LOUISIANA WATER RESOURCES RESEARCH INSTITUTE

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CYCLIC STORAGE OF FRESH WATER IN SALINE AQUIFERS

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CYCLIC STORAGE OF FRESH WATER IN SALINE AQUIFERS

DESCRIPTORS: *Underground Storage, *Water Storage, Groundwater Recharge, Groundwater Movement, Artificial Recharge, *Saline Water-Freshwater Interfaces, *Mixing, *Diffusivity, Dispersion, Transmissivity, *Injection, *Injection Wells, Model Studies, Computer Programs, Aquifers, *Water Management (Applied)

IDENTIFIERS: Dispersivity, Gravitational Laydown, Miscible Displacement, Salaquifers, Isopotential Zones, Zones of Stagnation, Well-Field Design, Miniaquifers

ABSTRACT: The storage of fresh water in horizontal, confined saline aquifers is technically feasible. Experimental results with physical aquifer models involving single and multiple injection/production wells were used to verify computer programs for: prediction of the recovery efficiency (the ratio of retrieved-to-injected fresh water); frontal position of the fresh water bubble; position of the saline water-freshwater interface resulting from gravitational laydown, dispersion, and mixing; and calculation of the dispersivity. For salaquifers with pre-existing groundwater movement, an additional program must be used to design a system of injection/withdrawal bounding wells that will neutralize the movement and create a zone of stagnation for the storage project. Tests on tilted physical systems, with and without pre-existing flow, showed that deleterious mixing and water movement could occur in aquifers with more than a few degrees of dip. Besides the determination of the usual in-situ hydrogeologic factors for groundwater supply systems, additional tests for porosity, dispersivity, and pre-existing flow are required for the design of an Injection/Storage/Production well field. Each additional I/S/P cycle will result in improved recovery efficiency, regardless of the hydrogeologic parameters. All necessary computer programs are detailed in the appendices. Cost estimates for a hypothetical underground storage system compare favorably to an alternate tank storage project. The construction and calibration of the miniaquifers (artificial porous sandstones composed of uniformly-graded sand and epoxy resin) are described along with the test procedures, equipment, and mathematical bases.

REFERENCE: Kimbler, Oscar K., Kazmann, Raphael G., and Whitehead, Walter R., CYCLIC STORAGE OF FRESH WATER IN SALINE AQUIFERS, *Louisiana Water Resources Research Institute Bulletin No. 10*, Louisiana State University, Baton Rouge, October 1975

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I

SUMMARY

The storage of fresh water in an underground formation containing saline water is feasible if the formation is porous and permeable and conforms to the usual assumptions of ground-water engineering.

The theoretical and technical bases for accomplishing such storage projects require not only the field determination of storativity and transmissivity, hydrologic boundaries, and initial potentiometric gradients, but also the in-situ porosity and in-situ dispersivity. For determining the dispersivity, any existing potentiometric gradients must first be neutralized in the salaquifer in the area under consideration.

Before the construction of a full-scale project for cyclic underground storage of fresh water, it is desirable to test the applicability of the procedure by an actual injection/storage/production (I/S/P) sequence with a single well. This sequence will also provide information on potential chemical interactions between the injected water, the formation water, the bounding clay layers, and other minerals in the aquifer.

The studies discussed are based on the assumption of a horizontal salaquifer with the viscosity of the injected and native waters approximately equal. However, the density differences between the fresh water and the native saline water are taken into account.

The study results cannot be applied to salaquifers with secondary permeability and porosity, such as cavernous limestones or fractured igneous or metamorphic rocks.

The effect of aquifer dip is known to be deleterious, but the reduction in recovery efficiency of an I/S/P project in dipping formations has not yet been quantified. The smaller the dip and the smaller the density difference (the lower the total dissolved solids in the native water), the more likely it is that the computational and field procedures will apply.

After the mathematical and computational bases for the cyclic storage of fresh water in salaquifers are set forth, a review is

given of the laboratory procedures used to build and verify the miniature aquifers (miniaquifers), the primary role of which was to test the mathematical models.

The final section, dealing with the field test procedures, includes a long series of illustrative computations to show the effects of varying each parameter on the economic feasibility of an index project for the storage of one billion (10^9) gallons of potable water. The cost variations to be expected from changes in parameters such as transmissivity, permeability, density difference, aquifer thickness, and dispersivity are shown in the appropriate table.

Several computer programs are included in the appendix, together with numerical examples, so that the user may be assured that a program prepared from our listing will produce correct results. These programs include: the computation of the field dispersivity by a match-curve procedure; the design of a well field to create a zone of isopotential in a salaquifer in which the native water already is in motion; the computation of the frontal position at any time during the initial injection of fresh water; and, most important, the calculation of the recovery efficiency for either a single- or a multi-well field for fresh water storage in a salaquifer.

It should be noted in passing that one or more of the computer programs are applicable to the deep-well disposal of liquid wastes, the leach-mining of thin, deep, or lean ore bodies and, with some additional research, the storage of waste heat in aquifers.

II

INTRODUCTION

BACKGROUND

As our population increases, the need for more and more water manifests itself, with reliability of supply being of paramount importance. Each and every day, water must be supplied in response to varying daily, seasonal, and yearly demands. But rain and snow, the perennial water sources, occur at times and in places that are not reliably predictable, and in quantities even less predictable. So, just as Joseph recommended to the Pharaoh, present-day machine civilization must store its surplus water in times of plenty to use in times of drought.

In many parts of the country, until recently, reservoir sites were numerous. Land could be purchased at what was considered a reasonable price and dams and spillways could be constructed at advantageous sites at minimum expense. Although many of the existing reservoirs had a regrettable tendency to silt up, many additional impoundment areas were still available for development at only slightly greater unit costs to offset the lost storage capacities.

Within the past decade, matters have changed substantially. Many truly economical sites for dams and reservoirs no longer remain. They have been occupied, or so encroached upon, until the remaining areas control smaller and smaller watersheds. Thus the cost of water tends to be higher due to the decreased incremental yield obtainable from construction of a dam.

Even for remote areas with advantageous sites for the construction of dams and reservoirs, a significant and highly vocal fraction of the population opposes such construction in the name of environmental preservation. Granted that their arguments have some merit, such opposition too often turns into personal invective and almost intolerable exaggeration.

Historically, water for low-lying coastal areas (such as Los Angeles, San Francisco, and New York) has been brought in over great distances or supplied from nearby rivers (as in Philadelphia,

New Orleans, St. Louis, and Washington, D.C.). Even in the latter cases, water storage is necessary both to meet hourly and daily peak demands and to serve as an interim supply when accidents on the river or upstream industrial plants inadvertently contaminate the river water.

Inexorably, the population spread, while reducing the number of available reservoir sites, increases the water demand--hence the need for water storage.

Cities such as St. Louis, Pittsburgh, and New Orleans, which are located along major rivers or lakes where large impoundments have previously been unnecessary because of ample river flow even during droughts, are now confronted with the problem of producing safe drinking water despite possible river contamination due to accidents, sabotage, or enemy action. The personnel engaged in providing clean, safe water are being forced into finding some method of storing potable water during times of relative plenty for emergency use. Reservoir sites are not readily available, partly due to the urbanization of the flat areas in river valleys that once were potential reservoir sites.

Adding to the national water storage problem is the shortage of low-cost domestic energy. Thus the construction of offshore terminals becomes necessary for the economical importation of oil. The nearby attendant petrochemical plants will require water year round, even when the source is erratic and intermittent. Compounding the problem are yet-to-be-built steam-electric generating plants. Water will be needed for cooling towers and boiler-feed makeup. Where are the sites for the needed storage reservoirs?

Despite the implications of misfortune given in the foregoing paragraphs, we submit that much unused storage space is already at hand, usually near the points of use, in the extensive saline aquifers that underlie many of the most densely populated or industrialized areas as well as some isolated industrial sites. In addition, some cities possess surplus treatment capacity in their water supply systems that enables the waterworks to satisfy seasonal peak demands with possible margins for future expansion.

Without storage facilities, these water users are, during droughts or periods of low water, in the position of being surrounded by unusable, polluted or unsatisfactory water. And though aquifers filled with saline water might underlie the city, they had never been considered as potential storage reservoirs for fresh water.

Admittedly, there are also certain legal obstacles to the storage of water under private property and preventing withdrawal of

fresh water from wells drilled by any property owner who lives in the area overlying the storage aquifer.

Incidentally, the deep saline aquifers (salaquifers) in most of the country also have potential uses for the storage of natural gas (some are being used for this purpose now) and as permanent depositories for the storage of liquid wastes of many sorts (Warner, 1968; Kazmann et al., 1974). Serious suggestions have been made to use them for storage of waste heat (Meyer and Todd, 1973). This heat, later retrieved, could significantly alleviate the energy shortage.

Salaquifers must now be considered a major, yet virtually unused natural resource, not for the water they contain--water polluted by salts of one kind or another--but because they, by being usually close at hand, are potential storage space for liquids and gases.

REVIEW OF WORK BY OTHER RESEARCHERS

Several early attempts to store fresh water in saline aquifers have nearly all been empirical. In each instance, surplus water, usually from a municipal water plant, was injected into a sand or sandstone aquifer containing brackish water. The same well was subsequently pumped until the discharge stream became unusable.

Apparently, the first effort of this sort was made by Cederstrom (1947) in Virginia. Ten years later in Texas, Moulder and Frazor (1957) also performed a field test. Brown and Silvey (1973) reported on a similar experiment in Norfolk, Virginia. At this writing, the Florida Department of Natural Resources is conducting a test aimed at storing and retrieving water by use of a salaquifer under West Palm Beach.

The only conclusion to be drawn from our study of the published field data is that, under certain conditions, such storage in saline aquifers is feasible. However, Harpaz and Bear (1964) found that the injection of fresh water into a dolomite containing saline water was unsuccessful due to pre-existing movement of the native water.

From an engineering standpoint, design principles have generally been lacking. More importantly, a theoretical, generalized approach to the problem that includes all the parameters involved has not been developed. Even in the research reported in this bulletin, all the parameters have not yet been identified and quantified.

The principal theoretical work on the subject, that of miscible displacement, has traditionally been accomplished in the field of petroleum technology, where the need for secondary recovery, usually water flooding, has stimulated research. Koch and Slobod (1957),

Kern, Kimbler, and Wilson (1958), Raimondi, Gardner, and Petrick (1959), and Gardner, Downie, and Wyllie (1962) reported on miscible displacement of fluids in porous media. Caudle (1963) constructed a consolidated sand analog to study natural water drive in an oil reservoir. Pressure increases were produced by the injection of water. The primary interest was in the flow patterns by which oil is swept ahead of the injected water toward the producing wells.

Green and Cox (1966) produced a mathematical model of a system for storage of fresh water in salaquifers, but no physical verification was published. They later (1968) did accomplish some laboratory studies but did not attempt to extend the work to the parameters encountered in the field.

Gelhar et al. (1972) investigated the problem of density segregation both mathematically and experimentally. They used a mathematical description with the assumption of immiscible displacement. They computed the shape of the interface (gravitational lay-down of the front) and compared the results with observations obtained from a horizontal radial flow system in which the porous medium consisted of epoxy-cemented plastic beads. Their apparatus was 2.4 meters long and 7.46 cm thick and was a pie-wedge sector of 15°. They found small discrepancies between the computed and observed frontal shape but concluded that their computational technique gave conservative estimates (lower than observed) of recovery efficiency. They also concluded that recovery efficiencies increase after several complete cycles of injection-storage-production (I/S/P).

REVIEW OF WORK AT LOUISIANA STATE UNIVERSITY

Late in 1965, the first studies at LSU concerning miscible displacement were started in the Petroleum Engineering laboratories. The technique of building porous aquifer models (miniaquifers), by consolidating specially-graded blasting sand with epoxy resins, was first applied to linear systems.

In later experiments with single wells in circular horizontal aquifers, the boundary pressures were held constant throughout each injection and withdrawal cycle. Because a circular system is symmetrical, a 45-degree sector of the entire aquifer could be used to keep the model size large, but manageable.

Thin miniaquifers were used at first to reduce the effect of density differences on the frontal position--that is, to make the assumption of a vertical front for the injected fluid a reasonable approximation.

In 1967, Esmail and Kimbler made their first report on the

feasibility of fresh-water storage in salaquifers. Their conclusions were based on a mathematical model that had been verified by experimental observations.

The approach used in all the LSU studies consisted of: devising of a computational model for a system that had been simplified to the maximum extent; testing the results of the procedure by inserting the aquifer constants into the equations; making the required computer runs; and checking of the results against observations of the physical system. It was concluded that the use of saline aquifers on a cyclic basis would result in increased percentages of fresh-water recovery as the number of cycles is increased.

All the theoretical concepts were tested on at least one physical system. Every new additional physical system was used to test and evaluate all previous mathematical models. Because of variations in the grain-size distribution of the blasting sand, and possibly, differences in manual technique between the persons who mixed, cured, and finished the sand-resin aquifer models, each of the tested physical systems differed in characteristics--porosity, transmissivity, thickness, dispersivity, boundary locations, etc. Although this lack of replication appears unscientific, we took advantage of the variations in checking the mathematical models.

In 1970, Kimbler tentatively predicted the favorable outcome of the salaquifer storage of fresh water.

SCOPE OF REPORT

This bulletin is not concerned with the use of salaquifers except as reservoirs for fresh water. The storage of natural gas in aquifers is well established as a semi-empirical procedure and, although the computer programs developed as a part of our studies can be adapted by the natural gas industry, we have not digressed toward this application. The technology of disposing of wastes in deep saline aquifers could also utilize some of the computer programs developed, primarily those for predicting the location of the waste fronts after long periods of waste injection (Kazmann, Kimbler, and Whitehead, 1974), but waste disposal is also outside the scope of this report. The conservation of energy by the use of heat-storage wells (Meyer and Todd, 1973) is still hypothetical, as are the proposals, so we have not discussed the possibility.

In this exposition, attention is focused on the utilization of saline aquifers for the storage of fresh water. Our primary

interest was the computation of the ratio of fresh water obtained, after a certain storage time, to the fresh water injected; i.e., the recovery ratio. This report deals with experimental studies and computer programs to find the recovery ratios through individual wells and complete well fields that might be constructed in essentially horizontal aquifers which have negligible existing ground-water movement.

However, because most aquifers are dynamic systems in which the water is in motion, water movement appeared to be a restriction on the application of our experimental and theoretical work. To study the importance of this problem, we conducted experiments on the recovery ratios attainable under conditions of pre-existing ground-water movement (W. Esmail, 1973). Very unfavorable results forced us to undertake a theoretical study to determine if an isopotential surface (an area of stagnation) could be created in a horizontal aquifer that had a sloping potentiometric surface.

It was our à priori belief that such an area of stagnation could be created by operation of a well field consisting of strategically placed injection and production wells. After the preparation of this bulletin was well underway, the theoretical studies, based on work by Lin (1972), proved to be successful. A method for creating an isopotential surface of any desired size was devised and tested analytically with field parameters for transmissivity, storativity, and potentiometric gradients. Langhetee (1974) reported three main conclusions:

a. The effects of pre-existing ground-water movement in an aquifer can be effectively eliminated by positioning injection and production wells around a given storage area and operating them at calculated rates.

b. The rates of production and injection can be balanced so that there will be neither an excess of produced saline water nor a shortage of saline water to inject.

c. Once the potentiometric gradient within an area has been reduced to zero, the previously developed mathematical techniques can be used to predict the recovery efficiency of a fresh-water storage project in the stagnated portion of the salaquifer.

Inasmuch as ground-water theory, based on the Theis equation and superposition of effects, has been found to be valid in field applications, there is every reason to believe that areas of stagnation can be created in practice. Preliminary economic evaluations indicate that the cost of installing and operating an I/S/P well field (including the creation and maintenance of an isopotential area) will usually be financially more attractive than other methods of providing an equivalent storage volume. Thus the application of the methods and computer programs given in later sections will have

far wider applicability than that originally hoped by the authors when they began writing this report.

The main focus, however, has been to describe the development of our experimentally-verified computer model and its potential application to single wells and well fields for essentially unused horizontal aquifers in which the existing saline water is, or has been made, motionless. Because of this limited scope, the economic evaluations given in the final section should be considered only as preliminary engineering estimates, especially now during the current price fluctuations.

In outline, then, we will discuss the mathematics of the miscible displacement process, the creation of a zone of stagnation (should this be necessary), the effect of density difference, and the computation of recovery efficiency in the cyclic I/S/P of fresh water. We have detailed the construction and calibration of the basic tool for our physical verifications of the mathematical model, the properly designed and equipped miniaquifer, and reported all the experimental procedures and results. Finally, we have outlined the application of the results to field situations and included a series of economic computations to show the effect of each parameter on the costs.

ACKNOWLEDGMENTS

We have intended to give the definitive statement on the theoretical and experimental research concerning the storage of fresh water in saline aquifers that was begun at Louisiana State University in 1965. Much of the outcome has already been published piecemeal, primarily in the technical literature, as it became available. Portions of the work were funded separately under specific grants with only short, unpublished completion reports having been written. However, no broad review of all the work up to now has been issued for the interested professional who would want to examine the whole project and to make his own evaluations and use the results. Thus the following organizations and persons whose support, work, and assistance make up a long list may, even so, be incomplete.

Financial support for all the work described herein came from funds allotted by the Office of Water Research and Technology (formerly the Office of Water Resources Research) to the Louisiana Water Resources Research Institute. Various phases of the work included projects A-002-LA, A-011-LA, A-022-LA, A-023-LA, A-027-LA, A-031-LA, and A-034-LA. Work is still underway on projects

A-034-LA and A-037-LA.

Because a major purpose of OWRT funding is the training of graduate students, there is a long list of talented young men who did much of the experimental and computer work that comprises the basis for this publication. The diligence and assistance of the following graduate students are gratefully acknowledged: Omar J. Esmail, Anil Kumar, Joseph L. Francis, Thomas R. Painter, Walid J. Esmail, Edmond J. Langhetee, Joseph A. D'Amico, and Paul T. Tate, all of whom were graduate students in the Departments of Civil or Petroleum Engineering. A considerable contribution was made by Dr. Walter R. Whitehead, co-author of this bulletin, while he was working on his dissertation. Several students of Dr. J. S. Hanor (geochemistry), particularly R. K. Stoessell (1974), contributed to our understanding of the miscible displacement process insofar as chemical changes in injected water and the diffusion of ions from salt to fresh water are concerned. Other work, of great practical and theoretical importance on ion exchange between clays and injected water, is now (1975) in progress under Dr. Hanor's supervision. His findings must be used in conjunction with the results described herein when plans are made to apply our work to field situations.

We have been fortunate that research along complementary lines has been underway in the Department of Petroleum Engineering at the University of Texas/Austin under the direction of Dr. Ben H. Caudle. In particular we call attention to the assistance we received from our study of the dissertation of Dr. Caudle's student, Dr. J. Lin, in developing a procedure for the creation of a zone of stagnation to nullify the effects of pre-existing ground-water flow.

The assistance of Mary H. Alston, who typed final copy after helping to edit and proofread the numerous drafts of this publication, is acknowledged with gratitude. The drawings were prepared by Norma B. Duffy and the final manuscript was edited by Charles W. Hill. The theses and dissertations evolving from the studies described in this bulletin are marked by an asterisk in the references.

Although we are jointly responsible for the writing, the draft of the section on mathematical background was prepared by W. R. Whitehead, the draft of the introduction and applications by R. G. Kazmann, and the draft on miniaquifers and experimental procedures by O. K. Kimbler.

The help of all persons named was invaluable. The authors, however, are solely responsible for the mistakes (hopefully very few) that may be found here and, of course, for the conclusions.

III

MATHEMATICAL BACKGROUND

INTRODUCTION

In the storage of fresh water in saline aquifers, six factors affect the recovery efficiency: (1) mixing of the two fluids due to molecular diffusion and convective dispersion; (2) segregation of the two fluids due to density difference; (3) viscous fingering due to a difference in viscosities between the injected and native fluids; (4) aquifer heterogeneities; (5) aquifer dip; and (6) pre-existing ground-water movement in the aquifer.

Throughout this report, the following assumptions have been made:

1. The aquifer is horizontal, homogeneous, and isotropic, and is of infinite areal extent.
2. The viscosities of the injected and native fluids are the same.

Hence, the only factors affecting the recovery efficiency that have to be evaluated are: (1) molecular diffusion and convective dispersion; (2) segregation of the two fluids due to density difference; and (3) pre-existing ground-water movement (which can be counteracted through the use of bounding wells).

MIXING DUE TO MOLECULAR DIFFUSION AND CONVECTIVE DISPERSION

If two miscible fluids of different composition are in contact, a transfer of molecules will result. As time passes, the random movement of molecules creates a mixed zone where the two fluids have diffused into one another (molecular diffusion).

When one fluid miscibly displaces another fluid in a porous medium, the mixed zone will be greater than that due to molecular diffusion alone. The additional mixing, primarily dependent on pore geometry, results from variations in the velocity field and the constant intermingling of flow paths as displacement progresses. This additional mixing, convective dispersion, occurs both longitu-

dinally, in the direction of gross fluid movement and transversely, in a direction normal to the gross fluid movement. Other investigators have shown that longitudinal dispersion is 6 to 20 times greater than transverse dispersion (de Josselin de Jong, 1958; Pozzi and Blackwell, 1963); in this bulletin, transverse dispersion is considered to be negligible.

In the case of both molecular diffusion and longitudinal dispersion, Raimondi, Gardner, and Petrick (1959) stated that for a fluid displacing another fluid (of the same viscosity and density) radially outward from a well through a homogeneous, isotropic, porous medium, the amount of mixing can be described by the equation

$$\frac{\partial c_i}{\partial t} + \frac{Q}{r}\frac{\partial c_i}{\partial r} = \left(\frac{\alpha r}{Q} + D\frac{r^2}{Q^2}\right)\frac{\partial^2 c_i}{\partial t^2} \qquad (3.1)$$

where c_i = concentration of injected fluid at the radius, r, and time, t

r = radius (cm)

t = time (seconds)

$Q = q/(2\pi h\phi)$ (cm^2/sec)

q = volumetric flow rate (cm^3/sec)

h = aquifer thickness (cm)

ϕ = aquifer porosity (fraction)

D = coefficient of molecular diffusion of fluids in porous medium (cm^2/sec)

α = longitudinal dispersivity coefficient of porous medium (cm)

In the derivation of Equation 3.1, it was assumed that the coefficient of molecular diffusion, D, is a constant. This is not entirely true because the value of D is dependent on the concentration (Jost, 1960). However, because the dependence is small, for all practical purposes the value of D may be considered constant (Perkins and Johnston, 1963). Stoessell (1974) has shown experimentally that the value of D in a porous medium similar to that used in our experimental work is about 10^{-6} cm^2/sec.

The derivation of Equation 3.1 is also based on the assumption that the longitudinal dispersion is proportional to the first power of the average velocity of the fluid through the porous medium, with the proportionality constant being the longitudinal dispersivity

coefficient (α). Aris and Amundsen (1957) had shown that this assumption is valid if the velocity is low enough to allow diffusion to equalize the concentration within each pore space. If it is so high that concentration equalization cannot occur, dispersion will be proportional to the velocity raised to a power greater than unity. Most experimental data indicates that this exponent has an upper limit of about 1.2 (Brigham, Reed, and Dew, 1961; Perkins and Johnston, 1963). The authors believe that any errors due to the use of 1.0 for all cases are not large enough to invalidate the equation from an engineering viewpoint.

The value of the longitudinal dispersivity coefficient (α) is a characteristic of the porous medium and increases as the uniformity coefficient increases (a material containing a greater range of particle sizes) or the intrinsic permeability increases (Raimondi, Gardner, and Petrick, 1959). The value of α for our miniaquifers was found to range from 0.02 to 0.08 cm. (The computational procedure used to obtain the values will be discussed later.)

Experiments by Brigham, Reed, and Dew (1961) and Bentsen and Nielsen (1965) have shown that the value of α is also a function of the ratio of the viscosities (displaced fluid/displacing fluid). The larger this ratio, the larger the value of α. Throughout our investigations, a viscosity ratio of unity was maintained.

For the continuous injection of a fluid at a steady rate with a concentration c_o at $r = 0$, Raimondi, Gardner, and Petrick (1959) proposed a solution to Equation 3.1:

$$\frac{c_i}{c_o} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2/2 - Qt}{\left(4\alpha R^3/3 + DR^4/Q\right)^{\frac{1}{2}}}\right]$$

where R = radius (cm) of injected fluid at time t, with no mixing or gravitational segregation.

$\mathrm{erfc}(\xi)$ = complementary error function of ξ

$$\mathrm{erfc}(\xi) = \frac{2}{\sqrt{\pi}}\int_{\xi}^{\infty} e^{-w^2}\,dw$$

Equation 3.2 satisfies the boundary conditions $c_i(r = 0,\ t > 0) = c_o$ and $c_i(r = \infty,\ t = 0) = 0$; however, it does not satisfy the initial condition, $c_i(r, t = 0) = 0$. This is due to the fact that in obtaining Equation 3.2, it was assumed that $\partial c/\partial t = 0$ at $t = 0$. Hoopes and Harleman (1967) state that this assumption is approximately true away from the immediate vicinity of the source; however, it is

not true within 10-20 particle diameters of the source. The validity of the solution given by Equation 3.2 has been demonstrated experimentally by Bentsen and Nielsen (1965), Hoopes and Harleman (1967) and Esmail and Kimbler (1967).

Equation 3.2 may be rewritten (Appendix A) as

$$\frac{c_i}{c_o} = \frac{1}{2}\,\text{erfc}\left[\frac{r^2 - R^2}{2\sqrt{f(t)}}\right] \quad \ldots\ldots\ldots\ldots \quad (3.3)$$

where $f(t) = \frac{4}{3}\alpha\,(2Qt)^{3/2} + D\,\frac{(2Qt)^2}{Q}$

In terms of the concentration of native fluid (c_n), it can be shown that Equation 3.3 may also be written as

$$\frac{c_n}{c_o} = \frac{1}{2}\,\text{erfc}\left[\frac{R^2 - r^2}{2\sqrt{f(t)}}\right] \quad \ldots\ldots\ldots\ldots \quad (3.4)$$

Gardner, Downie, and Wyllie (1962) extended the solution given by Equation 3.4 to apply to successive injection and production half-cycles. The following equations are essentially those given by them except for rearrangement, simplification, and subscripts. The concentration at any radius (r) and for any injection or production half-cycle may be computed from the appropriate form of the solution

$$\frac{c_n}{c_o} = \frac{1}{2}\,\text{erfc}\left[\frac{R^2 - r^2}{DNOM_{(I\text{ or }P),j}}\right] \quad \ldots\ldots\ldots\ldots \quad (3.5)$$

where

First Injection Half-Cycle

$$DNOM_{I,1} = 2[f_{I,1}(t_1)]^{1/2}$$

First Production Half-Cycle

$$DNOM_{P,2} = 2[f_{P,2}(t_2) - f_{P,2}(t_1) + f_{I,1}(t_1)]^{1/2}$$

Second Injection Half-Cycle

$$DNOM_{I,3} = 2[f_{I,3}(t_3) - f_{I,3}(t_2) + f_{P,2}(t_2) - f_{P,2}(t_1) + f_{I,1}(t_1)]^{1/2}$$

Second Production Half-Cycle

$$DNOM_{P,4} = 2[f_{P,4}(t_4) - f_{P,4}(t_3) + f_{I,3}(t_3) - f_{I,3}(t_2) + f_{P,2}(t_2) - f_{P,2}(t_1) + f_{I,1}(t_1)]^{1/2}$$

etc.

in which

$$f_{I,j}(t_k) = \frac{4}{3}\alpha(2Q_{I,j}t_k)^{3/2} + D\frac{(2Q_{I,j}t_k)^2}{Q_{I,j}}$$

$$f_{P,j}(t_k) = \frac{4}{3}\alpha(2Q_{P,j}t_k)^{3/2} + D\frac{(2Q_{P,j}t_k)^2}{Q_{P,j}}$$

$$Q_{(I \text{ or } P),j} = \left|\frac{r^2_{final} - r^2_{initial}}{2\Delta t}\right| \qquad (cm^2/sec)$$

where

Δt = time for fluid to travel from $r_{initial}$ to r_{final} (sec)

$t_1, t_2, t_3, \ldots$ = time measured from start of first injection half-cycle (sec)

I,P = subscripts for injection and production respectively

j,k = integers

Equation 3.5 has been experimentally verified by Esmail and Kimbler (1967) for two complete cycles.

SEGREGATION DUE TO DENSITY DIFFERENCE

When fluids of unequal densities are in contact in a porous medium, gravity causes the less dense fluid to rise relative to the more dense. The interface will assume a progressively greater angle with respect to the vertical (gravitational segregation or laydown). Gardner, Downie, and Wyllie (1962) have shown mathematically and Esmail (1966) has shown experimentally that, for a viscosity ratio of unity, the interface may be treated as a plane surface in linear systems. Kumar (1968) found experimentally that this also may be done for radial systems.

Segregation in porous media can be separated into two cases.

The so-called "static" case involves no bulk flow of fluids except that arising from convective currents attributable to gravity. The second case, dynamic gravitational segregation, occurs in the presence of bulk flow (Esmail and Kimbler, 1967). An example of the latter would be the gravitational segregation that occurs during the displacement of a fluid by an injected fluid of different density. Esmail (1966) assumed and Kumar (1968) later verified that for a viscosity ratio of unity, dynamic and static gravitational segregation are practically equal. The results obtained from linear systems have been directly related to radial systems by correcting the linear results to radial geometry.

Gardner, Downie, and Kendall (1962) studied static gravitational segregation of miscible fluids in linear, horizontal systems. They reported that the projection of the interface on the horizontal can be approximated by the equation

$$\left(\frac{2XL}{h}\right)^2 = \frac{16}{3} F^2 \frac{k_H}{k_V} \frac{(t/t_o)^2}{(1 + t/t_o)} \quad \ldots\ldots\ldots\ldots \quad (3.6)$$

where

$2XL$ = projection of the interface on the horizontal (cm)

h = aquifer thickness (cm)

k_H = horizontal intrinsic permeability (cm^2)

k_V = vertical intrinsic permeability (cm^2)

F = a dimensionless factor dependent on viscosity ratio ($F = 1.0$ for a ratio of one)

t = time (sec)

$t_o = \frac{4}{3} \frac{\phi h \bar{\mu} F}{k_V g \Delta\rho}$ (sec)

ϕ = porosity (fraction)

$\bar{\mu}$ = average viscosity of the two fluids (poises)

g = acceleration due to gravity (cm/sec^2)

$\Delta\rho$ = density difference between the fluids (gm/cc)

Equation 3.6 is based on the assumption of a sharp interface between the two fluids. Esmail (1966) contended that, in practice, a mixed zone would be present due to diffusion and dispersion. He further reasoned that this zone would retard the effect of gravitational segregation, with a smaller rate of interface laydown than

the earlier sharp interface equation would predict.

