comparison, however, is not between the apparent unit water cost of the corporate and community projects, but between the cost to the company of purchasing water from the community project and the cost of supplying its own needs. It will be recalled that the company's cost of purchasing water from the community project is equal to one minus the corporate tax rate multiplied by the price which the community project charges. Thus the comparison made in Table 3 is between the effective cost to the company (51.84% of the unit cost of the community project) and the company's own cost. It is to be noted, that the corporation's cost of purchasing water is lower than its cost of producing water at every rate of discount. Moreover, the difference between the costs of the two alternatives increases as the rate of discount rises--

ranging from a low of \$3.94 per million gallons at 4% to a high of \$11.88 at 10%. This in turn means that if both projects had a 4% cost of capital, the corporation would save 12.7% of the total cost by purchasing from the community project; if the costs of capital were both 10% the cost saving would be 24.3%.

TABLE 3. Corporation Cost of Water: Comparison between Producing Its Own Supply and Purchasing from the Community Project at Selected Rates of Discount

Discount Rate	Dollar Cost of One Million Gallons of Water to the Corporation			Percentage Difference
	Producing Own Supply	Purchasing from Community Project	Difference	
.04	30.95	27.01	3.94	12.7
.06	36.41	30.08	6.33	17.4
.08	42.43	33.45	8.98	21.2
.10	48.93	37.05	11.88	24.3

The effect of different rates of discount, or costs of capital, on the saving that the corporation realizes by purchasing from the community project rather than by supplying its own needs is shown in Table 4. It will be recalled from the first section that if the community project is an arm of a state or municipal government and can issue tax exempt revenue bonds, the cost of capital to the community project will be lower than that of the corporate project. Thus the more relevant entries in Table 4 are those below the principal diagonal. An effective interest rate of 4% is perhaps a reasonable figure for a state or municipally sponsored project, in which case column 1 of Table 4 is relevant for unit cost comparisons; showing the

cost saving per million gallons for selected corporate costs of capital. If the cost of capital to the corporation is 8%, for example, the corporation would realize a saving of \$15.42 per million gallons by purchasing water from the community project rather than by producing its own replacement supply.

TABLE 4. Corporation's Savings, in Dollars per Million Gallons, Resulting from the Purchase of Water from the Community Project at Different Rates of Discount

Corporate Cost of Capital	Community Cost of Capital			
	.04	.06	.08	.10
.04	3.94	0.90	-2.47	- 6.07
.06	9.40	6.33	2.96	- 0.64
.08	15.42	12.35	8.98	5.38
.10	21.82	18.75	15.38	11.88

The basic conclusion that it is less expensive for the corporation to purchase water from the community project rather than to supply its own needs is incorrect only if the cost of capital is substantially lower to the corporation than to the community project. Table 4 shows that only if the cost of capital to the community project is over 6% will a 4% cost of capital to the corporation make the corporation's cost of producing water lower than the effective cost of purchasing water from the community project. Alternatively, with a 6% corporation cost of capital, the cost of capital to the community project must be over 8% to make the corporate project feasible. These cases are indeed unlikely, and it may be concluded that such differences in the costs of capital as are likely to exist will increase rather than diminish the comparative advantage of the community project.

The analysis thus far has been conducted under the assumption of constant returns to outlay. It has been noted that economies of scale, or increasing returns to outlay, will strengthen the comparative advantage of the community

project. The possibility of diseconomies of scale bears some examination. How strong must diseconomies of scale be in order to overturn the basic conclusion? Figure 1 provides an answer to this question. The underlying assumption is that the corporate project is constructed at a total cost of \$10 million. Construction costs for the community project are measured along the vertical axis and costs of capital for the corporate project are measured along the horizontal axis. Each curve is drawn for a given cost of capital to the community project, showing what relationship between the cost of capital to the corporate project and the construction costs of the community project would equalize the unit cost of water to the corporation from the two alternatives. It should be noted that since annual maintenance costs are assumed to be 2% of construction costs, a higher construction cost implies proportionately higher maintenance costs. However, operating costs are assumed to be unaffected. The curves in Figure 1 are drawn to reflect the higher maintenance costs.