Esmail (1966) introduced a variable, density gradient (S), which he defined as the ratio of the density difference to the length of the mixed zone. Assuming a horizontal, homogeneous, isotropic porous medium, he used dimensional analysis and experimental data from several linear systems to obtain the following equation that best described gravitational segregation in linear systems:

$$\frac{2XL}{h} = f(\psi) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.7)$$

where

$$\psi = \left[\frac{k\ g\ \Delta\rho\ t}{\bar{\mu}\ \phi\ h}\right]\left[\frac{(\bar{\mu})^{2/3}\ S}{(\Delta\rho)^{5/3}\ (g)^{1/3}}\right]^{1/2}$$

In each experimental run, Esmail started with a known mixed zone length, assumed to remain constant throughout the experiment. He also started each experiment with the interface vertical at time = 0 and then recorded values of 2XL/h as time progressed.

To find the functional relationship between 2XL/h and ψ, we plotted the experimental data obtained by Esmail (1966). The curve (Fig. 3.1) was divided into two sections that were fitted separately.

The equation for the first part of the curve was

$$\frac{2XL}{h} = 20.0\psi \ ; \quad (0 \leq \psi \leq 0.1) \quad \ldots\ldots\ldots \quad (3.8)$$

and for the second part,

$$\frac{2XL}{h} = 0.8 + 12.5\psi - 4.8\psi^2 \ ; \quad (0.1 \leq \psi \leq 1.0) \quad \ldots \quad (3.9)$$

Because there were no data beyond $\psi = 1.0$, the functional relationship was arbitrarily taken to be

$$\frac{2XL}{h} = 6.5 + 2.0\psi \ ; \quad (\psi \leq 1.0) \quad \ldots\ldots\ldots \quad (3.10)$$

It will be shown later that this relationship yields recovery efficiencies on the safe side (lower than those actually observed).

In computing the values of ψ plotted in Figure 3.1, Esmail (1966) calculated the density gradient on the basis that the length of the mixed zone was the distance between the positions where the concentrations of native fluid were 3 and 97 percent. The same basis was used throughout our investigation for computing zone lengths.

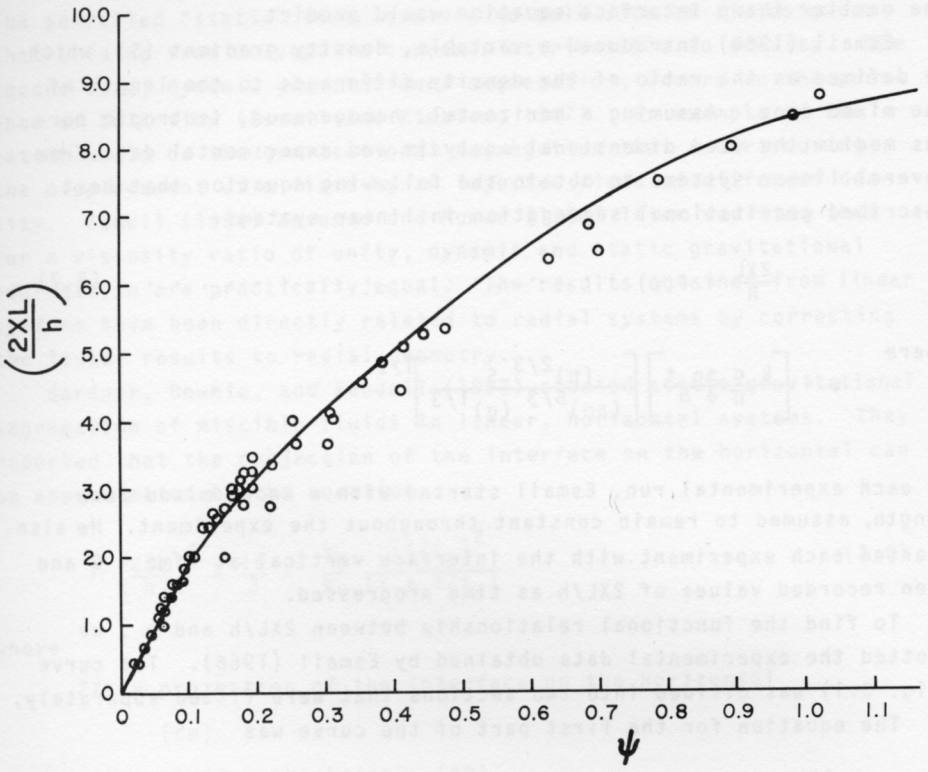


Figure 3.1 DIMENSIONLESS CORRELATION USED TO COMPUTE GRAVITATIONAL SEGREGATION (LAYDOWN)

COUNTERACTING THE EFFECTS OF PRE-EXISTING GROUND-WATER MOVEMENT

If fresh water is being injected into an aquifer in which saline ground water is already moving, the fresh-water bubble will not only move out radially from the injection well, but also tend to move in the direction of the natural ground-water flow. The injected water will continue to migrate in the natural-flow direction during the injection half-cycle, static storage period, and production half-cycle. As a consequence, during the production half-cycle, breakthrough of the native saline water will occur at the upstream side of the well before it can occur at the downstream side. Hence some of the stored fresh water (in addition to other losses) will be unrecovered at breakthrough. The volume of water lost will depend on

the rate of pre-existing ground-water movement.

Thus the storage of fresh water in a saline aquifer with the ground water in motion may not be economically feasible unless some method is devised to negate the effects of the ground-water movement. Langhetee (1974) proposed a method that uses bounding wells. This method, a modification and extension of an image well technique proposed by Lin (1972) for bounding various oil reservoir shapes, is outlined in the succeeding paragraphs.

In a horizontal, homogeneous, isotropic aquifer of infinite areal extent, saturated with a liquid at rest, the potential at any point in the aquifer is constant. If a well located at (x_i,y_i) on an arbitrary horizontal coordinate system is pumped at a rate (q), the steady-state potential at any other point (x,y) in the aquifer is

$$\Phi(x,y) = -\frac{\mu}{4\pi kh} q \cdot \ln[(x-x_i)^2 + (y-y_i)^2] + C \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.11)$$

where

$\Phi(x,y)$ = potential at point x,y (atmospheres)
μ = viscosity (centipoises, cP)
k = permeability (darcies)
h = thickness of aquifer (cm)
q = operating rate of well (cm^3/sec)
q > 0, injector; q < 0, producer
C = an arbitrary level setting constant (atmospheres)

For an injection well, it is assumed that the viscosities and densities of the injected fluid and the native fluid are the same. If N wells are operating in the aquifer, then, by the principle of superposition, the potential at any point (x,y) is given by

$$\Phi(x,y) = -\frac{\mu}{4\pi kh} \sum_{i=1}^{N} \left\{ q_i \cdot \ln[(x-x_i)^2 + (y-y_i)^2] \right\} + C \qquad (3.12)$$

If the aquifer has gross fluid movement in the decreasing Y direction and no wells are operating in the aquifer, then the potential at any point (x,y) is

$$\Phi_o(x,y) = my + C_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.13)$$

where m = potential gradient with respect to Y (atmospheres/cm)
C_o = the constant potential along the X axis (atmospheres)

Again, from the principle of superposition, the potential at any point (x,y) due to the combined effects of N wells operating in an aquifer with pre-existing ground-water movement is given by

$$\Phi(x,y) = -\frac{\mu}{4\pi kh}\sum_{i=1}^{N}\left\{q_i \ln[(x-x_i)^2 + (y-y_i)^2]\right\} + \Phi_o(x,y) \quad . \quad . \quad . \quad . \quad (3.14)$$

Let r be the radius of an imaginary circular boundary of the desired storage area (see Fig. 3.2) in a saline aquifer experiencing gross fluid movement (flux). The resultant velocity at any point (x_B, y_B) on the boundary will be as shown in Figure 3.2.

If injection and production wells were installed around the outside of the boundary (Fig. 3.2) and operated at the proper rates to produce at every point on the boundary a resultant velocity equal but opposite in magnitude and direction to that of the flux, the enclosed area would then become an isopotential. Because there could be no movement of fluid across it, the area would become a zone of stagnation with a potential equal to that along the boundary. If an injection well were operated at any point within the zone of stagnation, the injected fluid would move out radially from the well until it reached the boundary and would behave as if it were in an aquifer of infinite areal extent which had no pre-existing ground-water movement.

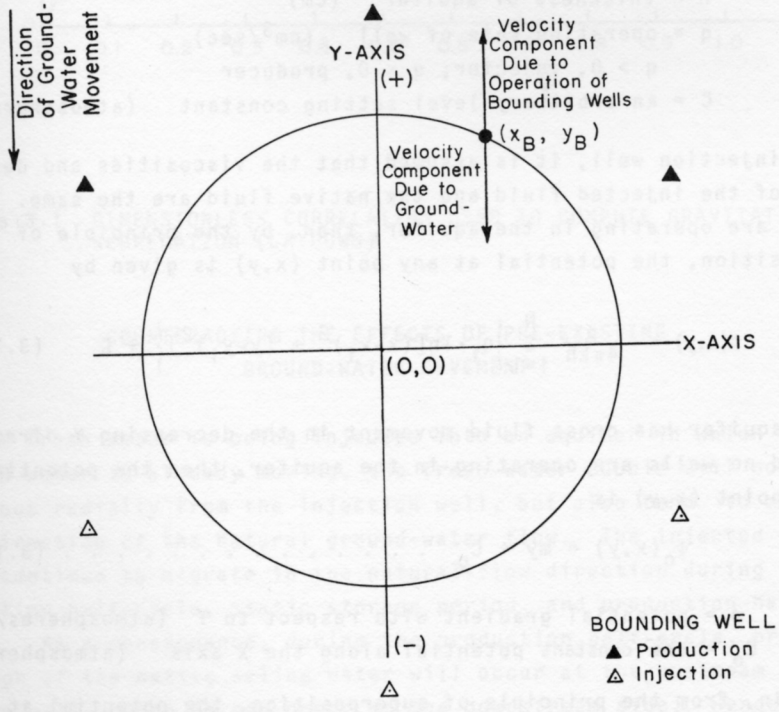


Figure 3.2 ARRANGEMENT OF BOUNDING WELLS TO COUNTERACT THE EFFECTS OF PRE-EXISTING GROUND-WATER MOVEMENT

In order to compute the rates at which the bounding wells should be operated, a desired potential for all points along the boundary and within the zone of stagnation must be selected. A good value is the potential that exists along the X axis (see Fig. 3.2) when no wells are operating in the zone of stagnation. If this value of potential is denoted by P_B, then

$$\Phi(x_B, y_B) = P_B$$

thus

$$P_B = -\frac{\mu}{4\pi Kh} \sum_{i=1}^{N} \left\{ q_i \cdot \ln[(x_B - x_i)^2 + (y_B - y_i)^2] \right\} + \Phi_o(x_B, y_B) \quad . \quad . \quad . \quad . \quad (3.15)$$

where

N = number of bounding wells

Equation 3.15 can be rearranged to obtain

$$\sum_{i=1}^{N} q_i \cdot \ln[x_B - x_i)^2 + (y_B - y_i)^2] = \frac{4\pi kh}{\mu} [\Phi_o(x_B, y_B) - P_B] \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.16)$$

With the selection of N different points along the boundary, N equations in N unknowns (the values of q_i) can be set up with Equation 3.16. These equations can then be solved for the flow rates of the bounding wells. Lin (1972) found, by taking only N points and generating N equations, that in some cases the generated matrix was, in mathematical terms, ill-conditioned. That is, it was difficult to obtain a solution vector using techniques such as Gauss elimination, Crout reduction, etc. He found that by taking 2N boundary points and generating 2N equations in N unknowns and finding the best solution vector, in the least-squares sense, to this set of equations, the problem of the ill-conditioned matrix was overcome.

The method for solving 2N equations in N unknowns to find the best solution vector in the least-squares sense can be found in any good matrix algebra text. Briefly, the method is this. Suppose we have a set of linear equations such that

$$AQ = B \quad . \quad . \quad . \quad . \quad . \quad .

tions mentioned in this section. A description and listing of the program, in FORTRAN IV language for use on an IBM 360/65 system, is given in Appendix B.

THE CALCULATION OF RECOVERY EFFICIENCIES

The following treatment assumes a horizontal, homogeneous, isotropic storage aquifer of infinite areal extent in which there is no pre-existing ground-water movement (or that pre-existing ground-water movement in the aquifer has been negated through the use of bounding wells). Additional assumptions are that the ratio of the viscosities of the injected and native fluids is unity and that the flow geometry is radial. The factors which influence recovery efficiency that must be considered are: (1) mixing due to molecular diffusion and longitudinal convective dispersion; and (2) gravitational segregation due to density difference.

Calculation of Mixing due to Molecular Diffusion and Longitudinal Convective Dispersion

Figure 3.3 idealizes the flow system during an injection half-cycle (gravitational segregation ignored). The injected fresh water displaces the native salt water away from the source. As the interface between the fresh water and salt water moves in the aquifer, the mixing between the two fluids will generate a transition or mixed zone in which the composition of either fluid will vary from 100 percent to 0 percent. The length of this mixed zone, as it moves in the aquifer, is dependent on the total distance traveled by the interface, the velocity of the interface, the total time of contact between the liquids, the properties of the liquids, and the properties of the porous medium.

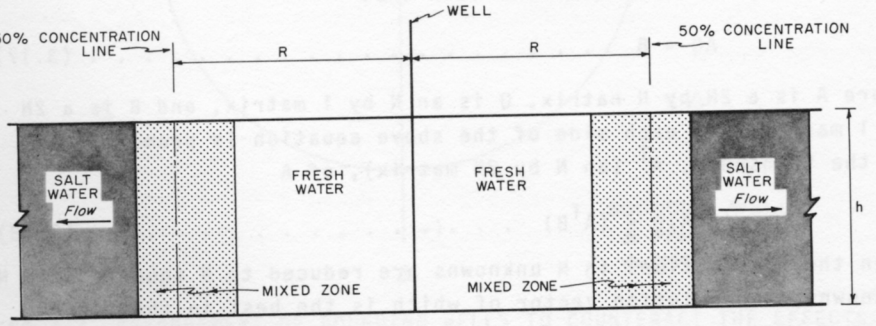


Figure 3.3 SCHEMATIC REPRESENTATION OF THE DISPLACEMENT PROCESS DURING AN INJECTION HALF-CYCLE (NO DENSITY DIFFERENCE)

In Figure 3.3, R is the radius of the injected fluid at any time t (no mixing assumed). Therefore, the average value of c_i/c_o or c_n/c_o at radius R is 0.5. The length of the mixed zone at any time (t) and about any radius (R) is computed from the appropriate form of Equation 3.5.

Calculation of Gravitational Segregation due to Density Difference

The density difference between the injected fresh water and the native salt water will cause the mixed zone between the two liquids to incline with respect to the vertical (see Fig. 3.4). The less dense fresh water will rise over the more dense salt water. The gravitational segregation between the two fluids at any time can be represented by the tangent of the angle, θ, that the 50-percent concentration line makes with respect to the vertical ($\tan \theta = 2XR/h$).

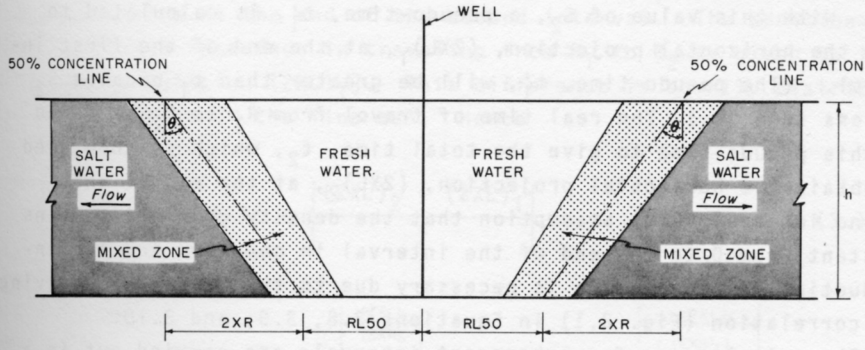


Figure 3.4 SCHEMATIC REPRESENTATION OF THE DISPLACEMENT PROCESS DURING AN INJECTION HALF-CYCLE DUE TO BOTH MIXING AND DENSITY DIFFERENCE.

Equations 3.8, 3.9, and 3.10 cannot be used directly to calculate gravitational segregation in a radial system because: they describe gravitational segregation in linear systems only; and the density gradient, S, is not a constant, but varies continuously as the injection, storage, and retrieval process progresses. In order for Equations 3.8, 3.9, and 3.10 to be used to compute gravitational segregation in a radial system, a stepwise procedure is first needed to allow for the continuously changing value of S. Secondly, a correction must be applied to approximate the computed values of gravitational segregation in radial geometry. The method by which the equations are applied is outlined below.

Consider Figure 3.5, which illustrates the stepwise procedure for calculating the value of gravitational segregation before

approximation to radial geometry. Note that only the 50 percent concentration lines are considered. Let the fresh water be injected to a radius of R_{max}. Divide the distance (R_{max}) into equal intervals, such as OR_1, R_1R_2, etc. The length of the mixed zone is calculated at the center, C_1, of the first interval from the appropriate form of Equation 3.5. This value of mixed zone length at C_1 is then used to compute the value of the density gradient, S_1, at C_1. (It is assumed that the density gradient has had a constant value of S_1 over the interval OR_1.) With the value of S_1 and the real time of travel, t_1, from 0 to R_1, the horizontal projection $(2XL)_1$ at the end of the first interval can be calculated from Equation 3.8, 3.9, or 3.10.

For the second interval, R_1R_2, the length of the mixed zone is computed at the center, C_2, of the interval. At C_2, this length will be longer than that at C_1; therefore S_2 will be less than S_1.

It is next assumed that S_2 was constant over the total interval OR_2. With this value of S_2, a pseudo-time, t_1', is calculated to give the horizontal projection, $(2XL)_1$, at the end of the first interval. (The pseudo-time, t_1', will be greater than t_1 because S_2 is less than S_1.) The real time of travel from R_1 to R_2 is added to this pseudo-time to give the total time, t_2, which is then used to obtain the horizontal projection, $(2XL)_2$, at the end of the second interval. The assumption that the density gradient remains constant from 0 to the end of the interval in question and the introduction of pseudo-time is necessary due to the manner of deriving the correlation (Fig. 3.1) in Equations 3.8, 3.9, and 3.10.

The calculations for subsequent intervals are carried out in a similar fashion. For the last interval of the injection half-cycle, the static storage time before the beginning of production is included in the total time.

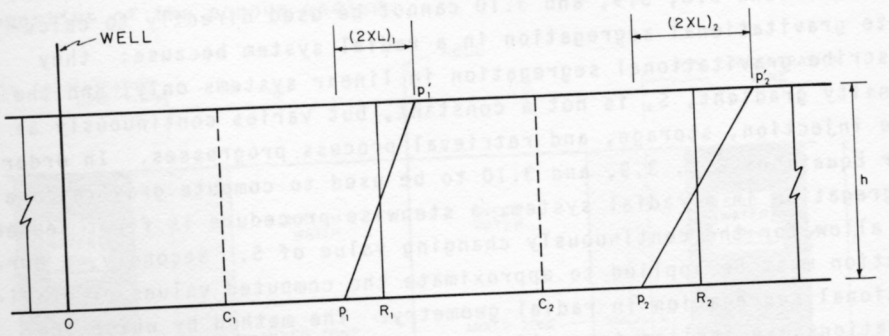


Figure 3.5 SCHEMATIC REPRESENTATION TO ILLUSTRATE THE CALCULATION OF GRAVITATIONAL SEGREGATION BEFORE APPROXIMATION TO RADIAL GEOMETRY.

In the preceding treatment, the projection of the interface was calculated on the basis of linear movement. In radial injection, the projection of the interface is less during injection half-cycles and greater for production half-cycles. The calculated projections shown in Figure 3.5 must be corrected for radial geometry.

In Figure 3.6, which illustrates the approximate conversion, lines p_1p_1' and p_2p_2' represent the 50-percent concentration lines at the ends of the first and second intervals of linear flow. Note that p_1p_1' and p_2p_2' in Figure 3.6 are the same as in Figure 3.5. For the first interval it is assumed that no correction is necessary; that is, $(2XL)_1 = (2XR)_1$. The corrected projection, $(2XR)_2$, at the end of the second interval is obtained in the following manner: Compute the radius to point (a) so that the annular area, $\pi[R_2^2 - a^2]$, is equal to the annular area, $\pi[R_1^2 - p_1^2]$. Compute the radius to point (a') so that the annular area, $\pi[(a')^2 - R_2^2]$, is equal to the annular area, $\pi[(p_1')^2 - R_1^2]$. Line aa' is line p_1p_1' converted to radial geometry at the end of the second interval. The additional tilting of the interface in traveling from R_1 to R_2 is given by the difference, $[(2XL)_2 - (2XL)_1]$. This difference is equally distributed on each side of the line aa'. Hence

$$ba = a'b' = \left| \frac{(2XL)_2 - (2XL)_1}{2} \right| \quad \ldots\ldots\ldots \quad (3.19)$$

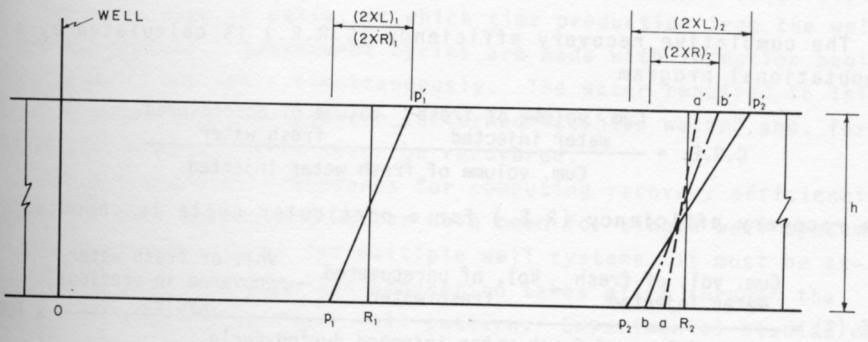


Figure 3.6 SCHEMATIC REPRESENTATION TO ILLUSTRATE THE APPROXIMATION OF GRAVITATIONAL SEGREGATION CALCULATIONS IN RADIAL GEOMETRY.

The horizontal projection, $(2XR)_2$, of the line bb' is the projection of the 50 percent concentration line in radial geometry at the end of the second interval. The calculations for subsequent intervals are carried out in a similar manner.

Recovery Efficiency

Stored fresh water is produced until the leading edge of the mixed zone reaches the breakthrough radius, RBT (see Fig. 3.7). For a single-well system, this would be the wellbore radius. For a multiple well system, it would be the radius from the center of the well pattern to the outer ring of wells. The volume of water contained in the frustrum of the cone having a height, h, and upper/lower radii of RU50/RL50, respectively, is the volume of unrecovered fresh water.

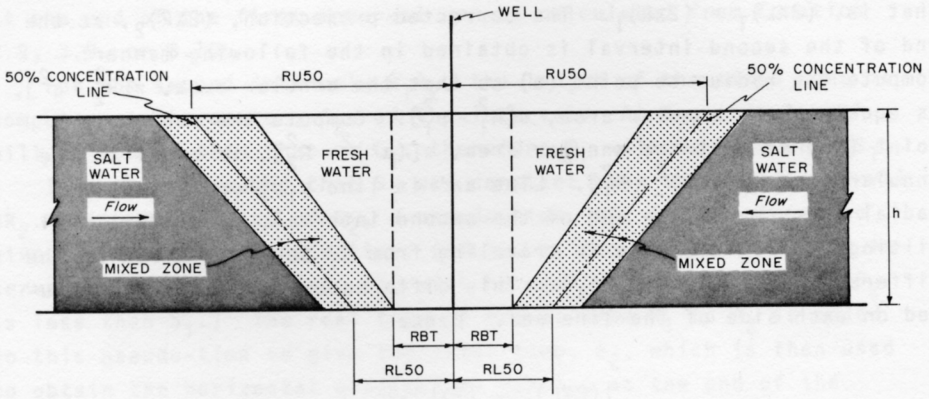


Figure 3.7 SCHEMATIC DIAGRAM ILLUSTRATING THE CALCULATION OF RECOVERY EFFICIENCY.

The cumulative recovery efficiency (C.R.E.) is calculated by a computational program

$$C.R.E. = \frac{\text{Cum. volume of fresh water injected} - \text{Volume of unrecovered fresh water}}{\text{Cum. volume of fresh water injected}}$$

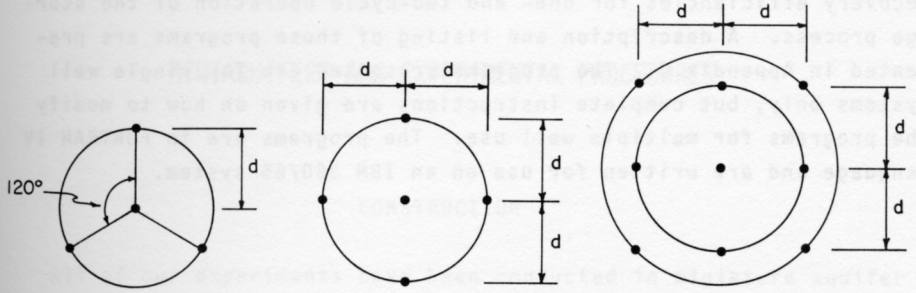
The recovery efficiency (R.E.) for a particular cycle is computed

$$R.E.(\%) = \frac{\text{Cum. vol. of fresh water injected} - \text{Vol. of unrecovered fresh water} - \text{Vol. of fresh water recovered in previous cycles}}{\text{Volume of fresh water injected during cycle}} \times 100$$

Multiple Well Systems

To this point our exposition has dealt with single wells. In a field application of the storage process, a well field likely will be needed instead of a single well. In this investigation it was assumed that these well fields would be symmetrical with one well

at the center of the pattern (see Fig. 3.8). It was also assumed that the fields would be operated so that the injected bubble of fresh water would remain essentially circular.



a) FOUR WELL PATTERN b) FIVE WELL PATTERN c) NINE WELL PATTERN

Figure 3.8 SOME POSSIBLE WELL FIELD PATTERNS

The operating procedure for a well field, such as that in Figure 3.8c, would be: (1) Inject into the center well until the lagging edge of the mixed zone passes the inner ring of wells. (2) Start injection in the inner wells (with injection continuing in the center well) until the lagging edge of the mixed zone passes the outer ring of wells. (3) Inject into all nine wells until the desired quantity is injected. (4) Allow the injected water to stand until needed. (5) Produce all nine wells until breakthrough occurs at the outer ring of wells, at which time production from the well field is stopped. Subsequent cycles are made with injection beginning in all nine wells simultaneously. The water required to initially "sweep out" the pattern is termed "cushion water" and, for practical purposes, will never be recovered.

The mathematical procedures for computing recovery efficiencies that have been proposed thus far have been for single well systems. To use these procedures for multiple well systems, it must be assumed that all injection and production takes place through the center well of the multiple well pattern. Experimental studies detailed later show that this is a valid assumption as long as the well field is symmetrical and is operated so that the injected bubble of fresh water remains essentially circular.

Computer Programs for Computing Recovery Efficiencies

Detailed computer programs have been developed for computing recovery efficiencies for one- and two-cycle operation of the storage process. A description and listing of these programs are presented in Appendix C. The programs presented are for single well systems only, but complete instructions are given on how to modify the programs for multiple well use. The programs are in FORTRAN IV language and are written for use on an IBM 360/65 system.

IV

MINIAQUIFERS AND EXPERIMENTAL PROCEDURES

CONSTRUCTION

All of our experiments have been conducted in miniature aquifer systems, herein termed "miniaquifers." These were constructed of synthetic sandstone in the laboratory with an epoxy resin as a bonding agent for unconsolidated sand or other granular material. Although general procedures for the construction of miniaquifers can be outlined, the production of a system of reasonable homogeneity is an art that requires both practice and a great deal of attention to detail.

All miniaquifers used in the present study were constructed from a white, rounded blasting sand, frequently called sugar sand. A sieve analysis is included in Table 4.1.

TABLE 4.1--Sieve Analyses of Sand Used in Miniaquifer Construction

Sieve No. (US)	Screen Opening (in.)	Analysis No. 1			Analysis No. 2			Analysis No. 3		
		Weight Retained (gm)	Percent Retained	Cum. % Retained	Weight Retained (gm)	Percent Retained	Cum. % Retained	Weight Retained (gm)	Percent Retained	Cum. % Retained
30	0.0328	0	0	0	0	0	0	0	0	0
40	0.0164	0.4	0.1	0.1	0.4	0.1	0.1	0.4	0.1	0.1
50	0.0116	2.0	0.5	0.6	2.3	0.6	0.7	2.4	0.6	0.7
70	0.0082	42.5	10.7	11.3	52.0	13.1	13.8	47.9	12.0	12.7
100	0.0058	184.5	46.5	57.8	185.4	46.7	60.5	186.6	47.0	59.7
140	0.0041	104.6	26.3	84.1	96.6	24.3	84.8	99.4	25.0	84.7
200	0.0029	57.9	14.6	98.7	53.9	13.6	98.4	53.4	13.5	98.2
Pan	--	5.3	1.3	100.0	6.5	1.6	100.0	7.1	1.8	100.0
Totals		397.2			397.1			397.2		

Small glass beads may be used in place of sand, if desired. The bonding agent was Armstrong C-7 epoxy resin mixed with 8% by weight of activator A, as recommended by the manufacturer. It is important to obtain thorough mixing of the activator and resin to avoid

points of weakness in the miniaquifer.

The quantity of epoxy to be mixed with the sand depends on the surface area of the sand and is therefore a function of grain-size distribution. For the sieve analyses shown, the amount of resin was equal to 6% of the sand weight. The objective is to use the smallest quantity of resin that will completely coat all grain surfaces lightly. If more resin is used, it will be forced out into the pore spaces during compaction and lead to unacceptable heterogeneity. The proper quantity and thorough mixing of resin with the sand results in formation of a pendular ring of bonding agent at each grain contact and produces a pore space configuration similar to that found in a lightly-consolidated natural sandstone.

The pot life of the activated resin is about one-half hour, depending on the ambient temperature. Because the reaction is highly exothermic, removal of the heat of reaction will extend this time appreciably. Consequently, if the epoxy is mixed with the sand as soon as possible, the sand will act as a heat sink and slow the rate of reaction. We could thus obtain a working time of about 1 hour for mixing and molding.

The following basic steps involved in miniaquifer construction are listed in the order in which they should be accomplished.

Mold Construction

Construct a mold from 3/4" plywood with the general shape and thickness desired in the finished product and at least 2 inches larger in both length and width to allow for trimming. The finished thickness will be about 1/16 inch less than the mold depth because the miniaquifer surfaces should be sanded before the application of epoxy sealing coats. The bottom of the mold should be covered with tightly-stretched polyethylene to serve as a release after curing. The sides of the mold should be covered with soap for the same purpose. Aluminum foil placed beneath the polyethylene is helpful in curing, but is not essential.

Epoxy-Sand Mixing Procedure

The quantity of sand or glass beads required to fill the mold should be determined with an additional 10% allowed for loss and waste. The material is then split into portions of 3000 grams each for batch mixing (and storage if required). The use of larger batches will probably result in incomplete mixing and consequent heterogeneity in the finished product. The activator and resin are carefully weighed, mixed thoroughly and added to the sand which has been spread in a thin layer over the bottom of a shallow porcelain pan. Pans measuring 12" x 20" x 2.5" are well suited for this purpose.

The sand and epoxy are mixed as quickly and thoroughly as possible by *hand kneading*. No mechanical mixing technique has been found to be successful. Thin plastic gloves should be worn to protect the hands. Mixing should continue until no small beads of epoxy remain, and all sand grains have been uniformly coated. This condition can be determined by packing a handful of the mix into a ball and striking successive portions away by hand to expose new surfaces for examination. Beads of unmixed resin will appear as

shiny spots. Uncoated sand will be evident from its color and texture.

The mixture should be placed directly in the mold if it is to be used immediately or quick-frozen if several batches must be accumulated for a large miniaquifer. If it is to be frozen, place the mixture immediately into a plastic bag, seal the bag with a twist tie, and flatten the mixture out to a thickness of no more than 1 inch. Place the bag on a layer of dry ice in a freezer and spread dry ice over the top surface to insure rapid freezing. Once frozen, the mixture can be stored at approximately 0°F for a period of about 10 days. When sufficient material has been accumulated, it is removed from the freezer, thawed, and packed into the mold as rapidly as possible.

Packing and Curing

Packing should begin as soon as the thawed mixture is readily workable, but before it has reached room temperature in order to provide time for all finishing operations in advance of the initial set. The mixture is carefully packed by hand with continuous tamping. When the mold is filled, sheet polyethylene is placed over the surface, which is then subjected to thorough rolling with a heavy steel roller. Throughout the rolling process, excess mix is removed ahead of the roller by means of a large spatula or trowel. An improperly rolled surface will require that large quantities of material be removed later by sanding. Rolling must not be continued beyond the time of initial set because planes of weakness will be introduced into the matrix.

The molded matrix should stand undisturbed at room temperature (70°-90°F) for at least 48 hours for the initial set. The top polyethylene sheet is then removed, the matrix transferred into the curing oven, and the temperature showly raised to 180°F. The time required to reach this temperature without introducing thermal strains will be 4-5 hours in the case of large miniaquifers. Hot air, heated externally and circulated through the oven, is recommended, as more direct heating methods will result in "hot spots" which will overcure and in cracks caused by thermal stresses. The matrix should be held at a curing temperature of 180-190°F for 5-24 hours, depending on its thickness. The assembly is then cooled very slowly to room temperature.

After cooling, the sides of the mold are removed and the matrix trimmed to size with an electric hand saw with a diamond blade. At this time the product should be carefully inspected visually for obvious inhomogeneities.

Coating and Well Installation

The matrix should be coated to seal all surfaces and edges except those through which flow is to take place. This must be accomplished in a manner that will preclude imbibition of the coating material into the matrix. On large miniaquifers this can most easily be done by applying a succession of very light coats of the same resin (C-7 and activator A) by means of a paint roller. Each coat is allowed to set before the next is applied. After enough coats have been applied to ensure a sealed surface, a thick coat is smoothed on with a plasterer's trowel. Two or more such "float coats" may be required. For flow fittings, such as "headers," to be constructed into the matrix, an "egg-beater" type hand drill should be used, with the cuttings removed by a vacuum pump. (Electric hand drills should not be used. The heat generated by rapid drilling will glaze the wellbore and produce partial [and non-uniform] plugging.) A block of phenolic containing a tubing fitting is cemented over the hole to form the flow connection to the well. A capacitance cell (described later) may be incorporated if

desired. Isopotentials may be incorporated, when required, by milling (plowing) a partially-penetrating groove into the matrix and covering it with a phenolic strip cemented into place with epoxy.

After all fittings have been attached, the system must be tested for leaks. This is most easily accomplished by charging of the miniaquifer with Freon-12 under a few psi pressure followed by a careful going-over of all surfaces and fittings with a Freon leak detector. All leaks must be carefully sealed with epoxy. Special attention should be given to all sharp changes in the surface, such as edges and corners.

DETERMINATION OF MINIAQUIFER PARAMETERS

Physical dimensions are, of course, readily determined by direct measurement before the coating of the miniaquifer. Porosity can be determined conveniently during the process of saturation. Reliable porosity and permeability values require that the system be completely saturated.

Saturation can usually be accomplished by: (1) flushing the air completely from the system with carbon dioxide; (2) evacuating the system as completely as possible with a good vacuum pump; and (3) carefully metering the saturating liquid into the system by means of a constant-rate positive displacement pump or other accurate metering device. A known quantity of fluid is pumped through the system and the effluent carefully measured. The pore volume in the miniaquifer is then obtained from the difference.

At least 1-1/2 pore volumes should be produced from the saturated miniaquifer under a small back pressure (5-10 psi) to ensure that any small amounts of carbon dioxide not removed during evacuation are dissolved and removed from the miniaquifer. The system may be tested for complete saturation by pumping a small, known quantity of liquid into the closed-off system and noting the rise in pressure. The system compressibility so noted may be used to estimate the degree of saturation.

Permeability should be measured where well-bore effects (dimensions or slight plugging) will not cloud the results, usually by making pressure measurements at points where injection and withdrawal are not taking place.

Instrumentation

The principal required instrumentation should have the capability to instantaneously and continuously detect and record the concentration of injected fluid in the effluent from a discharge well. A tracer added to the injected fluid has a dielectric constant that differs from that of the native fluid. Such a tracer

permits the use of a capacitance cell fitted to the well-bore to detect fluid composition. A detailed drawing of such a cell is shown in Figure 4.16. The cell is connected by a coaxial cable to an E.H. Sargent Chemical Oscillometer (Model V) where it forms a portion of a tank circuit in a high-frequency oscillator. The cell contents (fluid composition) determine the capacitance, with variations causing a departure from resonance in the circuit. The millivoltages that result are fed through a recording adapter, which permits sensitivity and zero control, and thence to a strip chart recorder. By prior calibration, the pen position at any instant may be interpreted in terms of fluid composition being discharged from the well and, thus, through the flow cell. Where a miniaquifer contains more than one well, each is fitted with a capacitance cell. Switching from one well to another is accomplished by a rotary coaxial switch.

At times, knowledge of pressures or pressure changes is desired. If the fluid volume required to actuate the sensing instrument is of no consequence (such as on an isopotential), ordinary Bourdon gauges have been used. Where the required volume of fluid must be minimal, a pressure transducer with readout, such as the Statham model, has been very satisfactory.

Fluids are metered into and out of all miniaquifer systems by means of constant-rate positive displacement pumps. These single-stroke piston pumps are driven through a gear train by a synchronous motor. The precision-honed barrels allow the use of a single calibration chart for all pumps of a given size. The pumping rate may be controlled over a wide range by proper gear selection. Several such pumps are shown in Figure 4.1 and the available pumping rates are given in Table 4.2.

In some instances it was desirable to photographically record the portion of the miniaquifer occupied by the injected fluid. This was accomplished by incorporating a dye in the fluid and taking either still or motion pictures during the displacement. Still photographs were made with a 35 mm camera (Beseler Topcon D); motion pictures were taken with a 16 mm camera (Bolex).

Validation of Homogeneity

Because the miniaquifers should be as homogeneous as possible, it is essential that some evaluation of the homogeneity be made. This can most easily be accomplished by injecting fluid into a single well (miniaquifer fluid with a dye added). Thus the injected and displaced fluids are miscible and are essentially

identical in density and viscosity. The dye allows visual tracking of the displacement front to check for any significant irregularities.

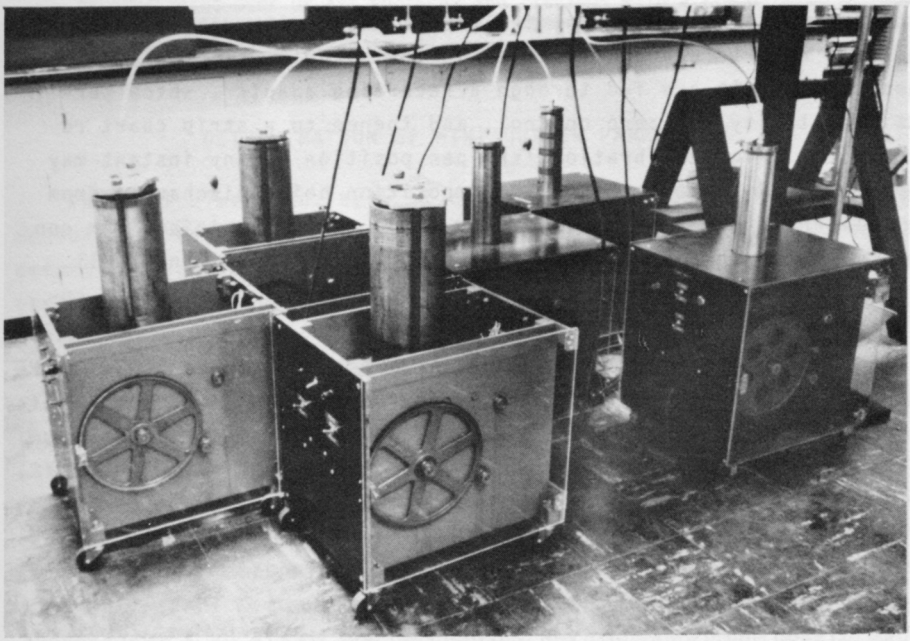


Figure 4.1 VIEW OF THREE LARGE-BARREL AND THREE SMALL-BARREL PUMPS USED IN EXPERIMENTS

TABLE 4.2 Pump Calibration, Constant Rate Pumps

x	Large Barrel (2800 cc) *Gears 200-20 12-x	Small Barrel (700 cc) *Gears 200-20 12-x
20	8.046 cc/min	2.005 cc/min
22	7.314	1.823
24	6.705	1.671
32	5.028	1.253
44	3.657	0.912
56	2.873	0.716
72	2.235	0.557
96	1.676	0.418
110	1.463	0.365
120	1.341	0.334

Values are ±0.07% at 72°F

*For gear ratios of 110-110, 12-x the above values should be multiplied by 10^{-1}.
For gear ratios of 20-200, 12-x the above values should be multiplied by 10^{-2}.

Because miniaquifers frequently contain non-homogeneities causing nonradial flow, a comparison between the observed and theoretical frontal shape must be made. This can be accomplished by comparing the calculated and observed positions of the injected fluid along several streamlines at various times.

EXPERIMENTAL PROCEDURES

Linear Systems

Linear systems of differing length and height were used to represent vertical sections through a linear aquifer in the study of gravitational laydown of a displacement front. All linear miniaquifers were thin enough (0.634 cm) to be translucent so that the frontal configuration could be observed by transmitted light. The properties of the two principal linear models (Fig. 4.2) are shown in Table 4.3.

TABLE 4.3 Physical Properties of Linear Miniaquifer

Miniaquifer	Length L, cm	Height h, cm	Width W, cm	Porosity ϕ, fraction	Permeability k_H, darcies
A	116.8	17.77	0.634	0.21	5.93
B	100.27	7.62	0.634	0.25	6.83

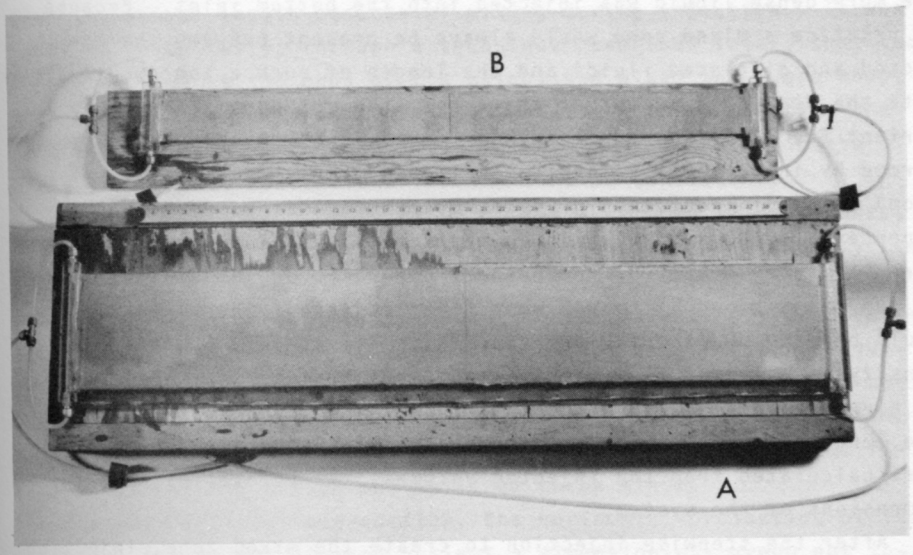


Figure 4.2 LINEAR MINIAQUIFERS

Water and sodium dichromate solutions were used as the two miscible fluids in all experimental runs to determine laydown. The natural red color of the dichromate solution facilitated visual observations of the frontal inclination. Viscosity ratios and density differences between the fluids were controlled by varying the concentration of the sodium dichromate. The fluid properties are shown in Table 4.4. In all cases, the fluid was injected with a syringe pump.

TABLE 4.4 Experimental Conditions--Gravity Segregation Study

Run No.	Model	Fluid No. 1	Fluid No. 2	$\bar{\mu}$ @ 30°C poise	$\Delta\rho$ @ 30°C gm/cc	Mixed Zone Length cm
1	A	Water	100 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.016	0.43	9.5
2	A	Water	100 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.016	0.43	19.0
3	A	Water	100 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.016	0.43	38.0
4	A	Water	100 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.016	0.43	60.0
5	B	Water	150 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.0215	0.55	14.9
6	B	Water	100 gm $Na_2Cr_2O_7 \cdot 2H_2O$/100 cc water	0.016	0.43	14.9

Each miniaquifer was initially evacuated by means of a vacuum pump and saturated with the less dense liquid (water). The miniaquifer was then positioned so that the long dimension was vertical. The more dense liquid was injected into the bottom inlet. Because in practice a mixed zone would always be present between the injected and displaced fluids and the length of such a zone would dictate the density gradient at the front, the one parameter deemed important for study was the length of mixed zone. Such a zone was formed by injecting the more dense liquid (sodium dichromate solution) in small increments of different concentration ranging from 5 percent to 100 percent. The number and sizes of the increments were varied from one run to another, depending on the desired size of the mixed zone. Although when first injected, the mixed zone would be made up of a fluid possessing small stepwise changes in density, subsequent flow within the porous medium resulted in mixing to give a relatively smooth change in density throughout the zone. The lengths of the mixed zone and the resulting density gradients were calculated from the injected volumes, the porosities, and the dimensions of the system.

After the stepwise injection to create the mixed zone, injection of the more dense fluid was continued until the zone approached the

center of the miniaquifer. Because of the vertical orientation of the system and the injection of the more dense fluid from the bottom, the result was a well-defined mixed zone perpendicular to the long edge of the miniaquifer.

After injection was stopped (with the edge of the mixed zone at about the center of the system), both the inlet and outlet valves were closed. The system was then placed horizontally on edge and zero time recorded. As the front was visually observed to "tilt" as a function of time, the angle of inclination of a fixed color intensity was recorded by tracing it with a grease pencil directly on the miniaquifer surface. The length of the projection of this line on the horizontal was measured with a scale and recorded with the corresponding time. Such data were taken about every 30 minutes until one edge of the mixed zone approached an end of the system. At this point, the run was discontinued and the system flushed with water in preparation for another experiment.

All experimental runs, together with the corresponding field properties, are listed in Table 4.4. Note that Runs 1 through 4 were made in the same miniaquifer with the same fluid, but with differing lengths of mixed zones. Runs 5 and 6 were made in another miniaquifer with different fluid compositions, but with the same lengths of mixed zones. Thus, all pertinent variables in Eq. 3.7 were changed.

It is evident that the length of the mixed zone, if calculated on the basis of total volume of all the increments, will be shorter than the actual length because of mixing during injection. It was found experimentally, through a technique described later, that the concentrations at the ends of the calculated mixed zone were about 3 percent and 97 percent. In all calculations for predicting the recovery efficiency, the density gradients were computed on the basis of these concentrations for the sake of consistency.

From the foregoing description of the experimental procedures, it is evident that no bulk flow of fluid from one end of the system to the other occurred during the runs. Fluid movement which gave rise to this inclination of the front consisted entirely of a rotational type of flow both toward and from each end of the miniaquifer. One would expect, therefore, that the data would be most reliable when the mix-zone was farthest from the ends of the system. However, any error introduced into the data as a result of the finite length of the system was believed to be minor.

For gravitational segregation, the horizontal projection of the interface (2y/h) was plotted against a dimensionless group not involving the density gradient (Fig. 4.3). The curves demonstrate

that the larger the mixed zone, the slower the rate of inclination of the front. When a similar plot is made against the dimensionless term ψ, which includes the density gradient as a parameter, all the data are brought into reasonable agreement. This correlation is shown in Figure 4.4.

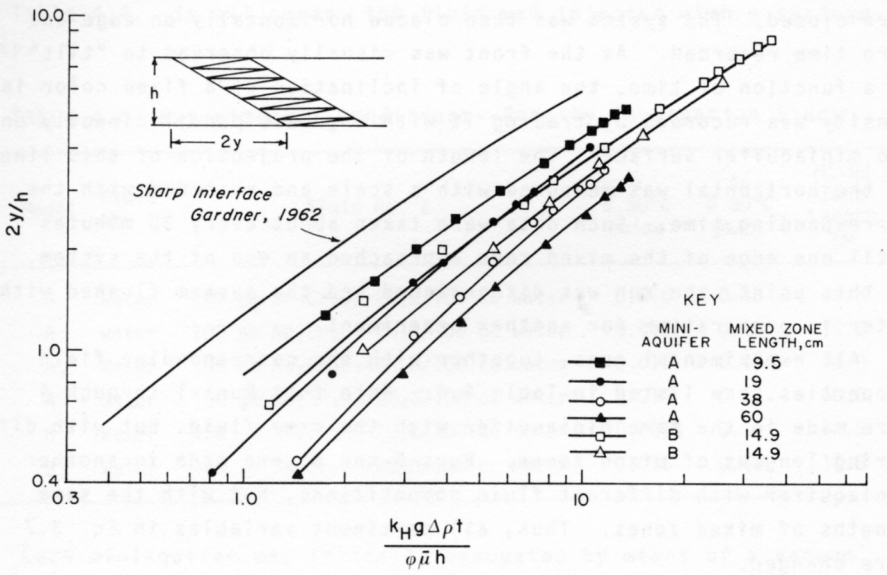


Figure 4.3 EFFECT OF MIXED-ZONE LENGTH ON GRAVITATIONAL LAYDOWN.

Wedge-Shaped Miniaquifers

To study a flow system consisting of a single well in a horizontal, homogeneous, porous medium, it is not necessary to construct a miniaquifer which forms a complete circle about the well. Because all streamlines emanate radially from the center, any radius constitutes a "no-flow boundary" across which flow does not occur. It is possible therefore to cut a sector out of the system along any two such radii and seal the edges without disturbing the characteristics of radial flow. Thus, a half-circle could be used, in which case the well would be considered a half-well from the standpoint of injection or production rates and volumes. Likewise, the radial system could be equally well represented by a 45° sector of a circle containing a well at the apex, which would be treated as 1/8 of a full well. A sector of any other reasonable angular measurement would be equally satisfactory.

Wedge-shaped miniaquifers are not valid, however, for dipping

systems when two fluids of different density are used. In this case, the only "no-flow boundary" is a line parallel to the dip. Thus a 180°-sector (or the full circle) should be used.

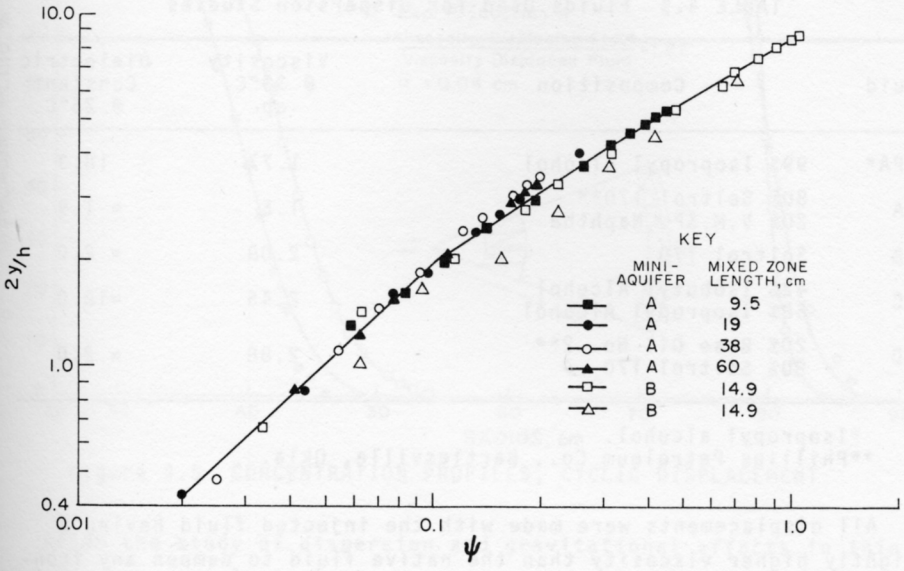


Figure 4.4 DIMENSIONLESS CORRELATION FOR GRAVITY LAYDOWN BASED ON FIGURE 4.3.

In view of these circumstances, initial work on dispersion in radial flow was undertaken on a thin (0.55 cm) wedge-shaped miniaquifer (a 45° sector). The radial distance was 91.4 cm. The matrix had a porosity of 0.24 and a permeability of 3.85 darcies. The wellbore, which had a radius of 3/8", was located at the apex. All other surfaces were sealed except for the circumference, which was fitted with an end piece so that fluids could be injected or produced from this face to produce an isopotential boundary.

The top surface of the miniaquifer was made smooth with a cemented-in-place sheet of acrylic plastic (1/16" Lucite). A sheet of 20-mesh copper screen was similarly cemented to the bottom surface. This screen, serving as one plate of a capacitor, permitted in-situ compositions to be determined from the dielectric constant of the fluid. A 1-inch diameter circular copper disk, which could be moved about on the upper surface, formed the second plate of the capacitor. By this arrangement, the miniaquifer and its associated fluid in the region being studied formed a capacitor in the tank circuit of a high-frequency oscillator (E. H. Sargent Chemical Oscillometer). The compositions were determined in a manner similar

to that described by Peffer (1962).

The fluids for dispersion studies were chosen on the basis of dielectric constants and viscosities (Table 4.5).

TABLE 4.5 Fluids Used For Dispersion Studies

Fluid	Composition	Viscosity @ 30°C cp	Dielectric Constant @ 25°C
IPA*	99% Isopropyl Alcohol	1.77	18.3
A	80% Soltrol 170** 20% V.M.&P. Naphtha	1.5	≈ 1.9
B	Soltrol 170	2.08	≈ 2.0
C	42% Isobutyl Alcohol 58% Isopropyl Alcohol	2.45	≈18.0
D	20% Base Oil No. 2** 80% Soltrol 170	2.88	≈ 2.0

*Isopropyl alcohol.
**Phillips Petroleum Co., Bartlesville, Okla.

All displacements were made with the injected fluid having a slightly higher viscosity than the native fluid to dampen any frontal irregularities in the system that might result from minor matrix heterogeneity. The fluid densities were sufficiently similar so that gravitational effects could be neglected in a miniaquifer of this thickness.

Because the objective of this part of the work was to test the validity of the Gardner equation during multi-cycle operation, it was first necessary to experimentally determine the dispersvity (α) for the system. Isopropyl alcohol was injected at a constant rate to displace fluid A (Table 4.5). The concentration profiles through the resulting mixed zone were measured at various frontal positions during one injection half-cycle. A trial-and-error procedure determined the value of α which gave the best agreement between the experimentally-determined profiles and those predicted by the Gardner equation.

The miniaquifer was then operated for two complete injection-production cycles. The excellent agreement between the observed concentration profiles and the calculated profiles (Fig. 4.5) was considered an experimental validation of the Gardner equation for multi-cycle use. As a consequence, this equation was adopted for use in all subsequent numerical models. Details of the experimental work were given by Esmail (1966).

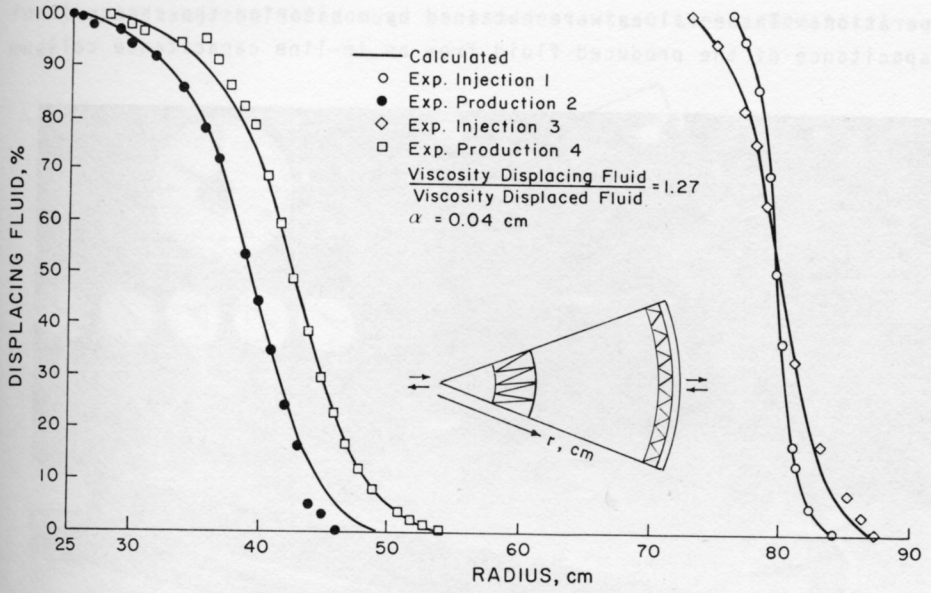


Figure 4.5 CONCENTRATION PROFILES, CYCLIC DISPLACEMENT

After the study of dispersion and gravitational effects in thin systems, the next logical step was to mathematically combine the two effects and predict the performance of a radial system in which both were taking place simultaneously. Such a procedure would also test the feasibility of applying the data on gravitational segregation from a linear system to radial systems.

A thick pie-shaped miniaquifer (Fig. 4.6) was next built. This miniaquifer, a 45°-sector, had a single injection-production well (at the apex) surrounded by a number of observation wells. The radius was 91 cm and the thickness, 7.5 cm. Because the epoxy resin used for sealing the surfaces was transparent, the flow of a dyed fluid through the matrix could be observed visually on three faces. (The fourth surface was, of course, obscured by the plywood base support.) The basic construction followed that described earlier except that provisions for in-situ composition determination were omitted because of the excessive thickness. Pressure taps (shown in Fig. 4.6) were added to provide detailed information on matrix permeability. Construction details and experimental procedures were described by Kumar (1967).

Because no provisions were included for the determination of composition profiles within the miniaquifer, the validity of the mathematical model rested entirely upon a comparison of the observed and predicted recovery efficiencies during single- and multi-cycle

operation. These values were obtained by monitoring the change in capacitance of the produced fluid from an in-line capacitance cell.

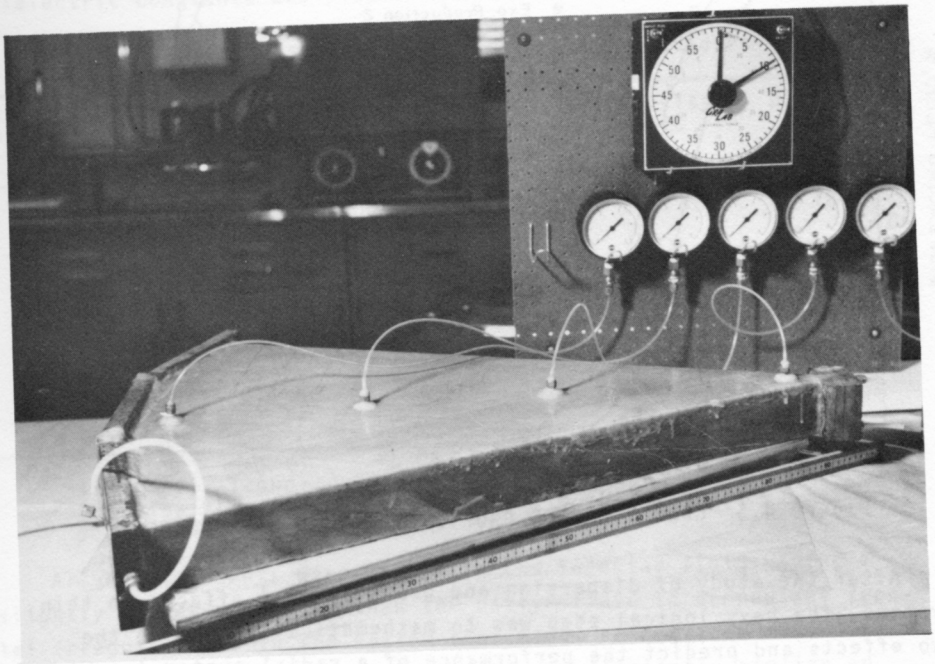


Figure 4.6 THE RADIAL MINIAQUIFER

In-place measurements showed an average porosity of 0.255 and an average horizontal permeability of 6.9 darcies. The actual measured permeability distribution was as shown in Figure 4.7. The average vertical permeability, 6.2 darcies, was determined from six core plugs drilled out of the discarded side trimmings obtained during construction of the miniaquifer.

The general arrangement of the miniaquifer and its associated equipment are shown in Figure 4.8. Fluid injection was accomplished by three constant-rate, positive-displacement pumps partially shown beneath the table in the foreground. The chemical oscillometer and strip-chart recorder used for obtaining the composition of the produced fluid may be seen on the small rolling table to the right.

Six cyclic injection/production runs were made in the thick miniaquifer with miscible fluids of varying densities, viscosities and dielectric constants (Table 4.6). In most cases, the viscosity ratio (injected fluid/native fluid) was slightly above unity to minimize the effect of minor inhomogeneities. Run No. 1 was used to determine a value of α (longitudinal dispersivity coefficient)

for the system; the other runs were made to test the mathematical model.