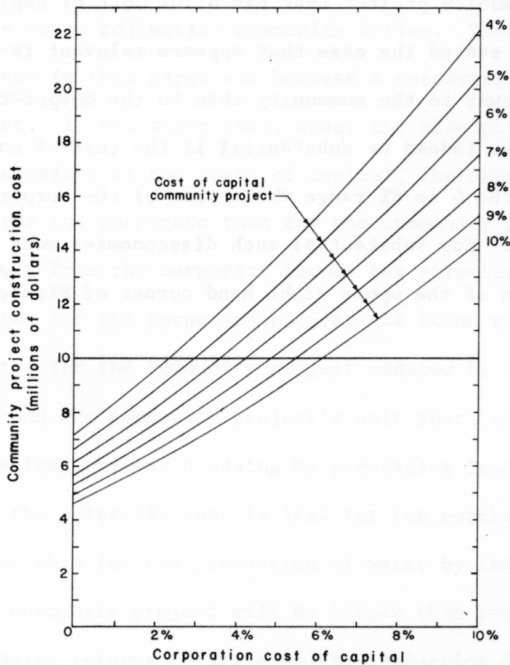


Fig. 1. Community Project Construction Cost Breakeven Points

At any given corporate cost of capital, the vertical coordinate of the curve representing a 4% cost of capital for the community project shows the level to which the community project's construction costs must rise before the cost saving is eliminated. Thus if the costs of capital are both 4%, a construction cost of \$12.2 million for the community project will equalize the effective unit water costs of both projects. Alternatively, a 10% cost of capital to the corporate project would, with a 4% community project cost of capital, require a construction cost of \$22.2 million to eliminate the cost saving. A similar interpretation attaches to each of the other curves.

The horizontal line representing a \$10 million construction cost for the community project shows that if construction costs for both projects are \$10 million (the case of constant returns to outlay), a 4% cost of capital to the community project requires a 2.4% corporate cost of capital to equalize unit costs. On the other hand, a 6.2% corporate cost of capital eliminates the advantage of the community project that has a 10% cost of capital.

For reasons already stated the case that appears relevant is that in which the cost of capital is lower to the community than to the corporate project. Diseconomies of scale must indeed be substantial if the cost of capital to the community project is in the 4 to 6% range while that of the corporate project ranges between 6 and 10%. How substantial such diseconomies must be is readily apparent upon examination of the upper right hand corner of Figure 1.

SUMMARY AND CONCLUSIONS

The basic proposition of this paper has been that cost is a primary determinant in deciding between alternative water replacement projects. Since water replacement projects are long-lived, and cost and output streams extending many years into the future are involved, costs and outputs must be discounted to obtain comparability. The cost figure relevant for comparison was called the present value unit cost of water, which was viewed as an internal price just sufficient to cover all costs associated with the water replacement project. It was further argued that the rate at which costs and outputs should be discounted is the 'cost of capital' to the organization undertaking the project. Although the 'cost of capital' has a definite meaning, the difficulties involved in its measurement require the use of a range of discount rates to approximate the 'true' cost of capital.

Water replacement projects may be undertaken by large water users on an individual basis or by collective community action. Thus the particular comparison undertaken in this paper was between a corporate project and a non-profit community project. It was shown that, under the assumptions of constant returns to outlay and equality of the costs of capital, the present value unit cost of water is lower for the corporate than for the community project. This conclusion was seen to result from the corporate income tax structure. However, since the unit cost of water for the corporate project was found to be greater than the unit cost of water for the community project reduced by the product of the corporate tax rate and the community project's unit cost, the basic conclusion was that the corporation achieves a saving by purchasing from the community project. In other words, the after-tax cost is less for the purchase of water from the community project than for the production of water by the corporate project. Inasmuch as the community project will be larger than individual corporate projects, increasing returns to scale in the production of water will strengthen the advantage of the community project. Decreasing returns to scale, on the other hand, will work to the advantage of the smaller corporate project. It was

further argued that such differences in the cost of capital as are likely to exist, will operate in favor of the community project.