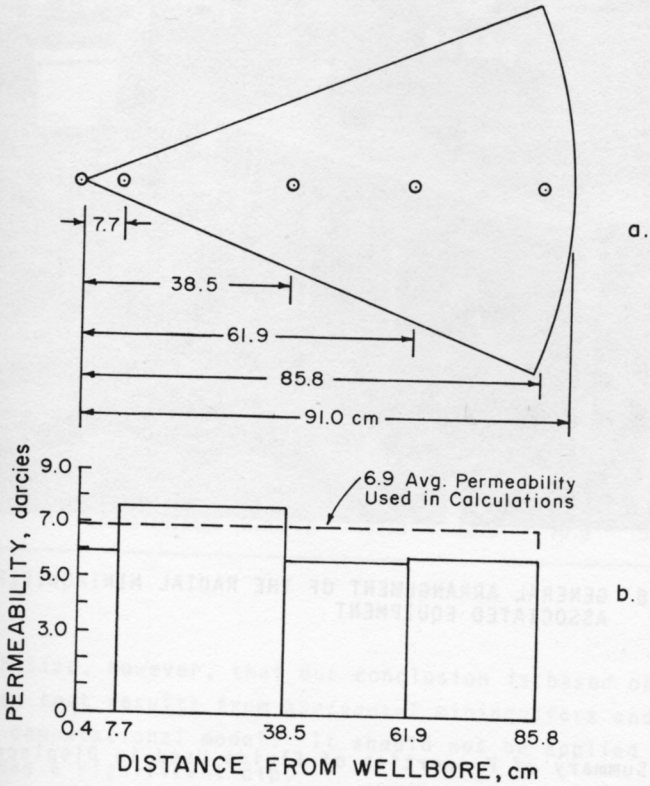


Figure 4.7 HORIZONTAL PERMEABILITY DISTRIBUTION IN THE RADIAL MINIAQUIFER

The operational variables, number of cycles, and a comparison of experimental and calculated recovery efficiencies are given in Table 4.7. The excellent correlation between the experimental and predicted efficiencies obtained by Kumar (1968) was fortuitous because the subsequently-improved mathematical program by Whitehead (1974), as listed in the appendix, gave predicted values with less agreement. Except for the anomalous Run No. 6, however, the revised predicted recovery efficiencies are less than the experimental values. Thus our procedure is on the safe, or conservative, side for field design purposes.

One of the important conclusions reached during this portion of the study was that, regardless of fluid properties and operating conditions, the recovery efficiency improves with multi-cycle operation. The result is due, at least in part, to the retarding effect of the mixed zone on the rate of gravitational segregation. We

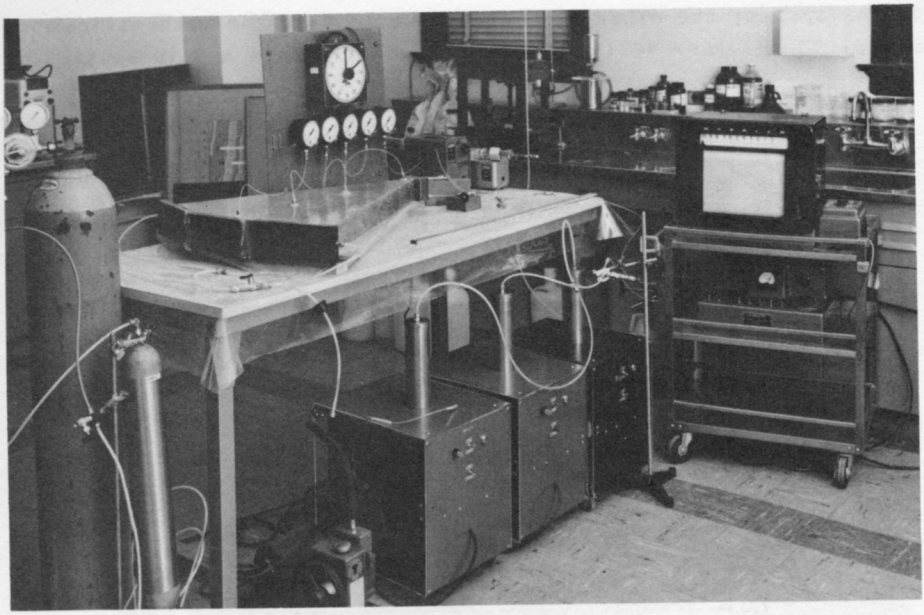


Figure 4.8 GENERAL ARRANGEMENT OF THE RADIAL MINIAQUIFER AND ASSOCIATED EQUIPMENT

TABLE 4.6 Summary of Properties of Fluids Used in Displacement Runs

Run Number	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6
Type of Run	One Cycle	One Cycle	One Cycle	Two Cycle	Two Cycle	One Cycle
Temp., °C	21.0	25.5	25.0	25.25	22.0	27.5
Fluid A Composition Volume %	100% Naphtha	55% Naphtha 45% CCl_4	80% Naphtha 20% CCl_4	80% Naphtha 14% CCl_4 6% Soltrol	93% Naphtha 7% CCl_4	90% Naphtha 10% CCl_4
Viscosity, cp	0.5621	0.7086	0.6465	0.6560	0.5920	0.5713
Density, gm/cc	0.7650	1.1288	0.9569	0.8701	0.8048	0.8234
Fluid B Composition Volume %	66% Naphtha 34% IPA*	58% Naphtha 42% IPA	60% Naphtha 40% IPA	60% Naphtha 40% IPA	70% Naphtha 30% IPA	70% Naphtha 30% IPA
Viscosity, cp	0.7026	0.8330	0.8012	0.7992	0.7287	0.6563
Density, gm/cc	0.7648	0.7653	0.7650	0.7642	0.7607	0.7514
Fluid C Composition Volume %	60% Naphtha 40% Soltrol	21% Naphtha 40% Soltrol 39% CCl_4	38% Naphtha 42% Soltrol 20% CCl_4	57% Naphtha 40% Soltrol 13% CCl_4	60% Naphtha 35% Soltrol 5% CCl_4	90% Naphtha 10% CCl_4
Viscosity, cp	0.8783	1.1294	1.009	1.9389	0.8992	0.5713
Density, gm/cc	0.7651	1.1315	0.9273	0.8704	0.8046	0.8234

*Isopropyl Alcohol

TABLE 4.7 Comparison of Experimental and Calculated Recoveries for Displacement Runs

Run Number		Run 2	Run 3	Run 4	Run 5	Run 6
Type of Run		One Cycle	One Cycle	Two Cycle	Two Cycle	One Cycle
Density Difference, gm/cc		0.3635	0.1919	0.1059	0.0440	0.072
Volume Injected cc	1st Half-Cycle	4528.0	3200.0	3150.0	3140.0	1974.0
	2nd Half-Cycle			1545.0	2590.0	
	Total	4528.0	3200.0	4695.0	5730.0	1974.0
Time of Injection sec	1st Half-Cycle	3630.0	8400.0	3000.0	2100.0	19740.0
	2nd Half-Cycle			1280.0	1800.0	
Time of Static Standing, sec	1st Half-Cycle	870.0	800.0	800.0	900.0	7260.0
	2nd Half-Cycle			820.0	600.0	
	3rd Half-Cycle			800.0	500.0	
Volume Produced cc	1st Half-Cycle	2933.0	1850.0	1325.0	2120.0	168.0
	2nd Half-Cycle			2250.0	2900.0	
	Total	2933.0	1850.0	3575.0	5020.0	168.0
Time of Production up to Breakthrough sec	1st Half-Cycle	1850.0	2200.0	1200.0	1800.0	1680.0
	2nd Half-Cycle			1837.0	2205.0	
Recovery, percent	Experimental	64.7	57.8	76.1	87.6	8.5
	Calculated (orig.)	64.2	55.2	74.5	86.8	14.1
	Recalculated (W.R.W.)	62.0	52.0	70.0	78.0	31.0

should emphasize, however, that our conclusion is based on comparisons between test results from *horizontal* miniaquifers and the output of the computational model. It should not be applied if the formation has a significant dip.

Thin Miniaquifers With Dip

The earlier experiments, including some with the small, slightly dipping radial and linear miniaquifers, provided general validation of the mathematical model so long as the system was essentially horizontal. As noted previously, the inclusion of aquifer dip as a parameter requires that the miniaquifer be either a 180°-sector or a complete circle.

In order to separate dip from the other parameters, a thin, rectangular miniaquifer (180°-sector) was constructed. A thickness of 1 cm was chosen to provide translucency and to minimize gravitational laydown of the front. The system (Fig. 4.9) was mounted in a wooden frame that could be tilted to various pre-selected angles (Fig. 4.10). Although the miniaquifer measured 110 cm x 230 cm, a semi-circular isopotential at a radius of 105 cm from the wellbore provided a radial flow system. The porosity was 0.25 and the

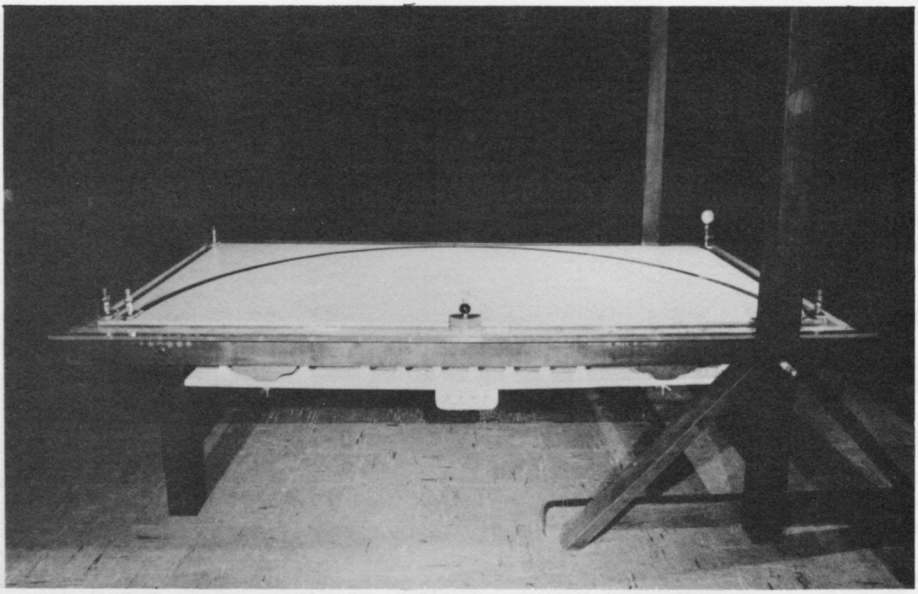


Figure 4.9 MINIAQUIFER USED DURING THE STUDY OF DIPPING SYSTEMS

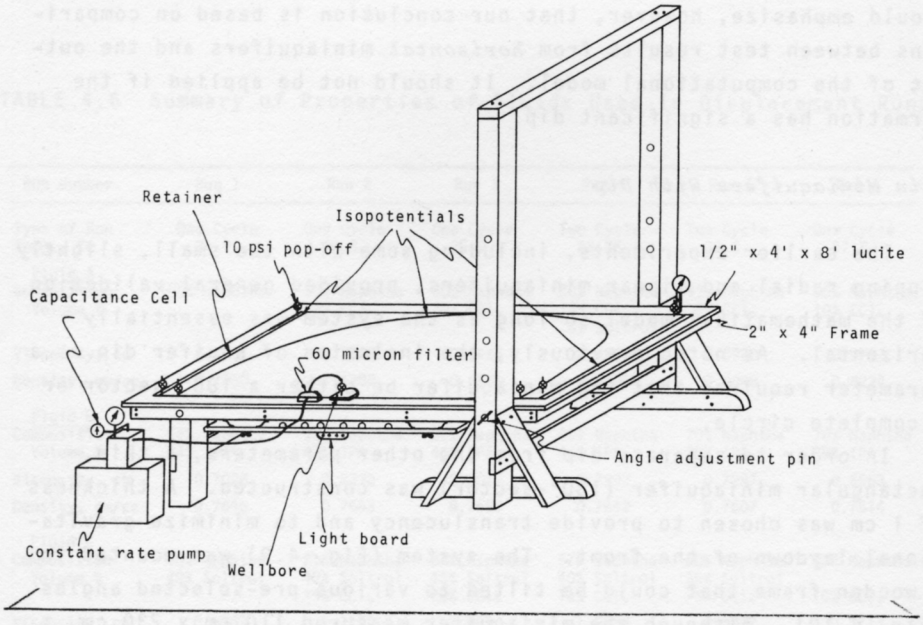


Figure 4.10 SCHEMATIC DIAGRAM OF THE MINIAQUIFER AND SUPPORTING STRUCTURE

average permeability, 5.09 darcies.

The wellbore was fitted with a capacitance cell for monitoring the composition of the produced fluid (the procedure described earlier). Six constant-rate pumps enabled us to inject and produce at preselected flows. Figure 4.11 shows the experimental set-up, which included a scaffold for taking still photographs and time-lapse motion pictures of the displacement.

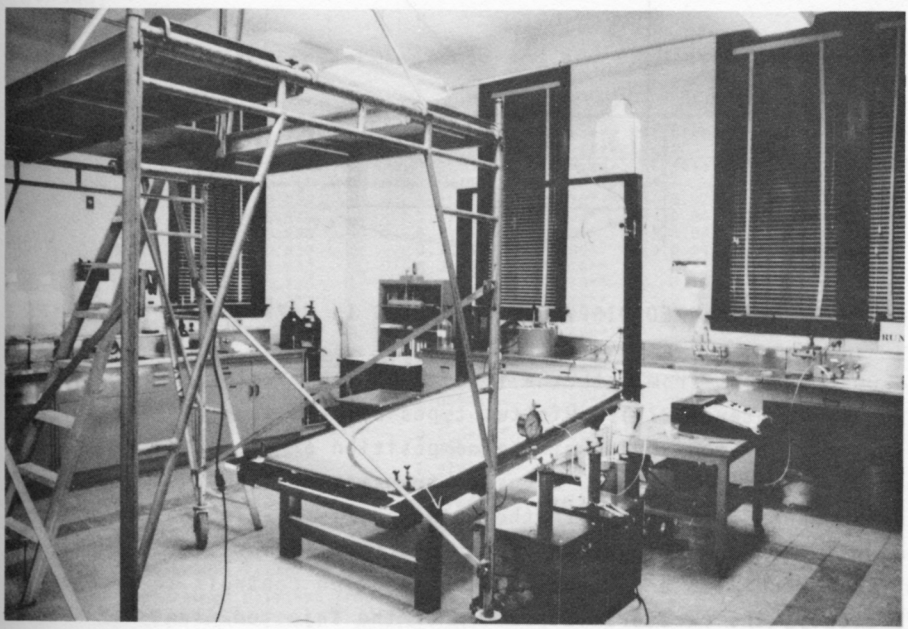


Figure 4.11 ARRANGEMENT OF EQUIPMENT

Experimental studies in the thin linear system (Painter, 1971) had shown that the displacement front would progress downdip to some maximum distance and then retreat toward the injection line. The mathematically-predicted positions of the updip and downdip fronts for a dipping linear system are shown in Figure 4.12. The experimental studies showed good agreement with the predicted positions.

In the miniaquifers used for linear studies the ends of the system were held at the same potential by virtue of being connected to a common reservoir. Fluid could enter or leave either end of the system, thus providing an opportunity for what might be considered a type of external circulation of fluid from one end of the injected fluid volume to the other. Since the injected fluid occupied the entire width of the miniaquifer, no opportunity existed for

fluid circulation within the matrix itself. It was reasoned, however, that in a dipping radial system an opportunity would exist for the circulation of native fluid to occur around the bubble of injected fluid.

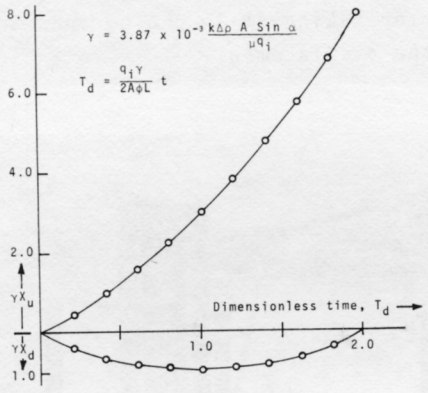


Figure 4.12 PREDICTED DISPLACEMENT FRONTS IN DIPPING LINEAR SYSTEMS

Observations from both the linear and radial miniaquifers were given by Painter (1971). His two types of data were: (1) recovery efficiency, by monitoring of the composition of the produced stream; and (2) frontal positions, by using transmitted light to trace the displacement of a red dye in the injected fluid. In the radial system, using a denser injected fluid, the updip frontal position did, in fact, stabilize and, because stabilization controls the recovery efficiency, the recoverable portion of the injected fluid was drastically reduced by significant dip.

A mathematical model to track streamlines was developed on the assumptions of a sharp front (no mixed zone) and a two-dimensional system. This model provided a usable approximation of the frontal configuration for reasonable density differences and dip angles. Typical experimental results from such displacements are compared with the results of the approximate mathematical model in Table 4.8. Predicted and observed frontal configurations are shown for one set of conditions in Figure 4.13. It is apparent that maximum disagreement between the observed and computer-predicted front occurred on the side that would correspond to the updip direction for freshwater storage or the downdip direction if a waste fluid denser than salt water were injected.

The experimental fronts in Figure 4.13 are seen to be at greater radii from the well than the predicted values. These errors are at least partially due to the failure of the thin miniaquifer to

respond as a two-dimensional system. Because the observations of the frontal position were made by transmitted light, the maximum progress of the dyed fluid would have been seen and the distance measurement distorted by any underrunning or overriding caused by density differences.

Thus a definitive description of the effects of dip could not be obtained from the thin system. A thicker miniaquifer would be necessary to include the inclination of the front due to gravity laydown.

TABLE 4.8 Comparison Between Observed and Predicted Recoveries

Run No.	Dip Angle (degrees)	Density Difference (gm/cc)	Rate (cc/min)	Recovery Efficiency (fraction)		Difference in Fractional Recovery
				Observed	Predicted	
1	30.0	0.830	3.25868	0.10*	0.07	-0.03
2	30.0	0.830	1.67116	0.03*	0.04	+0.01
3	30.0	0.830	0.71619	0.01*	0.02	+0.01
4	30.0	0.830	0.36467	0.00*	0.04	+0.04
5	30.0	0.830	6.01602	0.18**	0.21	+0.03
6	30.0	0.430	3.25868	0.15**	0.18	+0.03
7	30.0	0.210	1.67116	0.22**	0.22	+0.00
8	15.0	0.830	1.67116	0.04**	0.08	+0.04
11	15.0	0.830	3.11738	0.19**	0.17	-0.02
10	15.0	0.430	1.68369	0.19**	0.18	-0.01
9	15.0	0.210	0.83989	0.17**	0.21	+0.04
12	15.0	0.100	6.01602	0.72***	0.90	+0.18
13	15.0	0.100	2.00534	0.63***	0.76	+0.13
14	15.0	0.050	1.67116	0.68***	0.85	+0.17
15	15.0	0.050	0.20053	0.39***	0.60	+0.21
17	7.5	0.830	1.58756	0.15***	0.30	+0.15
16	7.5	0.100	2.91685	0.58***	0.93	+0.35
18	7.5	0.100	1.09381	0.56***	0.79	+0.23
19	7.5	0.100	3.07637	0.67***	0.87	+0.20
20	7.5	0.050	4.01068	0.86***	0.98	+0.12

*Breakthrough estimated visually.

**Breakthrough obtained from pressure sensor.

***Breakthrough obtained from oscillometric measurement.

[from Painter, 1971]

Following the experiments with the dipping thin miniaquifer, the circular isopotential was removed and the system altered to permit a flux (corresponding to a pre-existing ground-water movement) to be imposed on the system from either the up- or downgradient end. This modification permitted the study of an additional parameter, pre-existing ground-water movement. The model with flux could also accommodate dip if the direction of fluid movement coincided with the dip.

A series of experimental runs that incorporated flux and both dip and flux were reported in detail by W. Esmail (1973). Table 4.9 shows the effects of ground-water movement to be especially detrimental to the water-storage process, particularly when the adverse flow due to dip is in the same direction as the flux (Run 16).

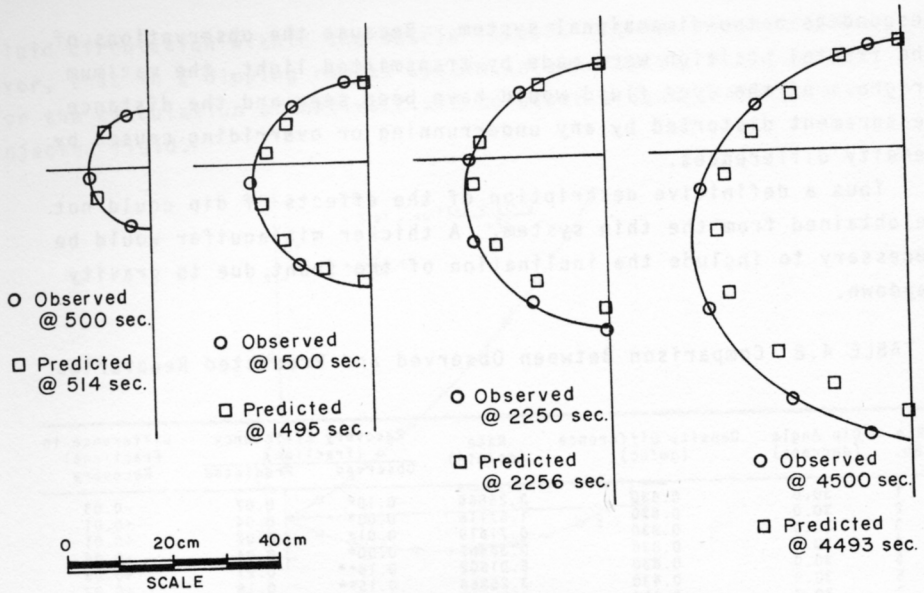


Figure 4.13 TYPICAL FRONTAL CONFIGURATIONS

A mathematical model for tracking streamlines was written to combine the effects of dip, flux, and dispersion to predict recovery efficiency. Because only the streamline that would break through to the producing well was of interest, no attempt was made to match the entire frontal configuration. A comparison of predicted and observed recovery efficiencies is given in Table 4.9. It is evident that the mathematical model agrees well with the experimental data in some cases, but poorly in others. This divergence is attributed to the failure of the mathematical model to adequately account simultaneously for both the effect of dip and gravitational laydown of the front. In other words, the thin miniaquifer was not an adequate representation of a dipping two-dimensional system and the mathematical model used for it could not adequately portray the gravitational effects in a dipping three-dimensional system.

Thick Miniaquifers

From the preceding discussion of experiments with thin miniaquifers, it is apparent that a large thick system would be required for adequate study of those situations in which gravitational forces would be significant. The decision was made for the study of a well field (multi-well system) in addition to a single-well installation. The thick miniaquifer shown in Figure 4.14 was then built.

TABLE 4.9 Comparison of Observed and Predicted Recovery Efficiency for Radial Systems with Pre-Existing Ground-Water Movement [from W. Esmail, 1973]

Run No.	Dip Angle (°)	Density Diff. (gm/cc)	Inj. & Prod. Rates (cc/min)	G-W Flow Rate (cm/sec)	Injection Time (sec)	Recovery Efficiency (Percent)	
						Observed	Predicted
8	0.0	0.02	6.016	7.6×10^{-4}	7,200	23	18
9	0.0	0.02	6.016	4.3×10^{-4}	7,200	37	38
10	0.0	0.02	6.016	2.2×10^{-4}	7,200	59	60
11	0.0	0.02	3.676	6.1×10^{-4}	10,500	17	18
12	0.0	0.017	4.010	0.84×10^{-4}	7,200	50	69
13	0.0	0.021	4.010	0.25×10^{-4}	7,200	80	76
14	15.0	0.210	4.010	0.84×10^{-4}	15,000	52	57
15	15.0	0.22	0.840	0.84×10^{-4}	25,000	46	47
*16	15.0	0.23	0.840	0.84×10^{-4}	15,000	14	19
19	15.0	0.22	0.840	1.21×10^{-4}	25,500	47	65
20	15.0	0.22	0.840	2.03×10^{-4}	25,500	25	40
21	15.0	0.22	0.840	1.52×10^{-4}	25,500	50	98
17	30.0	0.215	0.840	0.84×10^{-4}	15,200	20	28
18	30.0	0.21	0.840	1.9×10^{-4}	15,000	27	57

*Direction of flow is updip in all runs except Run 16, in which it is downdip.

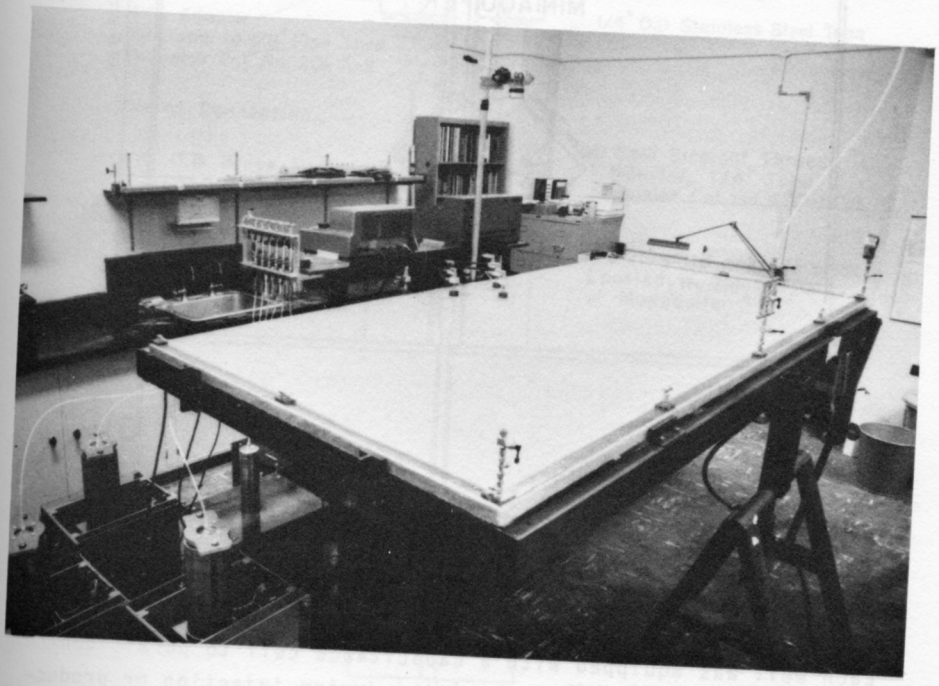


Figure 4.14. VIEW OF THICK MINIAQUIFER, PUMPS, AND INSTRUMENTATION. Note camera on support for photographing frontal position of injected fluid. In left foreground are three large-barrel pumps and one small-barrel pump. Chemical oscillometer and recorder are on stand just above well field. To left of oscillometer are individual controls for each well.

Figure 4.15 shows that the miniaquifer consisted of a well field composed of three wells and three half-wells. Thus, by symmetry, it represented a field of nine wells surrounded by a rectangular isopotential measuring 292 cm (9'-7") by 305 cm (10'-0"). The system was 3.81 cm (1.5 inches) thick and had a porosity of 0.25 and a permeability of 5.57 darcies. It was mounted on a heavy steel framework that could be inclined up to 30 degrees to simulate aquifer dip. The elevated position of the miniaquifer and the open construction of the supporting frame facilitated visual and photographic observation of both upper and lower surfaces.

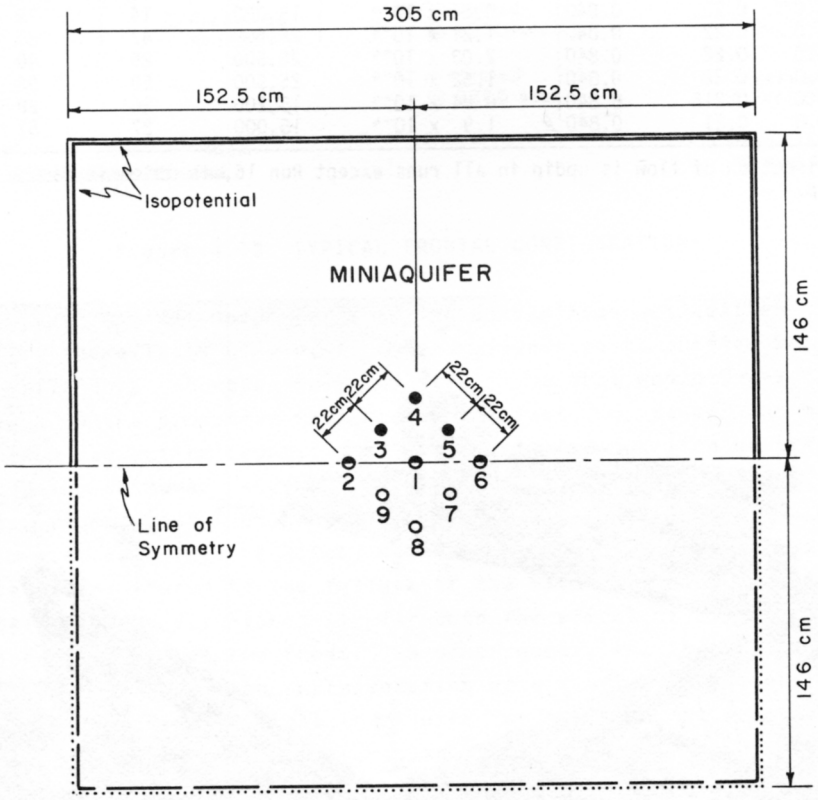


Figure 4.15 SYSTEM REPRESENTED BY THICK MINIAQUIFER

Each well was equipped with a capacitance cell to permit continuous monitoring of fluid composition during injection or production. Details of the cell construction and installation are shown in Figure 4.16.

The fluid in the annulus formed by the 1/4" stainless steel

tube and the 1/8" brass rod forms the dielectric of the capacitor. If two fluids of differing dielectric constant are chosen as native and injected fluid, any mixture of the two will yield a characteristic capacitance that can be related to fluid composition. (The most convenient means of continuously monitoring this capacitance was by means of a Sargent Chemical Oscillometer.) In effect, the cell becomes a variable capacitor in the tank circuit of a high-frequency oscillator.

The output from the oscillometer is fed through a recording adapter to a recording potentiometer. The proper choice of fluids and control settings produces a linear relationship between composition and recorder deflection (Fig. 4.17).