The method developed was then applied to a numerical problem. Under the assumptions of constant returns to outlay and equality of the costs of capital, the percentage savings resulting from the corporation's purchasing water ranged from 12.8% of unit water costs at a 4% cost of capital to 24.1% at a 10% cost of capital. For cases likely to be relevant -- lower costs of capital to the community than to the corporate project -- the cost savings were substantially greater. Finally, it was shown that construction and consequently maintenance costs for the community project must be a great deal higher than those of the corporate project to eliminate the advantage of the community project. Thus, although diseconomies of scale will weaken the conclusion that it is more economical for the corporation to purchase water from the community project, they must be quite strong indeed before the conclusion is reversed. In short, the community project is found to have an inherent cost advantage resulting from the corporation income tax. Moreover, this advantage is likely to be strengthened by a lower cost of capital for the community than for the corporate project.

APPENDIX

THE EFFECT OF SUM OF THE YEARS-DIGITS DEPRECIATION AND THE INVESTMENT TAX CREDIT

Corporate project unit water costs were calculated by using the double declining balance method of depreciation and without making allowance for the currently applicable 7% investment tax credit. The purpose of this Appendix is to demonstrate the sensitivity of the conclusions to changes in the method of depreciation and the investment tax credit. In all of the calculations that follow, the marginal tax rate of 48.16% is used. It should be pointed out again, however, that future changes in the marginal tax rate would affect the comparative advantage of the community project; an increase in the rate would widen and a decrease narrow this advantage.

The federal income tax code of 1954 allows the taxpayer to use an accelerated method of depreciation for tax purposes. If the objective of a depreciation policy for tax purposes is to maximize the present value of tax savings, the taxpayer, in effect, has a choice between two alternative methods of depreciation in computing his tax liability: the declining balance method with a switchover to straight line, which was used in the text; or the sum of the years-digits (SYD) method. The relative advantage of one method vis-a-vis the other depends on the corporate cost of capital, salvage value, and the service life of the asset [Davidson and Drake, 1961]. The double declining balance method will yield a greater present value of tax savings if the corporate cost of capital is sufficiently high, the salvage value is a large enough proportion of the original investment, or if the service life of the asset is short.

In the example provided in the text and indeed for conditions that seem generally applicable to water replacement projects, the SYD method yields a higher present value of tax savings than does the declining balance method. The effect on unit water costs of substituting SYD depreciation for the double declining balance method is shown in the first two columns of Table A-1.

The SYD method yields a lower present value unit cost at each rate of discount. This cost differential is relatively small, however, being less than 1.4% even at the highest rate of discount.

TABLE A-1. Corporate Project: Water Costs at Selected Rates of Discount

Discount Rate	<u>Present Value Unit Water Cost Per Million Gallons</u>			
	<u>Excluding the 7% Tax Credit</u>		<u>Including the 7% Tax Credit</u>	
	Double Declining Balance	SYD	Double Declining Balance	SYD
.04	\$30.95	\$30.57	\$29.08	\$28.70
.06	36.41	35.89	34.12	33.60
.08	42.43	41.81	39.69	39.07
.10	48.93	48.25	45.71	45.02

The federal tax code changes enacted in 1962 and 1964 provide that, subject to certain qualifications, 7% of the funds invested in new productive facilities during any year may be deducted from that year's corporate income tax liability. Moreover, the deduction does not alter the yearly depreciation allowances which may still be taken on the full amount of the investment.

Comparison of columns one and three or two and four of Table A-1 isolates the effect of the investment tax credit on unit cost calculations. The existence of the tax credit lowers the cost per million gallons by \$1.87 at a 4% rate of discount and by \$3.22 at a 10% rate of discount. The combined effects of the SYD method of depreciation and the investment tax credit can be determined by a comparison of columns one and four of Table A-1. At an 8% rate of discount, for example, the 7% investment tax credit and the SYD method of depreciation combine to lower the unit cost by \$3.36 per million gallons.

In general, a corporate water user will still realize a saving by purchasing from the community project even though the SYD method of depreciation and inclusion of the 7% credit reduce the unit cost to the corporation of providing its own supply. The cost savings per million gallons at selected rates of discount are presented in Table A-2. Again, the principal diagonal represents cost savings when costs of capital are the same for both the community and corporate projects. Although the entries in Table A-2 are lower than the corresponding entries in Table 4 (page 24), the relevant entries (those on and below the principal diagonal) still represent significant savings per million gallons.

TABLE A-2. Corporation's Savings in Dollars per Million Gallons Resulting from the Purchase of Water from the Community Project at Different Rates of Discount-- SYD Depreciation; 7% Tax Credit

Corporate Cost of Capital	Community Cost of Capital			
	.04	.06	.08	.10
.04	1.69	-1.38	-4.75	-8.35
.06	6.59	3.52	0.15	-3.45
.08	12.06	8.99	5.62	2.02
.10	18.01	14.94	11.57	7.97

The preceding comparisons were made under the assumption of constant returns to scale as was the case with the comparisons in the text. It now remains to examine the level to which construction costs of the community project must rise to eliminate its advantage. Table A-3 presents community project construction costs which, when considered with appropriately increased

maintenance costs but identical operation costs and salvage value, yield unit costs for the community project that are equal to those of the \$10 million corporate project. Examination of Table A-3 reveals that for realistic cost of capital combinations diseconomies of scale must still be quite substantial before the cost advantage of the community project is eliminated.

TABLE A-3. Community Project Construction Costs
Yielding Unit Water Costs Equal to
Those Incurred by \$10 Million Corporate
Project--SYD Depreciation; 7% Tax Credit

Corporate Cost of Capital	Community Cost of Capital			
	.04	.06	.08	.10
	[Millions of Dollars]			
.04	10.9	9.4	8.1	7.0
.06	13.6	11.6	10.1	8.8
.08	16.6	14.2	12.3	10.7
.10	19.9	17.0	14.7	12.8

In summary, an increase in the investment tax credit or the allowance of a more rapid rate of depreciation lessens the advantage of the community project. Under the most favorable method of depreciation currently allowed and with the present investment tax credit, however, a corporate water user still realizes a saving by purchasing from the community project rather than supplying its own needs. This conclusion would be overturned only if the cost of capital were higher to the community than to the water-using corporation, or if there were substantial diseconomies of scale associated with water supply.

REFERENCES

- Bierman, Harold Jr., and Seymour Smidt, The Capital Budgeting Decision, pp. 141-148, The Macmillan Company, New York, 1960.
- Cohen, Jerome B., and Sidney M. Robbins, The Financial Manager, Basic Aspects of Financial Administration, pp. 749-751, Harper and Row, New York, 1966.
- Davidson, Sidney, and David F. Drake, Capital budgeting and the 'best' tax depreciation method, The Journal of Business, 34, 442-453, 1961.
- Dean, Joel, Capital Budgeting, p. 43, Columbia University Press, New York, 1951.
- Donaldson, Elvin F., and John K. Pfahl, Corporate Finance, pp. 438-440, The Ronald Press Company, New York, 1963.
- Durand, David, The cost of capital, corporation finance, and the theory of investment: comment, The American Economic Review, 49, 639-655, 1959.
- Gordon, Myron J., The Investment Financing and Valuation of the Corporation, p. 131, Richard D. Irwin, Incorporated, Homewood, Illinois, 1962.
- Grant, Eugene L., and W. Grant Ireson, Principles of Engineering Economy, p. 50, The Ronald Press Company, New York, 1960.
- Hirshleifer, Jack, James C. DeHaven, and Jerome W. Milliman, Water Supply, Economics, Technology, and Policy, p. 276, The University of Chicago Press, Chicago, 1960.
- Meyer, R. R., and J. R. Rollo, Salt-Water Encroachment Baton Rouge Area, Louisiana, Water Resources Pamphlet No. 17, Department of Conservation, Louisiana Geological Survey and Louisiana Department of Public Works, Baton Rouge, November, 1965.
- Modigliani, F., and M. H. Miller, The cost of capital, corporation finance and the theory of investment, The American Economic Review, 48, 261-97, 1958.
- Morgan, C. O., and M. D. Winner, Jr., Salt-Water Encroachment in Aquifers of the Baton Rouge Area -- Preliminary Report and Proposal, Louisiana Department of Public Works, Baton Rouge, January, 1964.
- Solow, Robert M., Capital Theory and the Rate of Return, p. 15, Rand McNally and Company, Chicago, 1965.