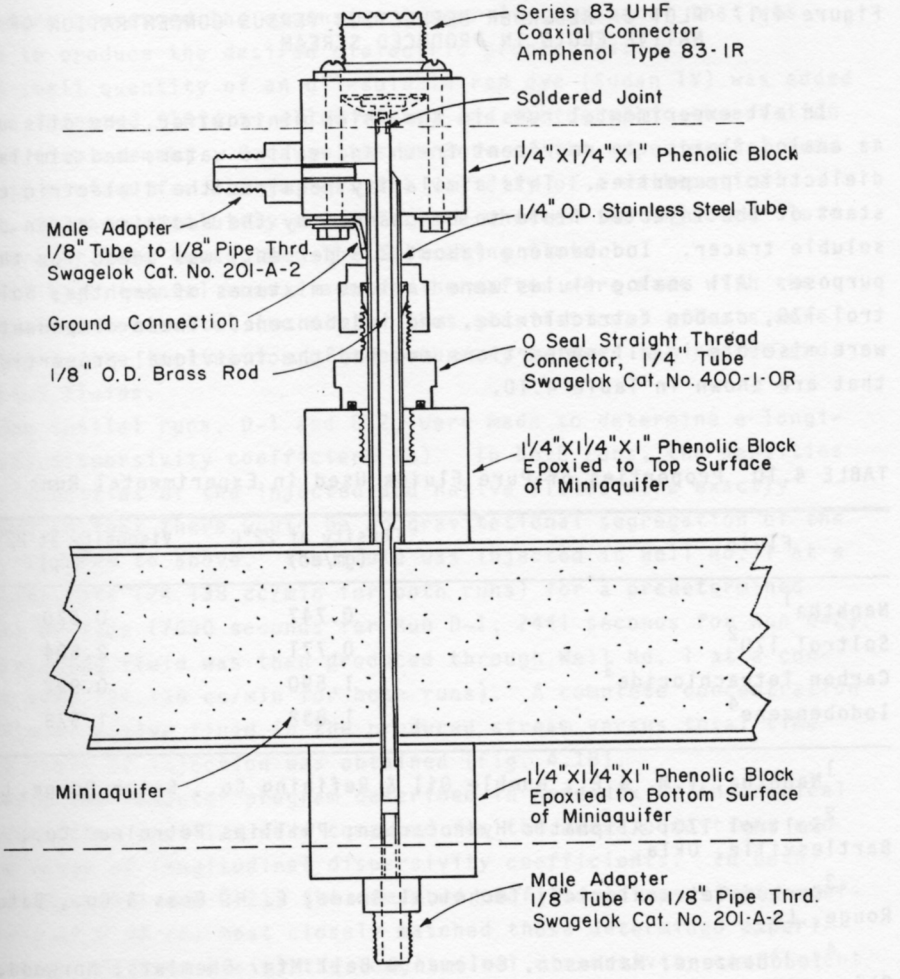


Figure 4.16 VIEW OF CAPACITANCE CELL USED TO DETECT CONCENTRATION CHANGES

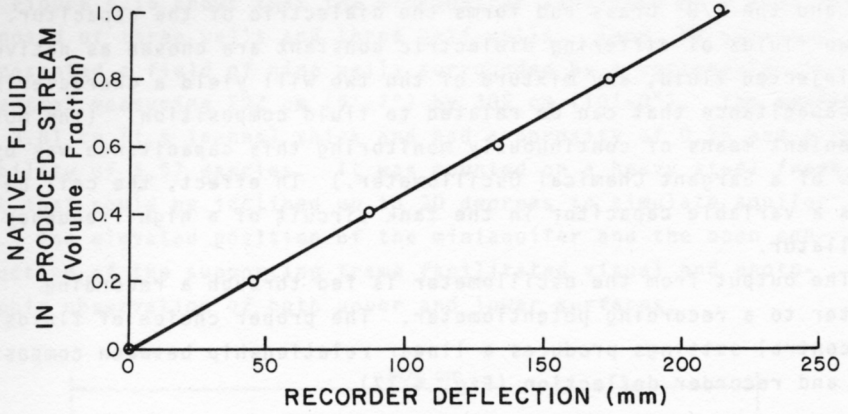


Figure 4.17 PLOT OF RECORDER DEFLECTION VERSUS CONCENTRATION OF NATIVE FLUID IN PRODUCED STREAM

In all experimental runs in the thick miniaquifer, the oils used as analog fluids, to represent fresh and saline water, had similar dielectric properties. This similarity required the dielectric constant of the injected fluid to be changed by the addition of an oil-soluble tracer. Iodobenzene (about 2.5 percent) was added for this purpose. All analog fluids were various mixtures of naphtha, Soltrol 170, carbon tetrachloride, and iodobenzene. These components were miscible in all proportions and had the individual properties that are shown in Table 4.10.

TABLE 4.10 Properties of Pure Fluids Used in Experimental Runs

Fluid	Density at 22°C (gm/cc)	Viscosity at 22°C (cp)
Naphtha[1]	0.747	0.570
Soltrol 170[2]	0.771	2.504
Carbon Tetrachloride[3]	1.590	0.992
Iodobenzene[4]	1.832	1.573

[1]Naphtha; V.M. & P.; Humble Oil & Refining Co., Baton Rouge, La.

[2]Soltrol 170; Aliphatic Hydrocarbon; Phillips Petroleum Co., Bartlesville, Okla.

[3]Carbon Tetrachloride; Technical Grade; F. H. Ross & Co., Baton Rouge, La.

[4]Iodobenzene; Matheson, Coleman & Bell Mfg. Chemists, Norwood, Ohio.

Analog fluids were used for two reasons: (1) they provide better control of properties than would fresh and salt water, and (2) they cause no detectable change in matrix properties or strength even after long (years) exposure. The latter is not always the case with aqueous fluids.

All displacements involved a fluid system in which the viscosity ratio was maintained at unity. Painter (1971) had found that a mixture of 45 percent Soltrol and 55 percent naphtha, by volume, had a viscosity equal to that of carbon tetrachloride. Thus, by the addition of any desired quantity of carbon tetrachloride to such a mixture, a more dense fluid could be obtained without an accompanying change in viscosity. Experimentally it was found desirable to use this more dense mixture as the injected fluid. In additon, this procedure conserved the expensive tracer (iodobenzene) that was added to produce the desired dielectric properties.

A small quantity of an oil-soluble red dye (Sudan IV) was added to the injected fluid to allow visual and photographic observation of the displacement. Because the purpose of the experimental runs in this miniaquifer was to test the validity of a mathematical model, it made absolutely no difference whether the injected fluid was more or less dense than the saturating fluid.

All experimental runs discussed hereafter were made with the miniaquifer horizontal (no dip). Thus gravitational effects were limited to the inclination of the boundary between the native and injected fluids.

The initial runs, D-1 and D-2, were made to determine a longitudinal dispersivity coefficient (α). In both runs, the densities and viscosities of the injected and native fluids were exactly matched so that there would be no gravitational segregation of the type referred to above. The fluid was injected in Well No. 1 at a constant rate (24.138 cc/min for both runs) for a predetermined length of time (7090 seconds for Run D-1; 7441 seconds for Run D-2). The injected fluid was then produced through Well No. 1 at a constant rate (24.138 cc/min for both runs). A complete concentration profile of native fluid in the produced stream versus total time since start of injection was obtained (Fig. 4.18).

From the computer program described in Appendix D, theoretical concentration profiles were computed for the above conditions and for a range of longitudinal dispersivity coefficients. In both cases (Runs D-1 and D-2), the computed profiles (dispersivity coefficient of 0.02 cm) most closely matched those determined experimentally. On this basis, the longitudinal dispersivity coefficient of the miniaquifer was taken to be 0.02 cm.

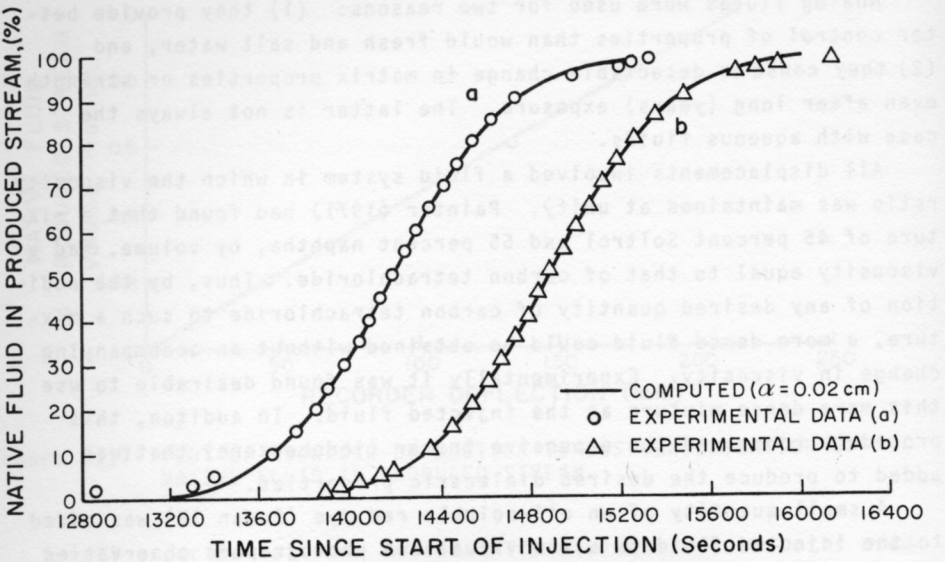


Figure 4.18 COMPARISONS OF COMPUTED AND OBSERVED CONCENTRATION PROFILES USED IN COMPUTING THE LONGITUDINAL DISPERSIVITY COEFFICIENT. (a) Comparisons for Run D-1; (b) Comparisons for Run D-2.

Seventeen experimental runs were made with a wide range of the parameters that affect recovery efficiency. (Complete descriptions and results of 16 of these runs are presented in Appendix E.) Because Run No. 7 had no quantitative value, the results for it are not given.

Table 4.11 compares the observed and predicted recovery efficiencies for single-well operation. All the predicted recovery efficiencies were less than the observed values. The average difference in recovery efficiency for the first cycle was 13 percent, with a range of 8 to 19 percent. For the second cycle, the average difference was 7 percent, with a range of 4 to 10 percent. The third cycle had an average difference of 7 percent, with a range of 5 to 8 percent. In all runs that were carried out for more than one cycle, the recovery efficiency improved with each cycle and the computed results more nearly approached the observed efficiency.

Table 4.12 compares the observed and predicted recovery efficiencies for multiple-well operation. All the predicted values of recovery efficiency were equal to or less than the observed values, except for the first cycle of Run No. 15. The average predicted values of recovery efficiency for the first cycle were 8 percent

lower than the observed values. The range of difference was from -1 to 14 percent. For the second cycle the average difference was 6 percent, with a range of 0 to 11 percent. The third cycle had an average difference of 9 percent, with a range of 5 to 11 percent. In all runs that were carried out for more than one cycle, the recovery efficiency improved with each cycle.

TABLE 4.11 Comparison of Observed and Predicted Recovery Efficiencies for Single-Well Operation

Run No.	First Cycle		Second Cycle		Third Cycle	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
1	93%	85%	--	--	--	--
2	94	86	95%	90%	97%	92%
3	91	77	94	85	--	--
6	74	61	87	77	92	85
8	93	78	94	86	96	88
11	77	64	--	--	--	--
12	77	64	89	85	92	87
14	68	58	83	79	89	81
16	39	20	--	--	--	--
17	17	0	--	--	--	--

TABLE 4.12 Comparison of Observed and Predicted Recovery Efficiencies for Multiple-Well Operation

Run No.	First Cycle		Second Cycle		Third Cycle	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
4	86%	82%	--	--	--	--
5	90	76	90%	83%	98%	88%
9	90	76	93	85	99	89
10	91	78	97	86	100	89
13	80	78	88	85	91	86
15	71	72	78	78	89	80

V

APPLICATIONS

FIELD TEST PROCEDURE

In order to evaluate the capability of a salaquifer to act as a storage reservoir for fresh water, a hydrogeological survey must be conducted. Inasmuch as a survey for this purpose is substantially different from the usual exploration accomplished before the construction of a ground-water supply system, the field procedure is outlined in detail in the following paragraphs, first with the determination of the potentiometric gradient, then the dispersivity, transmissivity, storativity, and finally, the porosity.

Potentiometric Gradient

Although the decision concerning the number of test wells to be drilled will be based on the aquifer thickness, the desired volume of storage, and the total area ultimately to be subsumed by the well field, at least three observation wells will be needed to determine the existence of any potentiometric gradient. These wells must be spaced widely enough apart so that the direction and slope of the gradient can be adequately defined. The data obtained during construction of these wells will, of course, include samples of the aquifer material and the overlying and underlying clays, electric logs in each well, and other geophysical information, such as a porosity log, that may seem appropriate. The test wells are to be screened through the full thickness of the target aquifer for later use in determining the progress of the fresh-water front during the large-scale injection test that must be accomplished before the main well field is constructed. A spinner survey will usually also be required in one of more of the test wells to determine if there is significant stratification in the target aquifer.

Dispersivity

One critical parameter of a water-storage project is the dispersivity of the aquifer. The higher the dispersivity, the lower the recovery ratio in the first cycle (although the recovery efficiency will improve in succeeding cycles).

Two methods for measuring the in-situ dispersivity are possible. Both eliminate the complications that would result from gravitational segregation due to density differences and would lead to incorrect values of α.

One method involves the use of native water for injection. A pump is installed in one observation well to obtain water to be injected into another observation well. The injected native water is tagged by the addition of a substance such as chloride ions, tritium, or other tracer that will enable the testing engineer to identify the injected water when it is pumped out.

In the second method, a few hundred thousand gallons of fresh water are first injected to drive the more dense salt water, and the resulting mixed and segregated zone, away from the well. Following this water are a few hundred thousand more gallons of suitably-tagged fresh water.

The S-shaped curve depicts the percentage of pumped tagged water and is based on the change in the concentration of the injected marker material, and can be compared with computed curves. The dispersivity can then be obtained by a best-match process.

It should be noted that for such a test there is an advantage in using water native to the aquifer if there is a significant fraction of dispersed clay or silt in the salaquifer. Under such circumstances, the use of fresh water can be expected to result in chemical reactions that can cause the clays to swell and consequently plug the aquifer in the vicinity of the test well.

If the potentiometric gradient in the salaquifer is zero or negligible, the injection/withdrawal test to determine dispersivity should be conducted with the minimum feasible storage time between injection and withdrawal. If a significant potentiometric gradient exists, the dispersivity test should probably be postponed until the other tests have been made, an isopotential surface (area of stagnation) has been created, and the transmissivity and storativity of the formation and the distance to the nearest boundary determined.

Porosity

Determination of the in-situ porosity of the salaquifer is one goal of the field studies because the value of this parameter is needed for final design of the system.

The porosity is computed on the basis of measurements of frontal position made during the previously-described injection test. In conjunction with this test, several miscible-displacement observation wells are used. These are located at different distances from the injection well and on different radii. With the time for the water front to arrive at each well being noted, the computation of the apparent porosity can be made from the known thickness (well logs) and the total amount of water injected. The procedure is repeated with the computation made each time on the basis of uniform flow outward from the injection well. The porosity, taken as the average determined from two or three observation wells, will make possible the computation of the approximate volume of cushion water required for any well field and the land area needed for a successful I/S/P project of any predetermined size.

Transmissivity, Storativity, and Distance to Boundaries

With the data obtained from the first three test wells, a site for an aquifer-performance test and an in-situ porosity determination is then selected.

A large-diameter well, ultimately to be used as part of the multi-well I/S/P system, is then constructed. It should be screened through the full thickness of the aquifer and gravel-packed. The aquifer should then be chemically stabilized for a distance of not less than five feet from the screen to minimize the well loss (increased drawdown) due to clay swelling or dispersion. Several wells for the observation of miscible displacement should be drilled at different distances from the central well and screened through the full thickness. Arrangements should be made to take water samples in each well from three or four different elevations in the salaquifer. For instance: if the salaquifer is 100 ft thick, then sampling sites might be located 10 ft, 40 ft, 70 ft, and 90 ft from the floor of the aquifer.

A standard aquifer performance test is then accomplished with the use of the large-diameter well and the observation wells. Transmissivity, storativity, and boundary effects, if any, are determined. By the time this test is completed, a significant quantity of geologic data should be available as an aid in the interpretation of the results.

After the test data have been analyzed, it will be possible to evaluate the potentiometric gradient in terms of water movement and to compute various configurations of injection and production wells to produce an isopotential surface (zone of stagnation) large enough to include under it an adequate storage volume in the sal-aquifer.

Creating an Isopotential Surface

The enemy of fresh-water storage in a horizontal salaquifer is pre-existing ground-water movement. Before the prototype I/S/P test can be made, the potentiometric gradient must be neutralized. This can be accomplished by construction of several small-diameter, small-capacity wells that can be used for either production or injection. The net water output of the system will be small. With these wells operating in the proper combination, a zone of stagnation can be created.

The diameter of the zone of stagnation will be determined by preliminary computations of the well-field size and configuration and the quantity of water to be stored, as well as the injection and production rates.

Once a zone of stagnation has been created, a dispersivity test in one of the observation wells would be made as previously described.

Injection/Storage/Production Test

After completion of the preliminary field test and the determination of the transmissivity, storativity, and the existence of boundaries, an I/S/P test with the large-diameter well is necessary to verify the recovery efficiency predicted on the basis of the aquifer parameters.

In general, a large-scale I/S/P test should be made prior to construction of a multi-well field. Water obtained from the city main for the test is injected into the large-diameter well after any needed pH adjustment or chlorine neutralization equipment has been installed in the water line. Injection is continued until a predetermined quantity of water has been inserted. Observations of pressure, temperature, and chemical composition of the injected water should be made as needed.

Porosity Determination.--During the injection phase of the test, samples would be taken from the various parts of the aquifer as the

fresh-water front advances. It will then be possible to compute the in-situ porosity of the aquifer by comparing the volume occupied by the injected water with the total injected volume.

Recovery Efficiency or Recovery Ratio.--As soon as the porosity has been determined by the in-situ measurements, the probable recovery efficiency of the injection well during the test may be computed and the preliminary design of the multi-well field can be initiated. Injection rates, pressures, storage periods, power requirements for production of stored water, water losses, etc., can all be computed.

The actual recovery efficiency of the test should be compared to the computed efficiency with the use of the same parameters. This is a check primarily on the dispersivity. Thus, if no unforeseen chemical or mineralogical problems show up during the test, the optimum configuration of the well field can be determined as well as the quantities of water to be stored and the rates of injection and production. At that time, the final layout of the well field and its appurtenances, including pipelines, meters, pumps, etc., can be accomplished and the project sent out for bids. When the bids are evaluated, the economics of subsurface storage can be compared with the cost of any other alternative to determine the feasibility of the proposed project.

ILLUSTRATIVE COMPUTATIONS

To show how the information may be used in design (and to enable the reader to check the output of the computer programs in the appendix), the parameters in Table 5.1 have been used to compute the recovery efficiency for a single and a multi-well system and show the economic comparison with an alternative storage method. Although the units are not consistent (metric and English units being used simultaneously), they are in common use in the industry. Purists may insert any desired conversion factors without altering the results (although to use the program in the appendix, the units must correspond to those listed here).

The impact of changes in the parameters is discussed in the next section, "Parametric Influences."

Single-Well Operation

The operation of a single well for I/S/P will prove feasible only for small volumes. Assume that water will be available for injection from surplus capacity in the water treatment plant for 160

days/yr, that the injected water will remain in storage for 120 days, and that the period of demand will average 85 days each year. Let us then assume that water will be injected at the rate of 1000 gpm for 160 days, that the well will be at rest for 120 days and that the demand will be at a rate of 1500 gpm. Our first trial is to determine the exact period of time that the 1500 gpm will be available. We find, on our first trial, that breakthrough occurs at 80 days, before the end of the specified 85 days. Therefore, we rerun the program, changing the rate of production incrementally and finally determine that 1400 gpm could be maintained for 85 days. The second and succeeding cycles will have better recovery ratios than the 75% of the first cycle, so the project as specified is feasible.

TABLE 5.1 Salaquifer Parameters

Thickness	100 ft
Permeability	400 gpd/ft^2
Storativity	10^{-4}
Porosity	3×10^{-1}
Dispersivity	1.0 cm
Coefficient of molecular diffusion	10^{-6} cm^2/sec
Static water level (Potentiometric level)	land surface
Depth to roof of salaquifer	800 ft
Density, water in salaquifer	1.0087 gm/cm^3
Viscosity, water in salaquifer	1.0 cP

Multi-Well Operation

For a project to store a much greater quantity of water, say one billion (10^9) gallons, a field of five wells, as shown in Figure 3.8b, would be needed. The distance d = 250 ft.

From the computational procedures described in Section III, we find that the injection rate must be 1000 gpm for 71 days into Well No. 1 before the lagging edge of the mixed zone (zone of mixed native and injected water) passes the outer ring of wells. The total volume of 102,000,000 gallons will be needed as cushion water--essential in permanent storage--and will be counted as part of the capital cost of the system.

At the end of 71 days, injection for storage will begin in all five wells at 1000 gpm per well. Again, from the computational procedure described in Section III, we find that if fresh water is injected in all five wells at 1000 gpm per well for 156 days, allowed to stand for 117 days, and then produced at 1500 gpm per well until

breakthrough occurs at the outer ring of wells, the volume produced until breakthrough will be 995,944,000 gallons--almost 1 billion gallons, slightly over 88% recovery (excluding cushion water). This recovery is adequate; therefore the well field will be able to supply the required quantity of water from storage because subsequent cycles will have better recovery rates. The maximum water loss per cycle will be the quantity injected (156 x 1440 x 1000 x 5 = 1,123,000,000 gal.) minus the quantity recovered (996,000,000 gal.) or 127,000,000 gallons. The cost of unrecovered water should be treated as an operating cost.

The Theis equation (Theis, 1935) is used to determine the pressure changes that will be encountered during injection. A head of 140 ft will have to be produced by the booster pump (220 HP) to inject 5000 gpm through the five-well field. During production, the drawdown will be about 325 ft. The power required to lift the water 325 ft at a rate of 1500 gpm and produce a pressure of 40 psi in the mains will be 200 HP. Thus each well must be equipped with a 200 HP motor.

ECONOMICS

In order to make an economic evaluation, a general design of the well field (Fig. 3.8b)--the individual wells, connecting piping, water-conditioning equipment, pumps, motors, controls--is needed.

For estimating purposes, each well is assumed to have an 18-inch diameter casing with 100 ft of 12-inch stainless steel screen surrounded by a 3-inch gravel wall. The total depth of each well from ground surface to the bottom of the screen will be 900 ft. In addition to the pump, motor, piping, valves, etc., that are normally associated with a water well, each well will be equipped with the following: (1) a piping manifold that will allow bypassing of the production string so the well can be used for injection; (2) flow control regulators that will accurately control the injection and production rates; and (3) instrumentation to monitor and record the rate and pressure fluctuations and the changes in water quality as injection or production proceeds.

The water to be injected is assumed to come from the city mains at a pressure of 40 psi and have a density of 1.0 gm/cc and a viscosity of 1.0 cP. The additional pressure needed to inject the water at the specified rates will be produced by a single booster pump for the entire well field. In addition to all the equipment listed above, it may be necessary to have a treatment facility to chlorinate and adjust the pH of the produced water before it is

re-introduced into the distribution system.

The well field will require a land area of 8 acres. The estimated cost of the well field and its appurtenances (including land at an assumed cost of $10,000/acre) is given in Table 5.2.

TABLE 5.2 Capital Cost of Well Field[1] (1973 Prices)

a) Hydrogeological Survey	=	$ 250,000
b) Land (8 acres @ $10,000/acre)	=	80,000
c) Wells (5 wells @ $75,000 each)	=	375,000
d) Motor and pump for wells ($20,000 per well)	=	100,000
e) Accessories; flow regulators, valves, instrumentation, etc., for wells ($10,000 per well)	=	50,000
f) Booster pump for injection	=	25,000
g) Water treatment facility	=	50,000
h) Cushion water[2] (102,240,000 gal @ 5¢/1000 gal)	=	5,000
i) Electric power to inject cushion water (63,000 kwhr @ 1.5¢)	=	1,000
j) Engineering & legal fees (25% of items c through g)	=	150,000
k) Contingency (20% of items c, d, e, f, g, and j)	=	150,000
Total		$1,236,000

(Use $1,240,000 in subsequent computation)

[1]The storage of fresh water in salaquifers is feasible in coastal areas and in such inland states as Illinois, Indiana, and Ohio that are underlain by brine-bearing sandstones.

[2]The cost of cushion water is based on the out-of pocket cost of operating a water-treatment plant close to its rated capacity. All costs that would have been paid, whether or not the additional water was treated, have not been apportioned. Consequently the cost of cushion water is the incremental cost of chemicals and power for the quantity involved. However, even doubling the cost of water, which would approximate the attributable cost of amortization, interest, maintenance and labor, would not significantly alter the capital cost of the project.

By way of comparison, the estimated cost of tank construction in metropolitan New Orleans in 1972 (obtained from P. J. Russo, Director of the Jefferson Parish, Louisiana, Water Department) was 10 cents per gallon of storage capacity, excluding the cost of land. The following table of costs has been prepared to show the overwhelming economic advantage of salaquifer storage, at least in the New Orleans area.

TABLE 5.2a Capital Cost, Steel Tanks, in Dollars

Tank farm, construction cost (10^9 gal.)	$ 100,000,000
Engineering and legal	1,000,000
Contingency (5 percent)	5,000,000
	$ 106,000,000

If we use tanks 160 ft in diameter and 50 ft high, we need 133 tanks. For a 200-ft center-to-center spacing, the land requirement is a minimum of 122 acres. If part of a built-up area must be purchased, with houses removed, etc., then a

figure of $10,000/acre is probably very much on the low side. Such a figure is used in this report to increase the economic advantage of the tankage alternate.

Land, 122 acres @ $10,000/acre	1,220,000
	$ 107,220,000
	(Say $107 million)

TABLE 5.2b Annual Cost of Tankage

Assuming 50-yr life, 8% interest, capital recovery (.0817 x 106,000,000)	$ 8,700,000
Operation and Maintenance, 1 percent/yr on $100,000,000	1,000,000
	$ 9,700,000

Thus, the annual cost of tankage, under the most favorable assumptions, will be $9,700,000 as compared to $164,000 (Table 5.3) for the salaquifer storage, giving salaquifer storage an advantage of more than 50:1.

The out-of-pocket operating cost of the well field used in storing water, together with interest and amortization, will constitute the annual charge.

One of the major items in any operating budget is the cost of labor. Fortunately, the operation of the storage facility can be automated and remotely controlled. Because the wells, pumps and motors can all be placed underground, the only surface facility will be the building for automated water treatment (chlorination or dechlorination and pH adjustment). In our analysis we assumed that no additional labor would be required. The available staff of the water department would be able to include the water storage operation in their routine. Thus no additional labor cost is included.

The power requirement for the cushion water was computed for one injection well for 71 days; i.e., 40 HP which, when allowance is made for motor efficiency, requires 63,000 kwhr. Of course, during the injection half-cycle, a period of 156 days will be involved. With an average pumping power requirement of 200 HP (and allowing for 90 percent efficiency) 0.57×10^6 kwhr will be needed. During the production half-cycle of 92 days, 1,100 HP are needed. Thus the total estimated power use during the production half-cycle will be 1.8×10^6 kwhr.

The capital cost of the land will not be amortized, but an annual charge for interest at 8 percent will be used.

The remaining capital costs will be amortized in 25 years at an interest rate of 8 percent. For level annual payments, this amortization would cost $.0936 per dollar of investment.

The water loss for the first I/S/P cycle is approximately 127 million gallons. Water loss will be a recurring operating expense,

but will not exceed this figure and will, on the contrary, decrease with each cyclic operation. Other cycle lengths may be visualized. Each of these would serve as a basis for a different economic computation.

Although no additional charges for labor are expected, an allowance of one percent/yr of the total capital cost (excluding cost of land and the hydrogeological survey) is chargeable, mostly for chemicals to make the injected water compatible with the native water and to adjust the pH of the produced water before it is placed back in the mains.

TABLE 5.3 Annual Cost of Salaquifer Storage, One Billion Gallons Recoverable (in thousands of dollars, 1973)

Interest and Capital Recovery	108
Interest on Cost of Land	6
Power	35
Water Losses	6
Operation and Maintenance	9
Totals	164

PARAMETRIC INFLUENCES

The entire calculation of recovery efficiency in a project of the type just described must be done by computer. The interrelationships are so complex, if not obscure, that no general rules can be laid down. In the following paragraphs, some of the most important parameters are changed, one by one, so that the reader can evaluate the impact of such variations on the capital and annual costs for the effective storage of one billion gallons of water.

Density (TDS) of Native Water

If the total dissolved solids in the native water are 1500 ppm instead of 8000 ppm, the density will be 1.0015 instead of 1.0087 gm/cm^3. (Figure 5.1 is a graph showing the relationship between total dissolved solids and density, based on field data.)

The required cushion water would be less, 96 million gallons instead of 102 million gallons, and the water loss in the first cycle, 56 million gallons instead of 130 million gallons.

These changes would *decrease* the annual operating costs by $4,000.

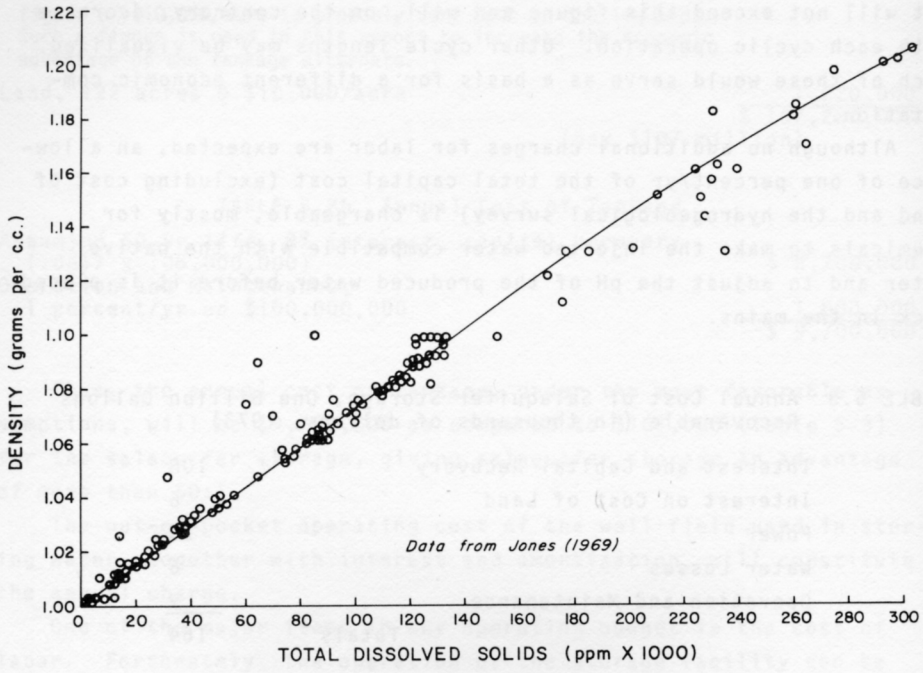


Figure 5.1 THE RELATION BETWEEN WATER DENSITY AND DISSOLVED SOLIDS CONTENT

On the other hand, if the native water has a TDS of 35,000 ppm (a density of 1.025), approximately equal to sea water, then the needed cushion water would be 114 million gallons as compared to 102 million gallons.

The water loss during the first cycle would be 168 million gallons, greater than the 130 million gallons shown previously.

These changes would *increase* the annual operating cost by $3,000.

Salaquifer Thickness

We originally assumed an aquifer thickness of 100 ft. If the aquifer is thicker (150 ft) or thinner (50 ft), with every other characteristic remaining the same, then the changes would be as shown in Table 5.4.

In this comparison, the well spacing and all operating rates remained constant. In practice, of course, it might be advantageous to use different well spacings and field areas to minimize the annual cost. It is also unlikely that the 18-inch wells used for the 100-ft aquifer will be adequate for the 50-ft salaquifer. Very

likely, a 24-inch casing and 18-inch gravel-packed screen would be required. Or possibly a seven-well field instead of a five-well field might be needed. Table 5.5 summarizes the project costs with only the aquifer thickness being changed.

TABLE 5.4 Operating Characteristics of Wells and Well Field Based on Salaquifer Thickness

	Salaquifer Thickness (ft)		
	150	100	50
Well depth (ft)	950	900	850
Screen length (ft)	150	100	50
Net HP for injection	115	220	510
Net HP for production	825	1000	1800
Power used, injection (kwhr)	0.3×10^6	0.6×10^6	1.4×10^6
Power used, production (kwhr)	1.4×10^6	1.8×10^6	3.0×10^6
Cushion water (MG)	161	102	49
Water loss/first cycle (MG)	168	127	78
Land required (acres)	8	8	8
Recovery efficiency, first cycle (%)	0.86	0.88	0.93

TABLE 5.5 Annual Cost of Salaquifer Storage Project Resulting from Different Thicknesses (Thousands of dollars, 1973 prices)

	Salaquifer Thickness (ft)		
	150	100	50
Interest and capital recovery	108	108	108
Interest on cost of land	6	6	6
Power	25	35	66
Water losses	8	6	4
Operation and maintenance	9	9	9
Totals	156	164	194

Permeability

Permeability is an important independent parameter. Even though the thickness is unchanged, the transmissivity (the product of permeability and thickness) changes the evaluation; other parameters, such as density difference and porosity, are not altered. Table 5.6 shows the results of increasing the permeability to 600 gpd/sq ft and decreasing it to 200 gpd/sq ft, with all other parameters

constant. Also included are the results of the original computations using 400 gpd/sq ft.

TABLE 5.6 Effect of Permeability Variations on Standard Multi-Well Project

	Salaquifer Permeability (meinzers)		
	200	400	600
Well depth (ft)	900	900	900
Screen length (ft)	100	100	100
Net HP for injection	510	220	115
Net HP for production	1800	1000	825
Power for injection half-cycle (kwhr)	1.4×10^6	0.6×10^6	0.3×10^6
Power for production half-cycle (kwhr)	3.0×10^6	1.8×10^6	1.4×10^6
Cushion water (MG)	97	102	108
Water loss, first cycle (MG)	78	127	168
Land required (acres)	8	8	8
Recovery efficiency, 1st cycle (%)	0.93	0.88	0.86
Annual cost, 1000's of dollars	194	164	156

Dispersivity

Dispersivity is a relatively new parameter in ground-water hydrology. Values for α have been obtained for laboratory-size physical systems and a few field values, by monitoring water composition in an observation well to determine the movement of liquid waste in aquifers. The field values are several orders of magnitude higher than our laboratory values and are not theoretically or practically comparable. Our values were the result of a sort of averaging process in the area surrounding an injection well. The effects of preexisting ground-water flow and density difference were carefully eliminated. Additionally, the published field dispersivities were taken at single points in aquifers with a down-gradient movement of a waste that was usually more dense than the native water. Thus they resulted from the composition of a "finger" of waste. It is not clear how such values can be used for predictive purposes. Our experimental values have proved to be of value in determining the recovery efficiency of a proposed I/S/P project.

A word of caution: one assumption is that the dispersivity is constant throughout the aquifer. This differs from reality as does the assumption of uniform transmissivity for the Theis equation. Variations in porosity and grain-size distribution occur within any aquifer. Lenticular deposits with permeabilities higher or

lower than the average are to be expected. However, our method of computing effective dispersivity, based on zero density difference, results in a useful average value. Table 5.7 illustrates the effect of differing values of dispersivity on recovery effeciency.

TABLE 5.7 Effect of Dispersivity Variation on the Recovery Efficiency of the Standard Multi-Well Project[1]

	Salaquifer Dispersivity (cm)		
	0.1	1.0	10
Permeability (meinzers)	400	400	400
Thickness (ft)	100	100	100
Well depth (ft)	900	900	900
Screen length (ft)	100	100	100
Net HP for injection	220	220	220
Net HP for production	1000	1000	1000
Power, injection half-cycle (kwhr)	0.6×10^6	0.6×10^6	0.6×10^6
Power, production half-cycle (kwhr)	1.8×10^6	1.8×10^6	1.8×10^6
Cushion water (MG)	102	102	102
Water loss, 1st cycle (MG)	210	200	220
Land required (acres)	8	8	8
Recovery efficiency, 1st cycle (%)	.83	.83	.82

[1]Unfortunately, for this example the dispersivity makes no difference. The table is included for completeness. Under other circumstances the difference in dispersivity would change the results appreciably.

PRACTICAL CONSIDERATIONS

The foregoing discussion of parametric influences must be considered elementary. In project design, one must start with the periods and frequency of water of satisfactory quality and adequate quantity. Simultaneously, the time, duration, and magnitude of the demand for stored water must be determined. This information, together with the excess capacity of the treatment plant to produce water for storage, i.e., treated water over and above the daily demand during the period of injection, must be analyzed in order to determine the overall rate of injection, duration of storage period, and rate of production.

A proper analysis of the storage project will resemble the hydrologic engineering studies performed on streams. The availability of excess treatment-plant capacity is the analog to the discharge record of a surface stream. It should be noted that this "excess" capacity must be constrained. It will probably decrease with time as the water demand approaches the full design capacity and there

may be periods where, although spare capacity is available, it may be only slightly greater than the daily demand.

Thus the task of selecting injection rates, although neither simple nor straightforward, is a special application of applied hydrology that will require the use of every available piece of data.

Although we have assumed that the I/S/P system would be used cyclically, in a precise manner, this mode of operation is unlikely. Most probably, after the first period of injection, the water would remain in storage until needed. For instance: the initial injection might be followed by an unusually wet year, with plenty of surface water available at all times. Thus there would be no point in injecting water and later pumping it out. Also, the well field might be used for "peaking" purposes during short periods of unusually high demand, and this water would then have to be replaced.

Again, there is no simple way of predicting the effect of storage period on recovery ratio. The quantity that is recoverable will depend on the density difference between native and injected water and the size of the mixed zone. Thus, should a long period of storage come after several complete I/S/P cycles, the recovery ratio would be higher than it would be after one or two cycles.

The total quantity of water stored, as well as the thickness of aquifer and density difference, must be taken into account in evaluating the effect of long periods of static storage. The larger the zone of mixed water that results from more cycles of operation, the better the recovery ratio for a given cycle. Because the mixed zone is a function of the distance travelled by the front of injected water, under conditions of high density difference (high TDS in the salaquifer) it might be worthwhile to run one or two I/S/P cycles, even though this might not be absolutely necessary, so that a long period of static storage during the third or subsequent cycles would benefit from the extra-large mixed zone. The quantity of water lost by this extended storage period would then be minimized. The position of the fresh-water front after the first half-cycle of injection may be computed through the use of the program in Appendix F as a check on the land area required for each scheme of operation based on total stored volume.

In view of the complexity of the phenomena, the designer of the salaquifer storage system would be well advised to study the effect of long storage periods on recovery ratios and devise operating schedules for the first few cycles to better preserve the quantity of water in storage for later use under emergency conditions.

The paragraphs dealing with parametric influences should be viewed as an attempt to give the reader a feeling for their relative importance.

VI

CONCLUSIONS

Among the conclusions drawn from the information presented in this report, the single most important one is this: the cyclic storage of fresh water in horizontal salaquifers that possess primary permeability and porosity is technically feasible. It is likely that the economic advantage of such a project will be substantial compared to any feasible alternative.

The economic feasibility of any particular project, however, can be determined only after detailed engineering studies of subsurface conditions have been completed and evaluated with reference to the availability of surface water and treatment plant capacity.

> Caution: Although aquifers consisting of sands, gravels or poorly consolidated sandstone are favorable for use of I/S/P projects, fractured rock and cavernous limestones are beyond evaluation by the field methods described herein. Consequently any computer program that is predicated upon aquifer parameters determined by in-situ tests is not applicable to formations whose permeability and porosity are "secondary."

Other conclusions of practical importance are:

1. The deleterious effect of pre-existing ground-water movement in the salaquifer upon the cyclic storage of fresh water can be counteracted. This can be accomplished by creating an isopotential zone, or zone of stagnation, within the volume of the salaquifer selected for storage of fresh water. A peripheral well field consisting of producing and injecting wells can be designed by use of the computer program included in the appendix so that the potentiometric surface within the desired area becomes essentially flat. The production and injection rates can be balanced so that little, if any, saline water must be discharged into the surface environment. The bounding wells must be operated at all times because the computed recovery efficiency of the I/S/P project is based on a motionless, exogenous ground-water body.

> Caution: The computer program is based on the classic Theis assumptions. Thus the zone of isopotential should be located far enough from any impermeable boundaries that the assumption of "infinite aquifer" is, for all practical purposes, correct. It may also prove desirable to design the isopotential area to its ultimate size so that future expansion in fresh-water storage can be accomplished without changing the bounding wells.

2. The recovery efficiency of the I/S/P cycle can be predicted by use of the computer program given in the appendix. Tests on small physical systems have shown that the actual recovery efficiencies are higher than those predicted:

a. The recovery efficiency in cyclic operation improves cycle-by-cycle even if the initial efficiency is low.

b. The larger the density difference between salaquifer water and fresh water, the poorer the recovery efficiency during the first cycle.

3. The dispersivity of the salaquifer must be determined in situ. However, the principal effect of a different dispersivity is to change the recovery effiency, and thus the cyclic water loss. If the density difference is small, the value of the dispersivity can range over three orders of magnitude without substantially changing the recovery efficiency. It is possible that after many field evaluations of dispersivity have been accumulated, it will be possible to assume a practical value and dispense with elaborate field tests.

4. To determine the size of the land area needed for the well field and, if needed, the isopotential zone, it is essential to make an in-situ determination of aquifer porosity.

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APPENDIX A

DERIVATION OF EQUATIONS

A.1 Derivation of Equation 3.3

Starting with Equation 3.2:

$$\frac{c_i}{c_o} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2/2 - Qt}{\left(\frac{4}{3}\ \alpha R^3 + D/Q\ R^4\right)^{1/2}}\right] \quad . \ . \ . \quad (3.2)$$

Where:

c_i = concentration of injected fluid at radius, r, and time, t. (volume fraction)

r = radius. (cm)

R = radius of injected fluid at time t, assuming no mixing or gravitational segregation. (cm)

$Q = q/(2\pi h\phi)$ (cm^2/sec)

q = volumetric flow rate. (cc/sec)
h = aquifer thickness. (cm)
ϕ = aquifer porosity. (fraction)

D = coefficient of molecular diffusion. (cm^2/sec)

α = longitudinal dispersivity coefficient of porous medium. (cm)

erfc = complementary error function.

Note that:

$$q \cdot t = \pi R^2 h\phi \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (A.1)$$

Hence:

$$\frac{q \cdot t}{\pi h\phi} = R^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (A.2)$$

Dividing both sides by 2:

$$\frac{q}{2\pi h\phi} \cdot t = \frac{R^2}{2} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (A.3)$$

Therefore:

$$Q \cdot t = \frac{R^2}{2} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (A.4)$$

Solving for R:

$$R = \sqrt{2 \cdot Q \cdot t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (A.5)$$

Substituting Equations A.4 and A.5 into Equation 2.2:

$$\frac{c_i}{c_o} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2/2 - R^2/2}{\left[\frac{4}{3}\alpha(2 \cdot Q \cdot t)^{3/2} + D\,\frac{(2 \cdot Q \cdot t)^2}{Q}\right]^{1/2}}\right] \qquad (A.6)$$

Or:

$$\frac{c_i}{c_o} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2 - R^2}{2\sqrt{f(t)}}\right] \qquad \ldots\ldots\ldots\ldots \quad (A.7)$$

Where:

$$f(t) = \frac{4}{3}\,\alpha\,(2 \cdot Q \cdot t)^{3/2} + D\,\frac{(2 \cdot Q \cdot t)^2}{Q}$$

Note that Equation A.7 is identical to Equation 3.2.

A.2 Derivation of Equation 3.4

Starting with Equation 3.3:

$$\frac{c_i}{c_o} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2 - R^2}{2\sqrt{f(t)}}\right] \qquad \ldots\ldots\ldots\ldots \quad (3.3)$$

Note that:

$$\frac{c_n}{c_o} = 1 - \frac{c_i}{c_o}$$

Hence:

$$\frac{c_n}{c_o} = 1 - \frac{1}{2}\,\mathrm{erfc}\left[\frac{r^2 - R^2}{2\sqrt{f(t)}}\right] \qquad \ldots\ldots\ldots\ldots \quad (A.8)$$

Multiplying through by 2:

$$\frac{2c_n}{c_o} = 2 - \mathrm{erfc}\left[\frac{r^2 - R^2}{2\sqrt{f(t)}}\right] \qquad \ldots\ldots\ldots\ldots \quad (A.9)$$

Rearranging:

$$\frac{2c_n}{c_o} = 1 + \left\{ 1 - \text{erfc} \left[\frac{r^2 - R^2}{2\sqrt{f(t)}} \right] \right\} \quad \ldots\ldots \quad (A.10)$$

Recall the identities:

$$\text{erfc}\ (\xi) = 1 - \text{erf}\ (\xi) \quad \ldots\ldots\ldots \quad (A.11)$$

$$\text{erf}\ (\xi) = 1 - \text{erfc}\ (\xi) \quad \ldots\ldots\ldots \quad (A.12)$$

$$\text{erf}\ (-\xi) = -\text{erf}\ (\xi) \quad \ldots\ldots\ldots \quad (A.13)$$

Using Equation A.12, note that Equation A.10 can be written as:

$$\frac{2c_n}{c_o} = 1 + \text{erf} \left[\frac{r^2 - R^2}{2\sqrt{f(t)}} \right] \quad \ldots\ldots\ldots \quad (A.14)$$

Using Equation A.13, note that Equation A.14 can be written as:

$$\frac{2c_n}{c_o} = 1 - \text{erf} \left[\frac{R^2 - r^2}{2\sqrt{f(t)}} \right] \quad \ldots\ldots\ldots \quad (A.15)$$

Using Equation A.11, note that Equation A.15 can be written as:

$$\frac{2c_n}{c_o} = \text{erfc} \left[\frac{R^2 - r^2}{2\sqrt{f(t)}} \right] \quad \ldots\ldots\ldots \quad (A.16)$$

Dividing through by 2:

$$\frac{c_n}{c_o} = \frac{1}{2} \text{erfc} \left[\frac{R^2 - r^2}{2\sqrt{f(t)}} \right] \quad \ldots\ldots\ldots \quad (A.17)$$

Note that Equation A.17 is identical to Equation 3.4.

APPENDIX B

COMPUTER PROGRAM FOR DETERMINING INJECTION OR PRODUCTION RATE OF EACH BOUNDING WELL TO CREATE A ZONE OF STAGNATION

The computer program listed on the following pages is used to calculate the injection or production rates of the bounding wells to create a zone of stagnation in an aquifer which has pre-existing ground-water movement. The positions of the bounding wells are specified by the program user. The program is in FORTRAN IV language and is written for use on an IBM 360/65 system.

A list of required input data is presented at the beginning of the program. Following this is a complete list of all the variable names used in the program along with their definitions.

```
C
C
C     PROGRAM TO CALCULATE BOUNDING-WELL RATES NECESSARY TO NEGATE PRE-
C      EXISTING GROUNDWATER MOVEMENT AND CREATE A ZONE OF STAGNATION.
C
C
C     PROGRAM 'BOUND'
C
C     ******************************************************************
C
C     DATA TO BE READ IN
C
C      FIRST CARD - FORMAT(1F12.0,1I3)
C       RBOUND   = RADIUS OF ISOPOTENTIAL PLATEAU. (FEET)
C       NBW      = NUMBER OF BOUNDING WELLS.
C
C      SECOND CARD - FORMAT(6F12.0)
C       XBWFT(I) = X COORDINATE OF ITH BOUNDING WELL. (FEET)
C       YBWFT(I) = Y COORDINATE OF ITH BOUNDING WELL. (FEET)
C
C      THIRD CARD - FORMAT(3F12.0)
C       PLYMEI   = AQUIFER PERMEABILITY. (MEINZERS)
C       HFT      = AQUIFER THICKNESS. (FEET)
C       PGRAD    = POTENTIAL GRADIENT CAUSING PRE-EXISTING GROUNDWATER
C                  MOVEMENT. (FEET/MILE)
C
C     ******************************************************************
C
C     DEFINITION OF VARIABLE NAMES USED IN PROGRAM.
C
C       A(I,J)   = ELEMENTS OF THE A MATRIX IN EQUATION 3.17.
C       ATA(I,J) = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                  THE A MATRIX BY ITS TRANSPOSE.
C       ATB(I)   = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                  THE B MATRIX BY THE TRANSPOSE OF THE A MATRIX.
C       B(I)     = ELEMENTS OF THE B MATRIX IN EQUATION 3.17.
C       CFFTCM   = CONVERSION FACTOR. (CM/FT)
C       CONST    = A CONSTANT USED IN COMPUTING THE ELEMENTS OF THE
C                  B MATRIX.
C       DTHETA   = INCREMENT BY WHICH THE ANGLE THETA IS INCREASED
C                  DURING COMPUTATION OF BOUNDARY POINT
C                  COORDINATES. (RADIANS)
C       H        = AQUIFER THICKNESS. (CM)
C       NBP      = NUMBER OF BOUNDARY POINTS.
C       P(I)     = PRE-EXISTING POTENTIAL AT THE ITH BOUNDARY
C                  POINT. (ATMOSPHERES)
C       PB       = VALUE OF POTENTIAL FOR THE ISOPOTENTIAL
C                  PLATEAU. (ATMOSPHERES)
C       PU       = PRE-EXISTING POTENTIAL AT THE MOST UPSTREAM POINT
C                  ON THE ISOPOTENTIAL PLATEAU. (ATMOSPHERES)
C       QBW(I)   = RATE OF THE ITH BOUNDING WELL. (CC/SEC)
C       QBWGPM   = RATE OF A BOUNDING WELL. (GPM)
C       R        = RADIUS OF ISOPOTENTIAL PLATEAU. (CM)
C       SOLVE    = A SUBROUTINE FOR SOLVING A N X N SET OF LINEAR
C                  EQUATIONS.
C       THETA    = ANGLE USED TO COMPUTE COORDINATES OF BOUNDARY POINTS.
C                  ANGLE IS MEASURED CLOCKWISE FROM THE POSITIVE Y
C                  AXIS. (RADIANS)
C       XBP(I)   = X COORDINATE OF ITH BOUNDARY POINT. (CM)
C       XBW(I)   = X COORDINATE OF ITH BOUNDING WELL. (CM)
C       XNBP     = FLOATING POINT VALUE OF NBP.
C       XNUM     = THE SQUARE OF THE DISTANCE BETWEEN A BOUNDARY POINT
C                  AND A BOUNDING WELL. (SQ CM)
C       YBP(I)   = Y COORDINATE OF ITH BOUNDARY POINT. (CM)
C       YBW(I)   = Y COORDINATE OF ITH BOUNDING WELL. (CM)
C
C     ******************************************************************
C
C
C
      DIMENSION XBWFT(20),YBWFT(20),XBW(20),YBW(20),XBP(60),
     1          YBP(60),A(60,20),B(60),P(60)
      COMMON ATA(20,20),ATB(20),QBW(20),ICHECK
```

```
C
C
C     READING IN DATA
C     ---------------
C
   20 READ(5,10000,END=65)RBOUND,NBW
      READ(5,11000)(XBWFT(I),YBWFT(I),I=1,NBW)
      READ(5,12000)PLYMEI,HFT,PGRAD
C
      NBP=3*NBW
C
C
C     PRINTING DATA
C     -------------
C
      WRITE(6,13000)RBOUND,NBW,PLYMEI,HFT,PGRAD
C
C
C     CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ------------------------------------------------
C
      CFFTCM=30.4801
      H=HFT*CFFTCM
      R=RBOUND*CFFTCM
      PU=PGRAD*RBOUND/89760.0
      DO25I=1,NBW
      XBW(I)=XBWFT(I)*CFFTCM
      YBW(I)=YBWFT(I)*CFFTCM
   25 CONTINUE
C
C
C     GENERATING COORDINATES OF BOUNDARY POINTS
C     -----------------------------------------
C
      XNBP=NBP
      DTHETA=6.283185/XNBP
      THETA=0.0
      DO30I=1,NBP
      XBP(I)=R*SIN(THETA)
      YBP(I)=R*COS(THETA)
      THETA=THETA+DTHETA
   30 CONTINUE
C
C
C     GENERATING THE 'A' MATRIX
C     -------------------------
C
      DO35I=1,NBP
      DO35J=1,NBW
      XNUM=(XBP(I)-XBW(J))**2+(YBP(I)-YBW(J))**2
      A(I,J)=ALOG(XNUM)
   35 CONTINUE
C
C
C     GENERATING THE 'B' MATRIX
C     -------------------------
C
      CONST=0.616545*PLYMEI*H
      PB=PU/2.0
      DO40I=1,NBP
      P(I)=YBP(I)*PU/(2.0*R)+PB
      B(I)=(P(I)-PB)*CONST
   40 CONTINUE
C
C
C     GENERATING 'A TRANSPOSE' * 'A'
C     ------------------------------
C
      DO50I=1,NBW
      DO50J=1,I
      ATA(J,I)=0.0
      DO45K=1,NBP
      ATA(J,I)=ATA(J,I)+A(K,I)*A(K,J)
   45 CONTINUE
      ATA(I,J)=ATA(J,I)
   50 CONTINUE
```

```
C
C
C     GENERATING 'A TRANSPOSE' * 'B'
C     ------------------------------
C
      DO55I=1,NBW
      ATB(I)=0.0
      DO55J=1,NBP
      ATB(I)=ATB(I)+B(J)*A(J,I)
   55 CONTINUE
C
C
C     SOLVING FOR THE BOUNDING WELL FLOW RATES
C     ----------------------------------------
C
      CALL SOLVE (NBW)
      IF(ICHECK.EQ.1)GOTO20
C
C
C     PRINTING THE RESULTS
C     --------------------
C
      WRITE(6,14000)
      DO60I=1,NBW
      QBWGPM=QBW(I)/63.0945778
      WRITE(6,15000)I,XBWFT(I),YBWFT(I),QBWGPM
   60 CONTINUE
      GOTO20
C
C
C     FORMAT STATEMENTS
C     -----------------
C
10000 FORMAT(1F12.0,1I3)
11000 FORMAT(6F12.0)
12000 FORMAT(3F12.0)
13000 FORMAT(1H1,29X,'DATA',/,30X,'----',///,
     1  6X,'RADIUS OF ISOPOTENTIAL PLATEAU (FEET)',8X,F7.2/,
     2  6X,'NUMBER OF BOUNDING WELLS',21X,I3/,
     3  6X,'AQUIFER PERMEABILITY (MEINZERS)',14X,F7.2/,
     4  6X,'AQUIFER THICKNESS (FEET)',21X,F7.2/,
     5  6X,'PRE-EXISTING FLOW GRADIENT (FEET/MILE)',7X,F7.2////)
14000 FORMAT(25X,'BOUNDING WELL DATA',/,25X,'-----------------',////,
     1  5X,'WELL NO.',5X,'X(FEET)',9X,'Y(FEET)',9X,'RATE(GPM)'//)
15000 FORMAT(9X,I3,7X,F8.2,8X,F8.2,10X,F6.1)
C
   65 STOP
      END


      SUBROUTINE SOLVE (N)
      DIMENSION A(20,21),ID(20)
      COMMON ATA(20,20),ATB(20),QBW(20),ICHECK
      ICHECK=0
      M=N
      NN=N+1
      DO1I=1,M
      A(I,NN)=ATB(I)
      ID(I)=I
      DO1J=1,N
    1 A(I,J)=ATA(I,J)
      K=1
    2 NR=K
      NC=K
      B=ABS(A(K,K))
      DO4I=K,M
      DO4J=K,N
      IF(ABS(A(I,J))-B)4,4,3
    3 NR=I
      NC=J
      B=ABS(A(I,J))
    4 CONTINUE
      IF(NR-K)7,7,5
    5 DO6J=K,NN
      CC=A(NR,J)
      A(NR,J)=A(K,J)
```

```
  6 A(K,J)=CC
  7 CONTINUE
    IF(NC-K)10,10,8
  8 DO9I=1,M
    CC=A(I,NC)
    A(I,NC)=A(I,K)
  9 A(I,K)=CC
    I=ID(NC)
    ID(NC)=ID(K)
    ID(K)=I
 10 CONTINUE
 11 IF(A(K,K))12,18,12
 12 KK=K+1
    DO14J=KK,NN
    A(K,J)=A(K,J)/A(K,K)
    DO14I=1,M
    IF(K-I)13,14,13
 13 A(I,J)=A(I,J)-A(I,K)*A(K,J)
 14 CONTINUE
    K=KK
    IF(K-N)2,11,15
 15 CONTINUE
    DO17I=1,N
    DO17J=1,N
    IF(ID(J)-I)17,16,17
 16 QBW(I)=A(J,NN)
 17 CONTINUE
    GOTO20
 18 WRITE(6,19)
 19 FORMAT(10X,'NO UNIQUE SOLUTION')
    ICHECK=1
 20 RETURN
    END
```

DATA

RADIUS OF ISOPOTENTIAL PLATEAU (FEET)	1684.34
NUMBER OF BOUNDING WELLS	8
AQUIFER PERMEABILITY (MEINZERS)	800.00
AQUIFER THICKNESS (FEET)	50.00
PRE-EXISTING FLOW GRADIENT (FEET/MILE)	8.00

BOUNDING WELL DATA

WELL NO.	X(FEET)	Y(FEET)	RATE(GPM)
1	2525.00	0.0	0.9
2	2525.00	2525.00	-226.0
3	0.0	2525.00	-109.3
4	-2525.00	2525.00	-225.4
5	-2525.00	0.0	1.6
6	-2525.00	-2525.00	222.2
7	0.0	-2525.00	112.0
8	2525.00	-2525.00	224.2

APPENDIX C

COMPUTER PROGRAMS FOR CALCULATING RECOVERY EFFICIENCIES

The two computer programs listed in the following pages are for predicting the recovery efficiencies of the storage process for one and two cycle operation of a single well system. The programs are in FORTRAN IV language and are written for use on an IBM 360/65 system.

A list of the required input data is presented at the beginning of each program. Following this is a complete list of all the variable names used in the program along with their definitions.

To illustrate the manner in which the programs have to be modified for use with a multiple well system, a well field pattern similar to that shown in Figure 3.8b will be assumed. The operating procedure would be as follows: (1) Inject into the center well until the lagging edge of the mixed zone passes the outer ring of wells. (2) Inject into all five wells until the desired quantity is injected. (3) Allow the injected water to stand until needed. (4) Produce all five wells until breakthrough occurs at the outer ring of wells, at which time production from the well field is stopped. Subsequent cycles are made with injection beginning in all five wells simultaneously.

As stated in Chapter III it is assumed that all injection and production takes place through the center well of the pattern. The programs listed in the following pages

are for single well systems and it is assumed that during any half-cycle the rate during that half-cycle remains constant. To use the programs for the operating procedure outlined above provisions would have to be made to allow a rate change during the first injection half-cycle. This can be accomplished as follows:

1) Change statement 21 to read:

```
21 READ(5,13000)QR1GM,QR1AGM,QR2GM
```

Where:

QR1GM = Injection rate in center well until mixed zone passes outer ring of wells. (gpm)

QR1AGM = Combined injection rate of all five wells. (gpm)

QR2GM = Combined production rate of all five wells. (gpm)

2) Immediately following statement 22 add the statement:

```
READ(5,15100)TIMDAY
```

Where:

TIMDAY = Time of injection in center well until the mixed zone passes the outer ring of wells and injection begins in all five wells. (days)

3) Change statement 24 to read:

```
24 WRITE(6,19000)QR1GM,QR1AGM,QR2GM
```

4) Immediately following statement 51 add the statement:

```
QR1A=QR1AGM*CGMCCS
```

Where:

QR1A = Combined injection rate of all five wells. (cc/sec)

5) Immediately following statement 57 add the statement:

```
TIMSEC=TIMDAY*CFDSEC
```

Where:

TIMSEC = Time of injection in center well until mixed zone passes outer ring of wells and injection begins in all five wells. (sec)

6) Immediately following statements 151 and 191 add the the statements:

```
TCHECK=TRT-TIMSEC

IF(TCHECK.GE.0.0)QR1=QR1A
```

Where:

TCHECK = A time check to see if time of injection has reached or exceeded TIMSEC. (sec)

7) Change format statement 13000 to read:

```
13000 FORMAT(3F12.0)
```

8) Immediately following format statement 15000 add the the format statement:

```
15100 FORMAT(1F12.0)
```

9) Make the following changes in the continuation statements of format statement 19000:

```
     2  9X, 'INJECTION RATES FOR FIRST INJECTION HALF-
        CYCLE'/,

     3 11X, 'RATE FOR FIRST STEP', 38X,F11.6/,

     4 11X, 'RATE FOR SECOND STEP', 37X,F11.6/,

     5  9X, 'PRODUCTION RATE FOR FIRST PRODUCTION
           HALF-CYCLE', 12X,F11.6/)
```

The above has shown how the programs can be modified to take care of one rate change in the first injection half-cycle. If there is more than one rate change in the first injection half-cycle or if there are rate changes in any subsequent half-cycle, either injection or production, they can be handled in a similar manner.

It should be noted that in the single well case and the multiple well case it is necessary to run the one cycle program to obtain the fluid produced during the first production half-cycle and the total number of computation intervals through the end of the first production half-cycle before the two-cycle program can be run. This procedure is necessary since these two values are input data for the two-cycle program.

```
C
C
C     PROGRAM TO CALCULATE THE RECOVERY EFFICIENCY OF THE PROCESS OF
C      STORING FRESH WATER IN SALINE AQUIFERS.
C
C
C     PROGRAM 'CYCL1AM' (SINGLE WELL - ONE CYCLE)
C
C     ******************************************************************
C
C     DATA TO BE READ IN
C
C      FIRST CARD - FORMAT(6F12.0)
C       RBTFT  = RADIUS AT WHICH BREAKTHROUGH IS COMPUTED. FOR A SINGLE
C                WELL SYSTEM THIS WILL NORMALLY BE THE WELLBORE
C                RADIUS. FOR A MULTIPLE WELL SYSTEM THIS WILL
C                NORMALLY BE THE RADIUS FROM THE CENTER OF THE SYSTEM
C                TO THE OUTER RING OF WELLS. (FT)
C       CBT    = ALLOWABLE CONCENTRATION OF NATIVE SALT WATER IN
C                PRODUCED STREAM. (VOLUME FRACTION)
C       TILIFT = INTERVAL LENGTH FOR CALCULATIONS DURING INJECTION
C                HALF-CYCLES. A GOOD STARTING VALUE FOR THIS VARIABLE
C                IS TEN FEET. THE VALUE OF RECOVERY EFFICIENCY COMPUTED
C                BY THE PROGRAM IS SENSITIVE TO THE TOTAL NUMBER OF
C                COMPUTATION INTERVALS. HENCE THE VALUE OF THIS
C                VARIABLE SHOULD BE DECREASED UNTIL THE COMPUTED VALUE
C                OF RECOVERY EFFICIENCY DOES NOT CHANGE SIGNIFICANTLY.
C                THESE STATEMENTS ALSO APPLY TO THE INTERVAL LENGTH
C                USED FOR CALCULATIONS DURING PRODUCTION
C                HALF-CYCLES. (FT)
C       TILPFT = INTERVAL LENGTH FOR CALCULATIONS DURING PRODUCTION
C                HALF-CYCLES. (FT)
C       RINCFT = LENGTH OF INCREMENT FOR CALCULATION OF MIXED ZONE
C                LENGTHS. THIS VARIABLE SHOULD NOT EXCEED ONE TENTH
C                THE VALUE USED FOR INTERVAL LENGTH FOR CALCULATIONS
C                DURING INJECTION HALF-CYCLES. (FT)
C       TINCFT = INCREMENT BY WHICH TILIFT IS INCREASED IF MIXED ZONE
C                INTERSECTS THE WELL DURING CALCULATIONS FOR FIRST
C                INTERVAL OF FIRST INJECTION HALF-CYCLE. THIS VARIABLE
C                SHOULD NOT EXCEED ONE TENTH THE VALUE USED FOR
C                INTERVAL LENGTH FOR CALCULATIONS DURING INJECTION
C                HALF-CYCLES. (FT)
C
C      SECOND CARD - FORMAT(5F12.0)
C       HFT    = AQUIFER THICKNESS. (FT)
C       PLYMEI = AQUIFER PERMEABILITY. (MEINZERS)
C       PR     = POROSITY. (FRACTION)
C       ALF    = LONGITUDINAL DISPERSIVITY COEFFICIENT. VALUES IN EXCESS
C                OF TEN CENTIMETERS SHOULD NOT BE USED. (CM)
C       DIFMOL = COEFFICIENT OF MOLECULAR DIFFUSION. ((SQ CM)/SEC)
C
C      THIRD CARD - FORMAT(4F12.0)
C       VISCP1 = VISCOSITY OF THE INJECTED FRESH WATER. (CP)
C       VISCP2 = VISCOSITY OF THE NATIVE SALT WATER. (CP)
C       DENS1  = DENSITY OF THE INJECTED FRESH WATER. (GM/CC)
C       DENS2  = DENSITY OF THE NATIVE SALT WATER. (GM/CC)
C
C      FOURTH CARD - FORMAT(2F12.0)
C       QR1GM  = INJECTION RATE FOR FIRST INJECTION HALF-CYCLE. (GPM)
C       QR2GM  = PRODUCTION RATE FOR FIRST PRODUCTION HALF-CYCLE. (GPM)
C
C      FIFTH CARD - FORMAT(1F12.0)
C       FLING1 = FLUID INJECTED IN FIRST INJECTION HALF-CYCLE. (GAL)
C
C      SIXTH CARD - FORMAT(1F12.0)
C       TST1D  = STATIC STORAGE TIME AT THE END OF THE FIRST INJECTION
C                HALF-CYCLE. (DAYS)
C
C     ******************************************************************
```

```
C
C      DEFINITION OF VARIABLE NAMES USED IN PROGRAM
C
C        A1      = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  FIRST INJECTION HALF-CYCLE. (SQ CM)
C        A2      = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  FIRST PRODUCTION HALF-CYCLE. (SQ CM)
C        B1      = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  FIRST INJECTION HALF-CYCLE. (SQ CM)
C        B2      = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  FIRST PRODUCTION HALF-CYCLE. (SQ CM)
C        CFCPP   = CONVERSION FACTOR. (POISE/CENTIPOISE)
C        CFMSCM  = CONVERSION FACTOR. ((SQ CM)/MEINZER)
C        CFDSEC  = CONVERSION FACTOR. (SEC/DAY)
C        CFFTCM  = CONVERSION FACTOR. (CM/FT)
C        CFGLCC  = CONVERSION FACTOR. (CC/GAL)
C        CGMCCS  = CONVERSION FACTOR. ((CC/SEC)/(GAL/MIN))
C        CONST1  = VALUE USED IN CHECKING FOR BREAKTHROUGH DURING A
C                  PRODUCTION HALF-CYCLE.
C        CONST2  = VALUE USED IN CHECKING FOR BREAKTHROUGH DURING A
C                  PRODUCTION HALF-CYCLE.
C        CRCEFF  = CUMULATIVE RECOVERY EFFICIENCY. (FRACTION)
C        CVLING  = CUMULATIVE VOLUME OF FLUID INJECTED. (GAL)
C        CVLRDG  = CUMULATIVE VOLUME OF INJECTED FLUID RECOVERED. (GAL)
C        CVOLIN  = CUMULATIVE VOLUME OF FLUID INJECTED. (CC)
C        CVOLRD  = CUMULATIVE VOLUME OF INJECTED FLUID RECOVERED. (CC)
C        C11     = COMPUTED CONCENTRATION AT THE RADIUS AND AT THE TIME
C                  BEING CONSIDERED. (VOLUME FRACTION)
C        DG      = DENSITY GRADIENT. ((GM/CC)/CM)
C        DM1     = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                  PARAMETER GIVEN BY EQUATION 3.7A.
C        DM2     = DIMENSIONLESS GROUP. SECOND GROUP ON RIGHT SIDE OF
C                  EQUATION 3.7A.
C        DM2A    = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                  PARAMETER GIVEN BY EQUATION 3.7A.
C        DNOMI1  = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR FIRST INJECTION HALF-CYCLE.
C        DNOMP2  = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR FIRST PRODUCTION HALF-CYCLE.
C        DSDF    = DENSITY DIFFERENCE BETWEEN INJECTED AND NATIVE
C                  FLUIDS. (GM/CC)
C        DSC2    = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  FIRST PRODUCTION HALF-CYCLE. (SQ CM)
C        FLINJ1  = FLUID INJECTED IN FIRST INJECTION HALF-CYCLE. (CC)
C        FLPRG2  = FLUID PRODUCED IN FIRST PRODUCTION HALF-CYCLE. (GAL)
C        FLPRN2  = FLUID PRODUCED IN FIRST PRODUCTION HALF-CYCLE. (CC)
C        H       = AQUIFER THICKNESS. (CM)
C        I       = SUBSCRIPT DESIGNATING COMPUTATION INTERVAL.
C        NINT1   = NUMBER OF COMPUTATION INTERVALS THRU THE END OF THE
C                  FIRST INJECTION HALF-CYCLE.
C        NINT2   = NUMBER OF COMPUTATION INTERVALS THRU THE FIRST
C                  PRODUCTION HALF-CYCLE.
C        PLY     = AQUIFER PERMEABILITY. (SQ CM)
C        PPP     = PRODUCT OF PI, POROSITY, AND THICKNESS. (CM)
C        PPP1    = 2*PPP. (CM)
C        QI1     = TWO DIMENSIONAL FLOW RATE FOR FIRST INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QP2     = TWO DIMENSIONAL FLOW RATE FOR FIRST PRODUCTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QR1     = FLOW RATE FOR FIRST INJECTION HALF-CYCLE. (CC/SEC)
C        QR2     = FLOW RATE FOR FIRST PRODUCTION HALF-CYCLE. (CC/SEC)
C        QSI1    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR FIRST INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QSP2    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR FIRST PRODUCTION
C                  HALF-CYCLE UP TO COMPUTATION INTERVAL AT WHICH
C                  BREAKTHROUGH CHECK IS BEING MADE. ((SQ CM)/SEC)
C        R(I)    = RADIUS OF INJECTED FLUID AT THE ITH COMPUTATION
C                  INTERVAL ASSUMING NO MIXING OR GRAVITATIONAL
C                  SEGREGATION. (CM)
C        RBT     = RADIUS AT WHICH BREAKTHROUGH IS COMPUTED. (CM)
C        RCBT(I)= LEAST RADIUS TO A VALUE OF CONCENTRATION OF CBT FOR THE
C                  LAST COMPUTATION INTERVAL OF THE LAST PRODUCTION
C                  HALF-CYCLE. (CM)
C        RCBTFT = LEAST RADIUS TO A VALUE OF CONCENTRATION OF CBT FOR THE
C                  LAST COMPUTATION INTERVAL OF THE LAST PRODUCTION
C                  HALF-CYCLE. (FT)
```

```
C       RCEFF   = CYCLE RECOVERY EFFICIENCY. (FRACTION)
C       RINC    = LENGTH OF INCREMENT USED FOR CALCULATION OF MIXED
C                 ZONE LENGTHS. (CM)
C       RINJ1   = RADIUS OF INJECTED FLUID AT THE END OF THE FIRST
C                 INJECTION HALF-CYCLE ASSUMING NO MIXING AND NO
C                 GRAVITATIONAL SEGREGATION. (CM)
C       RL50    = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION
C                 LINE. (CM)
C       RL50I1 = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION LINE
C                 AT THE START OF THE FIRST PRODUCTION HALF-CYCLE. (CM)
C       RU50    = RADIUS TO UPPER END OF 50 PERCENT CONCENTRATION
C                 LINE. (CM)
C       R1      = RADIUS AT WHICH CONCENTRATION IS BEING COMPUTED. ALSO
C                 INNER RADIUS OF MIXED ZONE. (CM)
C       R2      = OUTER RADIUS OF MIXED ZONE. (CM)
C       R3      = LENGTH OF MIXED ZONE. (CM)
C       R4      = TILI/2. (CM)
C       R5      = RADIUS TO THE MIDPOINT BETWEEN R(I) AND R(I-1). (CM)
C       R5SQ    = R5 SQUARED. (SQ CM)
C       R6      = TILP/2. (CM)
C       SDENOM = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                 IN EQUATION 3.5 USING PSEUDO RATES TO CHECK FOR
C                 BREAKTHROUGH DURING A PRODUCTION HALF-CYCLE.
C       TILI    = INTERVAL LENGTH FOR COMPUTATIONS DURING INJECTION
C                 HALF-CYCLES. (CM)
C       TILP    = INTERVAL LENGTH FOR COMPUTATIONS DURING PRODUCTION
C                 HALF-CYCLES. (CM)
C       TRT     = CUMULATIVE TRAVEL TIME OF FRESH WATER-SALT WATER
C                 INTERFACE. (SEC)
C       TRT1    = CUMULATIVE TRAVEL TIME OF INTERFACE THRU END OF FIRST
C                 INJECTION HALF-CYCLE. (SEC)
C       TRT2    = CUMULATIVE TRAVEL TIME OF INTERFACE THRU COMPUTATION
C                 INTERVAL AT WHICH BREAKTHROUGH CHECK IS BEING
C                 MADE. (SEC)
C       TST1    = STATIC STORAGE TIME AT THE END OF THE FIRST INJECTION
C                 HALF-CYCLE. (SEC)
C       TSP     = PSEUDO TIME USED IN GRAVITATIONAL SEGREGATION
C                 CALCULATIONS. (SEC)
C       TT      = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL PLUS THE
C                 PSEUDO TIME FOR THAT INTERVAL. (SEC)
C       T1      = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 FIRST INJECTION HALF-CYCLE. ALSO CUMULATIVE TRAVEL
C                 TIME OF INTERFACE THRU END OF FIRST INJECTION
C                 HALF-CYCLE. (SEC)
C       T2      = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 FIRST PRODUCTION HALF-CYCLE. (SEC)
C       T11     = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL. (SEC)
C       VIS     = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (POISE)
C       VISCP   = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (CP)
C       VOLNR   = TOTAL VOLUME OF INJECTED FLUID NOT RECOVERED. (CC)
C       X       = VALUE OF DIMENSIONLESS PARAMETER GIVEN BY
C                 EQUATION 3.7A.
C       XL(I)   = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                 LINE FOR LINEAR GEOMETRY. (CM)
C       XR(I)   = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                 LINE FOR RADIAL GEOMETRY. (CM)
C       XX      = ARGUMENT OF COMPLEMENTARY ERROR FUNCTION FOR
C                 EQUATION 3.5.
C       YL(I)   = RATIO OF HORIZONTAL PROJECTION OF 50 PERCENT
C                 CONCENTRATION LINE TO AQUIFER THICKNESS FOR LINEAR
C                 GEOMETRY.
C
C     ***********************************************************************
C
C
C
C
      DIMENSION R(1000),YL(1000),XL(1000),XR(1000),RCBT(1000)
C
C
C     PART 1 - READING DATA
C     ------   ------------
C
C
```

```
   20 READ(5,10000,END=1380)RBTFT,CBT,TILIFT,TILPFT,RINCFT,TINCFT
      READ(5,11000)HFT,PLYMEI,PR,ALF,DIFMOL
      READ(5,12000)VISCP1,VISCP2,DENS1,DENS2
   21 READ(5,13000)QR1GM,QR2GM
      READ(5,14000)FLING1
   22 READ(5,15000)TST1D
C
C     CALCULATION OF DENSITY DIFFERENCE AND MEAN VISCOSITY OF FLUIDS
C
      DSDF=ABS(DENS1-DENS2)
      VISCP=(VISCP1+VISCP2)/2.0
C
C
C     PART 2 - PRINTING DATA
C     ------   -------------
C
C
      WRITE(6,16000)HFT,PLYMEI,PR,ALF,DIFMOL
      WRITE(6,17000)VISCP1,VISCP2,VISCP
      WRITE(6,18000)DENS1,DENS2,DSDF
   24 WRITE(6,19000)QR1GM,QR2GM
      WRITE(6,20000)FLING1
      WRITE(6,21000)TST1D
C
      GOTO50
   30 TILIFT=TILIFT+TINCFT
      GOTO60
C
C
C     PART 3 - CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ------   ------------------------------------------------
C
C
   50 CFFTCM=30.4801
      CFMSCM=0.4842154E-9*VISCP2
      CFGLCC=3785.434
      CGMCCS=63.0906
      CFDSEC=86400.0
      CFCPP=0.01
      RBT=RBTFT*CFFTCM
      RINC=RINCFT*CFFTCM
      H=HFT*CFFTCM
      PPP=3.1416*PR*H
      PPP1=2.0*PPP
      PLY=PLYMEI*CFMSCM
      FLINJ1=FLING1*CFGLCC
   51 QR1=QR1GM*CGMCCS
   52 QR2=QR2GM*CGMCCS
   57 TST1=TST1D*CFDSEC
      VIS=VISCP*CFCPP
      DM1=(PLY*981.**DSDF)/(PR*VIS*H)
      DM2A=VIS**0.6667/(DSDF**1.6667*981.**0.3333)
      TILP=TILPFT*CFFTCM
      R6=TILP/2.0
   60 TILI=TILIFT*CFFTCM
      R4=TILI/2.0
C
C
C     PART 4 - CALCULATIONS FOR FIRST INJECTION HALF-CYCLE
C     ------   -------------------------------------------
C
C
C     CALCULATIONS OF INTERVALS AND RADIUS OF INJECTION
C
      RINJ1=SQRT(FLINJ1/PPP)
      NINT1=RINJ1/TILI
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
      I=1
      TRT=0.0
      R(I)=TILI
   61 R5=R(I)-R4
      R5SQ=R5*R5
      R1=R5-RINC
      T1=PPP*R5SQ/QR1
```

```
      QI1=QR1/PPP1
      DNOMI1=2.0*SQRT(1.333*ALF*(2.0*QI1*T1)**1.5+DIFMOL*(2.0*QI1*T1)**2
     1       /QI1)
   70 XX=(R5SQ-R1*R1)/DNOMI1
      C11=ERFC(XX)/2.0
      IF(C11-0.5)80,80,120
   80 IF(C11-0.03)90,90,100
   90 R2=R1
      GOTO110
  100 R1=R1-RINC
      IF(R1)30,30,70
  110 R1=R5+RINC
      GOTO70
  120 IF(C11-0.97)130,140,140
  130 R1=R1+RINC
      GOTO70
  140 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGATION
C
      IF(I-1)150,150,190
  150 DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*R(I)**2/QR1
  151 TRT=TRT+T11
      X=DM1*T11*DM2**0.5
      IF(X-0.1)160,160,165
  160 YL(I)=20.0*X
      GOTO180
  165 IF(X-1.0)170,175,175
  170 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO180
  175 YL(I)=6.5+2.0*X
  180 XL(I)=YL(I)*H
      GOTO290
  190 DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR1
  191 TRT=TRT+T11
      IF(YL(I-1)-2.0)195,195,200
  195 X=YL(I-1)/20.0
      GOTO215
  200 IF(YL(I-1)-6.5)205,210,210
  205 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO215
  210 X=(YL(I-1)-6.5)/2.0
  215 TSP=X/(DM1*DM2**0.5)
  220 TT=T11+TSP
      IF(I-NINT1)240,230,330
  230 TT=TT+TST1
  240 X=DM1*TT*DM2**0.5
      IF(X-0.1)250,250,255
  250 YL(I)=20.0*X
      GOTO270
  255 IF(X-1.0)260,265,265
  260 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO270
  265 YL(I)=6.5+2.0*X
  270 XL(I)=YL(I)*H
      GOTO300
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
  290 RU50=R(I)+XL(I)/2.0
      RL50=R(I)-XL(I)/2.0
      IF(RL50)1350,310,310
  300 A1=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B1=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      RU50=SQRT((3.142*R(I)**2+A1)/3.142)+(XL(I)-XL(I-1))/2.0
      RL50=SQRT((3.142*R(I)**2-B1)/3.142)-(XL(I)-XL(I-1))/2.0
  310 XR(I)=RU50-RL50
      I=I+1
      IF(I-NINT1)320,320,330
  320 R(I)=R(I-1)+TILI
      GOTO61
```

```
  330 TRT1=TRT
      RL50I1=RL50
      QSI1=(RL50I1*RL50I1)/(2.0*TRT1)
      CONST1=(2.0*QSI1*TRT1)**1.5
      CONST2=(2.0*QSI1*TRT1)**2/QSI1
C
C
C     PART 5 - CALCULATIONS FOR FIRST PRODUCTION HALF-CYCLE
C     ------   ------------------------------------------
C
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
  340 R(I)=R(I-1)-TILP
      IF(R(I))550,550,345
  345 R5=R(I)+R6
      R5SQ=R5*R5
      R1=R5-RINC
      T1=PPP*R(NINT1)**2/QR1
      T2=T1+PPP*(R(NINT1)**2-R5SQ)/QR2
      QP2=QR2/PPP1
      DNOMP2=2.0*SQRT(1.333*ALF*((2.0*QP2*T2)**1.5-(2.0*QP2*T1)**1.5+(2.
     1        0*QI1*T1)**1.5)+DIFMOL*((2.0*QP2*T2)**2/QP2-(2.0*QP2*T1)**2
     2        /QP2+(2.0*QI1*T1)**2/QI1))
  350 XX=(R5SQ-R1*R1)/DNOMP2
      C11=ERFC(XX)/2.0
      IF(C11-0.5)360,360,400
  360 IF(C11-0.03)370,370,380
  370 R2=R1
      GOTO390
  380 R1=R1-RINC
      IF(R1)550,350,350
  390 R1=R5+RINC
      GOTO350
  400 IF(C11-0.97)410,420,420
  410 R1=R1+RINC
      GOTO350
  420 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGRATION
C
      DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QP2
      TRT=TRT+T11
      TRT2=TRT
      IF(YL(I-1)-2.0)430,430,440
  430 X=YL(I-1)/20.0
      GOTO470
  440 IF(YL(I-1)-8.5)450,460,460
  450 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO470
  460 X=(YL(I-1)-6.5)/2.0
  470 TSP=X/(DM1*DM2**0.5)
  480 TT=T11+TSP
      X=DM1*TT*DM2**0.5
      IF(X-0.1)490,490,495
  490 YL(I)=20.0*X
      GOTO510
  495 IF(X-1.0)500,505,505
  500 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO510
  505 YL(I)=6.5+2.0*X
  510 XL(I)=YL(I)*H
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
      A2=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B2=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      DSC2=3.142*R(I)**2-B2
      IF(DSC2)550,550,520
  520 RL50=SQRT(DSC2/3.142)-(XL(I)-XL(I-1))/2.0
      IF(RL50)550,550,530
  530 RU50=SQRT((3.142*R(I)**2+A2)/3.142)+(XL(I)-XL(I-1))/2.0
C
```

```
C      CHECKING FOR BREAKTHROUGH
C
       RL50SQ=RL50*RL50
       QSP2=(RL50I1*RL50I1-RL50SQ)/(2.0*(TRT2-TRT1))
       SDENOM=2.0*SQRT(1.333*ALF*((2.0*QSP2*TRT2)**1.5-(2.0*QSP2*TRT1)**1
      1        .5+CONST1)+DIFMOL*((2.0*QSP2*TRT2)**2/QSP2-(2.0*QSP2*TRT1)*
      2        *2/QSP2+CONST2))
       R1=RL50-RINC
       IF(R1-RBT)550,540,540
  540  XX=(RL50SQ-R1*R1)/SDENOM
       C11=ERFC(XX)/2.0
       IF(C11-CBT)542,542,541
  541  R1=R1-RINC
       IF(R1-RBT)550,540,540
  542  RCBT(I)=R1
       XR(I)=RU50-RL50
  543  I=I+1
       GOTO340
  550  I=I-1
       NINT2=I
C
C
C      PART 6 - CALCULATION OF RECOVERY EFFICIENCY
C      ------   ----------------------------------
C
C
 1350  WRITE(6,32000)
C
       CVOLIN=FLINJ1
       IF(I.GT.NINT1)GOTO1360
       FLPRN2=0.0
       CVOLRD=FLPRN2
       GOTO1370
 1360  A2=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
       B2=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
       RU50=SQRT((3.142*R(I)**2+A2)/3.142)+(XL(I)-XL(I-1))/2.0
       RL50=SQRT((3.142*R(I)**2-B2)/3.142)-(XL(I)-XL(I-1))/2.0
       VOLNR=PPP*((RU50*RU50+RU50*RL50+RL50*RL50)/3.0-(RCBT(I)*RCBT(I)-
      1       RBT*RBT))
       FLPRN2=CVOLIN-VOLNR
       CVOLRD=FLPRN2
 1370  RCEFF=FLPRN2/FLINJ1
       CRCEFF=CVOLRD/CVOLIN
       FLPRG2=FLPRN2/CFGLCC
       CVLRDG=CVOLRD/CFGLCC
       WRITE(6,33000)FLPRG2,CVLRDG,I,RCEFF,CRCEFF,X,TILIFT,TILPFT
C
       GOTO20
C
C
C      PART 7 - FORMAT STATEMENTS
C      ------   -----------------
C
C
10000  FORMAT(6F12.0)
11000  FORMAT(5F12.0)
12000  FORMAT(4F12.0)
13000  FORMAT(2F12.0)
14000  FORMAT(1F12.0)
15000  FORMAT(1F12.0)
16000  FORMAT(1H1,35X,'DATA'/,36X,'----'////,6X,'POROUS MEDIUM'//,
      1  9X,'THICKNESS OF THE MEDIUM (FT)',30X,F10.6/,
      2  9X,'PERMEABILITY OF THE MEDIUM (MEINZERS)',20X,F11.6/,
      3  9X,'POROSITY OF THE MEDIUM (FRACTION)',27X,F8.6/,
      4  9X,'LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)',15X,F9.6/,
      5  9X,'COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)',14X,F8.6///
      6  )
17000  FORMAT(6X,'FLUID PROPERTIES'//,8X,'VISCOSITY OF THE FLUIDS (CP)'/,
      1  9X,'VISCOSITY OF THE INJECTED FLUID',29X,F8.6/,
      2  9X,'VISCOSITY OF THE NATIVE FLUID',31X,F8.6/,
      3  9X,'MEAN VISCOSITY OF THE TWO FLUIDS',28X,F8.6)
18000  FORMAT(8X,'DENSITY OF THE FLUIDS (GM/CC)'/,
      1  9X,'DENSITY OF THE INJECTED FLUID',31X,F8.6/,
      2  9X,'DENSITY OF THE NATIVE FLUID',33X,F8.6/,
      3  9X,'DENSITY DIFF. BETWEEN THE FLUIDS',28X,F8.6///)
```

```
19000 FORMAT(6X,'OPERATING CONDITIONS'//,
     1  8X,'INJECTION AND PRODUCTION RATES (GAL/MIN)'/,
     2  9X,'INJECTION  RATE FOR FIRST  INJECTION  HALF-CYCLE',9X,F11.6/,
     3  9X,'PRODUCTION RATE FOR FIRST  PRODUCTION HALF-CYCLE',9X,F11.6/)
20000 FORMAT(8X,'VOLUME OF FLUID INJECTED OR PRODUCED (GALLONS)'/,
     1  9X,'FLUID INJECTED IN FIRST  INJECTION  HALF-CYCLE',4X,F12.0/)
21000 FORMAT(8X,'TIME OF STATIC STORAGE (DAYS)'/,
     1  9X,'AT THE END OF FIRST  INJECTION HALF-CYCLE',16X,F11.6)
32000 FORMAT(1H1,24X,'CALCULATION OF RECOVERY EFFICIENCY'/,
     1  25X,'-----------------------------------'///)
33000 FORMAT(6X,'FLPRG2=',E13.6,6X,'CVLRDG=',E13.6,6X,'I=',I3/,7X,
     1  'RCEFF=',E13.6,6X,'CRCEFF=',E13.6,6X,'X=',E13.6/,6X,
     2  'TILIFT=',E13.6,6X,'TILPFT=',E13.6)
C
 1380 STOP
      END
```

DATA

POROUS MEDIUM

THICKNESS OF THE MEDIUM (FT)	100.000000
PERMEABILITY OF THE MEDIUM (MEINZERS)	400.000000
POROSITY OF THE MEDIUM (FRACTION)	0.300000
LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)	1.000000
COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)	0.000001

FLUID PROPERTIES

VISCOSITY OF THE FLUIDS (CP)	
VISCOSITY OF THE INJECTED FLUID	1.000000
VISCOSITY OF THE NATIVE FLUID	1.000000
MEAN VISCOSITY OF THE TWO FLUIDS	1.000000
DENSITY OF THE FLUIDS (GM/CC)	
DENSITY OF THE INJECTED FLUID	1.000000
DENSITY OF THE NATIVE FLUID	1.025000
DENSITY DIFF. BETWEEN THE FLUIDS	0.025000

OPERATING CONDITIONS

INJECTION AND PRODUCTION RATES (GAL/MIN)	
INJECTION RATE FOR FIRST INJECTION HALF-CYCLE	1000.00000
PRODUCTION RATE FOR FIRST PRODUCTION HALF-CYCLE	1500.00000
VOLUME OF FLUID INJECTED OR PRODUCED (GALLONS)	
FLUID INJECTED IN FIRST INJECTION HALF-CYCLE	250000000.
TIME OF STATIC STORAGE (DAYS)	
AT THE END OF FIRST INJECTION HALF-CYCLE	100.000000

CALCULATION OF RECOVERY EFFICIENCY

```
FLPRG2= 0.166365E+09      CVLRDG= 0.166365E+09      I= 85
 RCEFF= 0.665461E+00      CRCEFF= 0.665461E+00      X= 0.179927E+00
TILIFT= 0.100000E+02      TILPFT= 0.100000E+02
```

```
C
C
C     PROGRAM TO CALCULATE THE RECOVERY EFFICIENCY OF THE PROCESS OF
C      STORING FRESH WATER IN SALINE AQUIFERS.
C
C
C     PROGRAM 'CYCL2AM' (SINGLE WELL - TWO CYCLES)
C
C
C     ********************************************************************
C
C     DATA TO BE READ IN
C
C      FIRST CARD - FORMAT(6F12.0)
C       RBTFT  = RADIUS AT WHICH BREAKTHROUGH IS COMPUTED. FOR A SINGLE
C                WELL SYSTEM THIS WILL NORMALLY BE THE WELLBORE
C                RADIUS. FOR A MULTIPLE WELL SYSTEM THIS WILL
C                NORMALLY BE THE RADIUS FROM THE CENTER OF THE SYSTEM
C                TO THE OUTER RING OF WELLS. (FT)
C       CBT    = ALLOWABLE CONCENTRATION OF NATIVE SALT WATER IN
C                PRODUCED STREAM. (VOLUME FRACTION)
C       TILIFT = INTERVAL LENGTH FOR CALCULATIONS DURING INJECTION
C                HALF-CYCLES. A GOOD STARTING VALUE FOR THIS VARIABLE
C                IS TEN FEET. THE VALUE OF RECOVERY EFFICIENCY COMPUTED
C                BY THE PROGRAM IS SENSITIVE TO THE TOTAL NUMBER OF
C                COMPUTATION INTERVALS. HENCE THE VALUE OF THIS
C                VARIABLE SHOULD BE DECREASED UNTIL THE COMPUTED VALUE
C                OF RECOVERY EFFICIENCY DOES NOT CHANGE SIGNIFICANTLY.
C                THESE STATEMENTS ALSO APPLY TO THE INTERVAL LENGTH
C                USED FOR CALCULATIONS DURING PRODUCTION
C                HALF-CYCLES. (FT)
C       TILPFT = INTERVAL LENGTH FOR CALCULATIONS DURING PRODUCTION
C                HALF-CYCLES. (FT)
C       RINCFT = LENGTH OF INCREMENT FOR CALCULATION OF MIXED ZONE
C                LENGTHS. THIS VARIABLE SHOULD NOT EXCEED ONE TENTH
C                THE VALUE USED FOR INTERVAL LENGTH FOR CALCULATIONS
C                DURING INJECTION HALF-CYCLES. (FT)
C       TINCFT = INCREMENT BY WHICH TILIFT IS INCREASED IF MIXED ZONE
C                INTERSECTS THE WELL DURING CALCULATIONS FOR FIRST
C                INTERVAL OF FIRST INJECTION HALF-CYCLE. THIS VARIABLE
C                SHOULD NOT EXCEED ONE TENTH THE VALUE USED FOR
C                INTERVAL LENGTH FOR CALCULATIONS DURING INJECTION
C                HALF-CYCLES. (FT)
C
C      SECOND CARD - FORMAT(5F12.0)
C       HFT    = AQUIFER THICKNESS. (FT)
C       PLYMEI = AQUIFER PERMEABILITY. (MEINZERS)
C       PR     = POROSITY. (FRACTION)
C       ALF    = LONGITUDINAL DISPERSIVITY COEFFICIENT. VALUES IN EXCESS
C                OF TEN CENTIMETERS SHOULD NOT BE USED. (CM)
C       DIFMOL = COEFFICIENT OF MOLECULAR DIFFUSION. ((SQ CM)/SEC)
C
C      THIRD CARD - FORMAT(4F12.0)
C       VISCP1 = VISCOSITY OF THE INJECTED FRESH WATER. (CP)
C       VISCP2 = VISCOSITY OF THE NATIVE SALT WATER. (CP)
C       DENS1  = DENSITY OF THE INJECTED FRESH WATER. (GM/CC)
C       DENS2  = DENSITY OF THE NATIVE SALT WATER. (GM/CC)
C
C      FOURTH CARD - FORMAT(4F12.0)
C       QR1GM  = INJECTION RATE FOR FIRST INJECTION HALF-CYCLE. (GPM)
C       QR2GM  = PRODUCTION RATE FOR FIRST PRODUCTION HALF-CYCLE. (GPM)
C       QR3GM  = INJECTION RATE FOR SECOND INJECTION HALF-CYCLE. (GPM)
C       QR4GM  = PRODUCTION RATE FOR SECOND PRODUCTION HALF-CYCLE. (GPM)
C
C      FIFTH CARD - FORMAT(3F12.0)
C       FLING1 = FLUID INJECTED IN FIRST INJECTION HALF-CYCLE. (GAL)
C       FLPRG2 = FLUID PRODUCED IN FIRST PRODUCTION HALF-CYCLE. (GAL)
C       FLING3 = FLUID INJECTED IN SECOND INJECTION HALF-CYCLE. (GAL)
C
C      SIXTH CARD - FORMAT(2F12.0)
C       TST1D  = STATIC STORAGE TIME AT THE END OF THE FIRST INJECTION
C                HALF-CYCLE. (DAYS)
C       TST3D  = STATIC STORAGE TIME AT THE END OF THE SECOND INJECTION
C                HALF-CYCLE. (DAYS)
C
```

```
C       SEVENTH CARD - FORMAT(1I3)
C        NINT2  = NUMBER OF COMPUTATION INTERVALS THRU THE FIRST
C                 PRODUCTION HALF-CYCLE.
C
C     ********************************************************************
C
C     DEFINITION OF VARIABLE NAMES USED IN PROGRAM
C
C       A1       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  FIRST INJECTION HALF-CYCLE. (SQ CM)
C       A2       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  FIRST PRODUCTION HALF-CYCLE. (SQ CM)
C       A3       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  SECOND INJECTION HALF-CYCLE. (SQ CM)
C       A4       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RU50 FOR
C                  SECOND PRODUCTION HALF-CYCLE. (SQ CM)
C       B1       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  FIRST INJECTION HALF-CYCLE. (SQ CM)
C       B2       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  FIRST PRODUCTION HALF-CYCLE. (SQ CM)
C       B3       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  SECOND INJECTION HALF-CYCLE. (SQ CM)
C       B4       = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  SECOND PRODUCTION HALF-CYCLE. (SQ CM)
C       CFCPP    = CONVERSION FACTOR. (POISE/CENTIPOISE)
C       CFMSCM   = CONVERSION FACTOR. ((SQ CM)/MEINZER)
C       CFDSEC   = CONVERSION FACTOR. (SEC/DAY)
C       CFFTCM   = CONVERSION FACTOR. (CM/FT)
C       CFGLCC   = CONVERSION FACTOR. (CC/GAL)
C       CGMCCS   = CONVERSION FACTOR. ((CC/SEC)/(GAL/MIN))
C       CONST1   = VALUE USED IN CHECKING FOR BREAKTHROUGH DURING A
C                  PRODUCTION HALF-CYCLE.
C       CONST2   = VALUE USED IN CHECKING FOR BREAKTHROUGH DURING A
C                  PRODUCTION HALF-CYCLE.
C       CRCEFF   = CUMULATIVE RECOVERY EFFICIENCY. (FRACTION)
C       CVLING   = CUMULATIVE VOLUME OF FLUID INJECTED. (GAL)
C       CVLRDG   = CUMULATIVE VOLUME OF INJECTED FLUID RECOVERED. (GAL)
C       CVOLIN   = CUMULATIVE VOLUME OF FLUID INJECTED. (CC)
C       CVOLRC   = CUMULATIVE VOLUME OF INJECTED FLUID RECOVERED. (CC)
C       C11      = COMPUTED CONCENTRATION AT THE RADIUS AND AT THE TIME
C                  BEING CONSIDERED. (VOLUME FRACTION)
C       DG       = DENSITY GRADIENT. ((GM/CC)/CM)
C       DM1      = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                  PARAMETER GIVEN BY EQUATION 3.7A.
C       DM2      = DIMENSIONLESS GROUP. SECOND GROUP ON RIGHT SIDE OF
C                  EQUATION 3.7A.
C       DM2A     = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                  PARAMETER GIVEN BY EQUATION 3.7A.
C       DNOMI1   = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR FIRST INJECTION HALF-CYCLE.
C       DNOMI3   = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR FIRST PRODUCTION HALF-CYCLE.
C       DNOMP2   = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR FIRST PRODUCTION HALF-CYCLE.
C       DNOMP4   = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 FOR SECOND PRODUCTION HALF-CYCLE.
C       DSDF     = DENSITY DIFFERENCE BETWEEN INJECTED AND NATIVE
C                  FLUIDS. (GM/CC)
C       DSC4     = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF RL50 FOR
C                  SECOND PRODUCTION HALF-CYCLE. (SQ CM)
C       FLINJ1   = FLUID INJECTED IN FIRST INJECTION HALF-CYCLE. (CC)
C       FLINJ3   = FLUID INJECTED IN SECOND INJECTION HALF-CYCLE. (CC)
C       FLPRG4   = FLUID PRODUCED IN SECOND PRODUCTION HALF-CYCLE. (GAL)
C       FLPRN2   = FLUID PRODUCED IN FIRST PRODUCTION HALF-CYCLE. (CC)
C       FLPRN4   = FLUID PRODUCED IN SECOND PRODUCTION HALF-CYCLE. (CC)
C       H        = AQUIFER THICKNESS. (CM)
C       I        = SUBSCRIPT DESIGNATING COMPUTATION INTERVAL.
C       NINT     = INTERMEDIATE VALUE USED IN CALCULATING THE NUMBER OF
C                  COMPUTATION INTERVALS.
C       NINT1    = NUMBER OF COMPUTATION INTERVALS THRU THE END OF THE
C                  FIRST INJECTION HALF-CYCLE.
C       NINT3    = NUMBER OF COMPUTATION INTERVALS THRU THE SECOND
C                  INJECTION HALF CYCLE.
C       NINT4    = NUMBER OF COMPUTATION INTERVALS THRU THE SECOND
C                  PRODUCTION HALF-CYCLE.
C       PLY      = AQUIFER PERMEABILITY. (SQ CM)
```

```
C        PPP     = PRODUCT OF PI, POROSITY, AND THICKNESS. (CM)
C        PPP1    = 2*PPP. (CM)
C        QI1     = TWO DIMENSIONAL FLOW RATE FOR FIRST INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QI3     = TWO DIMENSIONAL FLOW RATE FOR SECOND INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QP2     = TWO DIMENSIONAL FLOW RATE FOR FIRST PRODUCTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QP4     = TWO DIMENSIONAL FLOW RATE FOR SECOND PRODUCTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QR1     = FLOW RATE FOR FIRST INJECTION HALF-CYCLE. (CC/SEC)
C        QR2     = FLOW RATE FOR FIRST PRODUCTION HALF-CYCLE. (CC/SEC)
C        QR3     = FLOW RATE FOR SECOND INJECTION HALF-CYCLE. (CC/SEC)
C        QR4     = FLOW RATE FOR SECOND PRODUCTION HALF-CYCLE. (CC/SEC)
C        QSI1    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR FIRST INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QSI3    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR SECOND INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QSP2    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR FIRST PRODUCTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QSP4    = TWO DIMENSIONAL PSEUDO FLOW RATE FOR SECOND PRODUCTION
C                  HALF-CYCLE UP TO COMPUTATION INTERVAL AT WHICH
C                  BREAKTHROUGH CHECK IS BEING MADE. ((SQ CM)/SEC)
C        R(I)    = RADIUS OF INJECTED FLUID AT THE ITH COMPUTATION
C                  INTERVAL ASSUMING NO MIXING OR GRAVITATIONAL
C                  SEGREGATION. (CM)
C        RBT     = RADIUS AT WHICH BREAKTHROUGH IS COMPUTED. (CM)
C        RCBT(I)= LEAST RADIUS TO A VALUE OF CONCENTRATION OF CBT FOR THE
C                  LAST COMPUTATION INTERVAL OF THE LAST PRODUCTION
C                  HALF-CYCLE. (CM)
C        RCBTFT = LEAST RADIUS TO A VALUE OF CONCENTRATION OF CBT FOR THE
C                  LAST COMPUTATION INTERVAL OF THE LAST PRODUCTION
C                  HALF-CYCLE. (FT)
C        RCEFF   = CYCLE RECOVERY EFFICIENCY. (FRACTION)
C        RINC    = LENGTH OF INCREMENT USED FOR CALCULATION OF MIXED
C                  ZONE LENGTHS. (CM)
C        RINJ1   = RADIUS OF INJECTED FLUID AT THE END OF THE FIRST
C                  INJECTION HALF-CYCLE ASSUMING NO MIXING AND NO
C                  GRAVITATIONAL SEGREGATION. (CM)
C        RINJ3   = RADIUS OF INJECTED FLUID AT THE END OF THE SECOND
C                  INJECTION HALF-CYCLE ASSUMING NO MIXING AND NO
C                  GRAVITATIONAL SEGREGATION. (CM)
C        RL50    = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION
C                  LINE. (CM)
C        RL50I1 = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION LINE
C                  AT THE START OF THE FIRST PRODUCTION HALF-CYCLE. (CM)
C        RL50I2 = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION LINE
C                  AT THE START OF THE SECOND PRODUCTION HALF-CYCLE. (CM)
C        RL50P1 = RADIUS TO LOWER END OF 50 PERCENT CONCENTRATION LINE
C                  AT THE START OF THE SECOND INJECTION HALF-CYCLE. (CM)
C        RU50    = RADIUS TO UPPER END OF 50 PERCENT CONCENTRATION
C                  LINE. (CM)
C        R1      = RADIUS AT WHICH CONCENTRATION IS BEING COMPUTED. ALSO
C                  INNER RADIUS OF MIXED ZONE. (CM)
C        R2      = OUTER RADIUS OF MIXED ZONE. (CM)
C        R3      = LENGTH OF MIXED ZONE. (CM)
C        R4      = TILI/2. (CM)
C        R5      = RADIUS TO THE MIDPOINT BETWEEN R(I) AND R(I-1). (CM)
C        R5SQ    = R5 SQUARED. (SQ CM)
C        R6      = TILP/2. (CM)
C        SDENOM = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                  IN EQUATION 3.5 USING PSEUDO RATES TO CHECK FOR
C                  BREAKTHROUGH DURING A PRODUCTION HALF-CYCLE.
C        TILI    = INTERVAL LENGTH FOR COMPUTATIONS DURING INJECTION
C                  HALF-CYCLES. (CM)
C        TILP    = INTERVAL LENGTH FOR COMPUTATIONS DURING PRODUCTION
C                  HALF-CYCLES. (CM)
C        TRT     = CUMULATIVE TRAVEL TIME OF FRESH WATER-SALT WATER
C                  INTERFACE. (SEC)
C        TRT1    = CUMULATIVE TRAVEL TIME OF INTERFACE THRU END OF FIRST
C                  INJECTION HALF-CYCLE. (SEC)
C        TRT2    = CUMULATIVE TRAVEL TIME OF INTERFACE THRU END OF FIRST
C                  PRODUCTION HALF-CYCLE. (SEC)
C        TRT3    = CUMULATIVE TRAVEL TIME OF INTERFACE THRU END OF SECOND
C                  INJECTION HALF-CYCLE. (SEC)
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C        TRT4   = CUMULATIVE TRAVEL TIME OF INTERFACE THRU COMPUTATION
C                 INTERVAL AT WHICH BREAKTHROUGH CHECK IS BEING
C                 MADE. (SEC)
C        TST1   = STATIC STORAGE TIME AT THE END OF THE FIRST INJECTION
C                 HALF-CYCLE. (SEC)
C        TST3   = STATIC STORAGE TIME AT THE END OF THE SECOND INJECTION
C                 HALF-CYCLE. (SEC)
C        TSP    = PSEUDO TIME USED IN GRAVITATIONAL SEGREGATION
C                 CALCULATIONS. (SEC)
C        TT     = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL PLUS THE
C                 PSEUDO TIME FOR THAT INTERVAL. (SEC)
C        T1     = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 FIRST INJECTION HALF-CYCLE. ALSO CUMULATIVE TRAVEL
C                 TIME OF INTERFACE THRU END OF FIRST INJECTION
C                 HALF-CYCLE. (SEC)
C        T2     = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 FIRST PRODUCTION HALF-CYCLE. ALSO CUMULATIVE TRAVEL
C                 TIME OF INTERFACE THRU END OF FIRST PRODUCTION
C                 HALF-CYCLE. (SEC)
C        T3     = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 SECOND INJECTION HALF-CYCLE. ALSO CUMULATIVE TRAVEL
C                 TIME OF INTERFACE THRU END OF SECOND INJECTION
C                 HALF-CYCLE. (SEC)
C        T4     = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                 SECOND PRODUCTION HALF-CYCLE. (SEC)
C        T11    = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL. (SEC)
C        VIS    = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (POISE)
C        VISCP  = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (CP)
C        VOLNR  = TOTAL VOLUME OF INJECTED FLUID NOT RECOVERED. (CC)
C        X      = VALUE OF DIMENSIONLESS PARAMETER GIVEN BY
C                 EQUATION 3.7A.
C        XL(I)  = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                 LINE FOR LINEAR GEOMETRY. (CM)
C        XR(I)  = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                 LINE FOR RADIAL GEOMETRY. (CM)
C        XX     = ARGUMENT OF COMPLEMENTARY ERROR FUNCTION FOR
C                 EQUATION 3.5.
C        YL(I)  = RATIO OF HORIZONTAL PROJECTION OF 50 PERCENT
C                 CONCENTRATION LINE TO AQUIFER THICKNESS FOR LINEAR
C                 GEOMETRY.
C
C     ***********************************************************************
C
C
C
C
      DIMENSION R(1000),YL(1000),XL(1000),XR(1000),RCBT(1000)
C
C
C     PART 1 - READING DATA
C     ------   ------------
C
C
   20 READ(5,10000,END=1380)RBTFT,CBT,TILIFT,TILPFT,RINCFT,TINCFT
      READ(5,11000)HFT,PLYMEI,PR,ALF,DIFMOL
      READ(5,12000)VISCP1,VISCP2,DENS1,DENS2
   21 READ(5,13000)QR1GM,QR2GM,QR3GM,QR4GM
      READ(5,14000)FLING1,FLPRG2,FLING3
   22 READ(5,15000)TST1D,TST3D
   23 READ(5,15200)NINT2
C
C     CALCULATION OF DENSITY DIFFERENCE AND MEAN VISCOSITY OF FLUIDS
C
      DSDF=ABS(DENS1-DENS2)
      VISCP=(VISCP1+VISCP2)/2.0
C
C
C     PART 2 - PRINTING DATA
C     ------   -------------
C
C
      WRITE(6,16000)HFT,PLYMEI,PR,ALF,DIFMOL
      WRITE(6,17000)VISCP1,VISCP2,VISCP
      WRITE(6,18000)DENS1,DENS2,DSDF
   24 WRITE(6,19000)QR1GM,QR2GM,QR3GM,QR4GM
      WRITE(6,20000)FLING1,FLPRG2,FLING3
      WRITE(6,21000)TST1D,TST3D
```

```
C
      GOTO50
   30 TILIFT=TILIFT+TINCFT
      GOTO60
C
C
C     PART 3 - CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ------   ---------------------------------------------
C
C
   50 CFFTCM=30.4801
      CFMSCM=0.4842154E-9*VISCP2
      CFGLCC=3785.434
      CGMCCS=63.0906
      CFDSEC=86400.0
      CFCPP=0.01
      RBT=RBTFT*CFFTCM
      RINC=RINCFT*CFFTCM
      H=HFT*CFFTCM
      PPP=3.1416*PR*H
      PPP1=2.0*PPP
      PLY=PLYMEI*CFMSCM
      FLINJ1=FLING1*CFGLCC
      FLPRN2=FLPRG2*CFGLCC
      FLINJ3=FLING3*CFGLCC
   51 QR1=QR1GM*CGMCCS
   52 QR2=QR2GM*CGMCCS
   53 QR3=QR3GM*CGMCCS
   54 QR4=QR4GM*CGMCCS
   57 TST1=TST1D*CFDSEC
   58 TST3=TST3D*CFDSEC
      VIS=VISCP*CFCPP
      DM1=(PLY*981.*DSDF)/(PR*VIS*H)
      DM2A=VIS**0.6667/(DSDF**1.6667*981.**0.3333)
      TILP=TILPFT*CFFTCM
      R6=TILP/2.0
   60 TILI=TILIFT*CFFTCM
      R4=TILI/2.0
C
C
C     PART 4 - CALCULATIONS FOR FIRST INJECTION HALF-CYCLE
C     ------   ------------------------------------------
C
C
C     CALCULATIONS OF INTERVALS AND RADIUS OF INJECTION
C
      RINJ1=SQRT(FLINJ1/PPP)
      NINT1=RINJ1/TILI
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
      I=1
      TRT=0.0
      R(I)=TILI
   61 R5=R(1)-R4
      R5SQ=R5*R5
      R1=R5-RINC
      T1=PPP*R5SQ/QR1
      QI1=QR1/PPP1
      DNOMI1=2.0*SQRT(1.333*ALF*(2.0*QI1*T1)**1.5+DIFMOL*(2.0*QI1*T1)**2
     1       /QI1)
   70 XX=(R5SQ-R1*R1)/DNOMI1
      C11=ERFC(XX)/2.0
      IF(C11-0.5)80,80,120
   80 IF(C11-0.03)90,90,100
   90 R2=R1
      GOTO110
  100 R1=R1-RINC
      IF(R1)30,30,70
  110 R1=R5+RINC
      GOTO70
  120 IF(C11-0.97)130,140,140
  130 R1=R1+RINC
      GOTO70
  140 R3=R1-R2
C
```

```
C      CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGATION
C
       IF(I-1)150,150,190
  150  DG=DSDF/R3
       DM2=DM2A*DG
       T11=PPP*R(I)**2/QR1
  151  TRT=TRT+T11
       X=DM1*T11*DM2**0.5
       IF(X-0.1)160,160,165
  160  YL(I)=20.0*X
       GOTO180
  165  IF(X-1.0)170,175,175
  170  YL(I)=0.7958+12.5238*X-4.8196*X**2
       GOTO180
  175  YL(I)=6.5+2.0*X
  180  XL(I)=YL(I)*H
       GOTO290
  190  DG=DSDF/R3
       DM2=DM2A*DG
       T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR1
  191  TRT=TRT+T11
       IF(YL(I-1)-2.0)195,195,200
  195  X=YL(I-1)/20.0
       GOTO215
  200  IF(YL(I-1)-8.5)205,210,210
  205  X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
       GOTO215
  210  X=(YL(I-1)-6.5)/2.0
  215  TSP=X/(DM1*DM2**0.5)
  220  TT=T11+TSP
       IF(I-NINT1)240,230,330
  230  TT=TT+TST1
  240  X=DM1*TT*DM2**0.5
       IF(X-0.1)250,250,255
  250  YL(I)=20.0*X
       GOTO270
  255  IF(X-1.0)260,265,265
  260  YL(I)=0.7958+12.5238*X-4.8196*X**2
       GOTO270
  265  YL(I)=6.5+2.0*X
  270  XL(I)=YL(I)*H
       GOTO300
C
C      APPROXIMATION TO RADIAL GEOMETRY
C
  290  RU50=R(I)+XL(I)/2.0
       RL50=R(I)-XL(I)/2.0
       GOTO310
  300  A1=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
       B1=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
       RU50=SQRT((3.142*R(I)**2+A1)/3.142)+(XL(I)-XL(I-1))/2.0
       RL50=SQRT((3.142*R(I)**2-B1)/3.142)-(XL(I)-XL(I-1))/2.0
  310  XR(I)=RU50-RL50
       I=I+1
       IF(I-NINT1)320,320,330
  320  R(I)=R(I-1)+TILI
       GOTO61
  330  TRT1=TRT
       RL50I1=RL50
       QSI1=(RL50I1*RL50I1)/(2.0*TRT1)
       CONST1=(2.0*QSI1*TRT1)**1.5
       CONST2=(2.0*QSI1*TRT1)**2/QSI1
C
C
C      PART 5 - CALCULATIONS FOR FIRST PRODUCTION HALF-CYCLE
C      ------   ------------------------------------------
C
C
C      CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
  340  R(I)=R(I-1)-TILP
       R5=R(I)+R6
       R5SQ=R5*R5
       R1=R5-RINC
       T1=PPP*R(NINT1)**2/QR1
       T2=T1+PPP*(R(NINT1)**2-R5SQ)/QR2
```

```
      QP2=QR2/PPP1
      DNOMP2=2.0*SQRT(1.333*ALF*((2.0*QP2*T2)**1.5-(2.0*QP2*T1)**1.5+(2.
     1        0*QI1*T1)**1.5)+DIFMOL*((2.0*QP2*T2)**2/QP2-(2.0*QP2*T1)**2
     2        /QP2+(2.0*QI1*T1)**2/QI1))
  350 XX=(R5SQ-R1*R1)/DNOMP2
      C11=ERFC(XX)/2.0
      IF(C11-0.5)360,360,400
  360 IF(C11-0.03)370,370,380
  370 R2=R1
      GOTO390
  380 R1=R1-RINC
      GOTO350
  390 R1=R5+RINC
      GOTO350
  400 IF(C11-0.97)410,420,420
  410 R1=R1+RINC
      GOTO350
  420 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGRATION
C
      DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR2
      TRT=TRT+T11
      IF(YL(I-1)-2.0)430,430,440
  430 X=YL(I-1)/20.0
      GOTO470
  440 IF(YL(I-1)-8.5)450,460,460
  450 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO470
  460 X=(YL(I-1)-6.5)/2.0
  470 TSP=X/(DM1*DM2**0.5)
  480 TT=T11+TSP
      X=DM1*TT*DM2**0.5
      IF(X-0.1)490,490,495
  490 YL(I)=20.0*X
      GOTO510
  495 IF(X-1.0)500,505,505
  500 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO510
  505 YL(I)=6.5+2.0*X
  510 XL(I)=YL(I)*H
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
      A2=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B2=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      RU50=SQRT((3.142*R(I)**2+A2)/3.142)+(XL(I)-XL(I-1))/2.0
      RL50=SQRT((3.142*R(I)**2-B2)/3.142)-(XL(I)-XL(I-1))/2.0
      XR(I)=RU50-RL50
  543 I=I+1
      IF(I-NINT2)340,340,560
  560 TRT2=TRT
      RL50P1=RL50
      QSP2=(RL50I1*RL50I1-RL50P1*RL50P1)/(2.0*(TRT2-TRT1))
      CONST1=CONST1+(2.0*QSP2*TRT2)**1.5-(2.0*QSP2*TRT1)**1.5
      CONST2=CONST2+(2.0*QSP2*TRT2)**2/QSP2-(2.0*QSP2*TRT1)**2/QSP2
C
C
C     PART 6 - CALCULATIONS FOR SECOND INJECTION HALF-CYCLE
C     ------   ------------------------------------------
C
C
C     CALCULATION OF INTERVALS AND RADIUS OF INJECTION
C
      RINJ3=SQRT((FLINJ1-FLPRN2+FLINJ3)/PPP)
      NINT=(RINJ3-R(NINT2))/TILI
      NINT3=NINT2+NINT
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
  580 R(I)=R(I-1)+TILI
      R5=R(I)-R4
      R5SQ=R5*R5
      R1=R5-RINC
```

```
      T2=T1+PPP*(R(NINT1)**2-R(NINT2)**2)/QR2
      T3=T2+PPP*(R5SQ-R(NINT2)**2)/QR3
      QI3=QR3/PPP1
      DNOMI3=2.0*SQRT(1.333*ALF*((2.0*QI3*T3)**1.5-(2.0*QI3*T2)**1.5+(2.
     1        0*QP2*T2)**1.5-(2.0*QP2*T1)**1.5+(2.0*QI1*T1)**1.5)+DIFMOL*
     2        ((2.0*QI3*T3)**2/QI3-(2.0*QI3*T2)**2/QI3+(2.0*QP2*T2)**2/QP
     3        2-(2.0*QP2*T1)**2/QP2+(2.0*QI1*T1)**2/QI1))
  590 XX=(R5SQ-R1*R1)/DNOMI3
      C11=ERFC(XX)/2.0
      IF(C11-0.5)600,600,640
  600 IF(C11-0.03)610,610,620
  610 R2=R1
      GOTO630
  620 R1=R1-RINC
      GOTO590
  630 R1=R5+RINC
      GOTO590
  640 IF(C11-0.97)650,660,660
  650 R1=R1+RINC
      GOTO590
  660 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGATION
C
      DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR3
  661 TRT=TRT+T11
      IF(YL(I-1)-2.0)665,665,670
  665 X=YL(I-1)/20.0
      GOTO685
  670 IF(YL(I-1)-8.5)675,680,680
  675 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO685
  680 X=(YL(I-1)-6.5)/2.0
  685 TSP=X/(DM1*DM2**0.5)
  690 TT=T11+TSP
      IF(I-NINT3)710,700,750
  700 TT=TT+TST3
  710 X=DM1*TT*DM2**0.5
      IF(X-0.1)715,715,720
  715 YL(I)=20.0*X
      GOTO740
  720 IF(X-1.0)725,730,730
  725 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO740
  730 YL(I)=6.5+2.0*X
  740 XL(I)=YL(I)*H
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
      A3=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B3=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      RU50=SQRT((3.142*R(I)**2+A3)/3.142)+(XL(I)-XL(I-1))/2.0
      RL50=SQRT((3.142*R(I)**2-B3)/3.142)-(XL(I)-XL(I-1))/2.0
      XR(I)=RU50-RL50
      I=I+1
      IF(I-NINT3)580,580,750
  750 TRT3=TRT
      RL50I2=RL50
      QSI3=(RL50I2*RL50I2-RL50P1*RL50P1)/(2.0*(TRT3-TRT2))
      CONST1=CONST1+(2.0*QSI3*TRT3)**1.5-(2.0*QSI3*TRT2)**1.5
      CONST2=CONST2+(2.0*QSI3*TRT3)**2/QSI3-(2.0*QSI3*TRT2)**2/QSI3
C
C
C     PART 7 - CALCULATIONS FOR SECOND PRODUCTION HALF-CYCLE
C     ------   ------------------------------------------
C
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
  760 R(I)=R(I-1)-TILP
      IF(R(I))940,940,765
  765 R5=R(I)+R6
      R5SQ=R5*P5
      R1=R5-RINC
```

```
      T3=T2+PPP*(R(NINT3)**2-R(NINT2)**2)/QR3
      T4=T3+PPP*(R(NINT3)**2-R5SQ)/QR4
      QP4=QR4/PPP1
      DNOMP4=2.0*SQRT(1.333*ALF*((2.0*QP4*T4)**1.5-(2.0*QP4*T3)**1.5+(2.
     1        0*QI3*T3)**1.5-(2.0*QI3*T2)**1.5+(2.0*QP2*T2)**1.5-(2.0*QP2
     2        *T1)**1.5+(2.0*QI1*T1)**1.5)+DIFMOL*((2.0*QP4*T4)**2/QP4-(2
     3        .0*QP4*T3)**2/QP4+(2.0*QI3*T3)**2/QI3-(2.0*QI3*T2)**2/QI3+(
     4        2.0*QP2*T2)**2/QP2-(2.0*QP2*T1)**2/QP2+(2.0*QI1*T1)**2/QI1)
     5        )
  770 XX=(R5SQ-R1*R1)/DNOMP4
      C11=ERFC(XX)/2.0
      IF(C11-0.5)780,780,820
  780 IF(C11-0.03)790,790,800
  790 R2=R1
      GOTO810
  800 R1=R1-RINC
      IF(R1)940,770,770
  810 R1=R5+RINC
      GOTO770
  820 IF(C11-0.97)830,840,840
  830 R1=R1+RINC
      GOTO770
  840 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGATION
C
      DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR4
      TRT=TRT+T11
      TRT4=TRT
      IF(YL(I-1)-2.0)845,845,850
  845 X=YL(I-1)/20.0
      GOTO865
  850 IF(YL(I-1)-8.5)855,860,860
  855 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO865
  860 X=(YL(I-1)-6.5)/2.0
  865 TSP=X/(DM1*DM2**0.5)
  870 TT=T11+TSP
      X=DM1*TT*DM2**0.5
      IF(X-0.1)875,875,880
  875 YL(I)=20.0*X
      GOTO900
  880 IF(X-1.0)885,890,890
  885 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO900
  890 YL(I)=6.5+2.0*X
  900 XL(I)=YL(I)*H
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
      A4=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B4=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      DSC4=3.142*R(I)**2-B4
      IF(DSC4)940,940,910
  910 RL50=SQRT(DSC4/3.142)-(XL(I)-XL(I-1))/2.0
      IF(RL50)940,940,920
  920 RU50=SQRT((3.142*R(I)**2+A4)/3.142)+(XL(I)-XL(I-1))/2.0
C
C     CHECKING FOR BREAKTHROUGH
C
      RL50SQ=RL50*RL50
      QSP4=(RL50I2*RL50I2-RL50SQ)/(2.0*(TRT4-TRT3))
      SDENOM=2.0*SQRT(1.333*ALF*((2.0*QSP4*TRT4)**1.5-(2.0*QSP4*TRT3)**1
     1        .5+CONST1)+DIFMOL*((2.0*QSP4*TRT4)**2/QSP4-(2.0*QSP4*TRT3)*
     2        *2/QSP4+CONST2))
      R1=RL50-RINC
      IF(R1-RBT)940,930,930
  930 XX=(RL50SQ-R1*R1)/SDENOM
      C11=ERFC(XX)/2.0
      IF(C11-CBT)932,932,931
  931 R1=R1-RINC
      IF(R1-RBT)940,930,930
  932 RCBT(I)=R1
      XR(I)=RU50-RL50
```

```
  933 I=I+1
      GOTO760
  940 I=I-1
      NINT4=I
C
C
C     PART 8 - CALCULATION OF RECOVERY EFFICIENCY
C     ------   ----------------------------------
C
C
 1350 WRITE(6,32000)
C
      CVOLIN=FLINJ1+FLINJ3
      IF(I.GT.NINT3)GOTO1360
      FLPRN4=0.0
      CVOLRD=FLPRN2+FLPRN4
      GOTO1370
 1360 A4=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B4=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      RU50=SQRT((3.142*R(I)**2+A4)/3.142)+(XL(I)-XL(I-1))/2.0
      RL50=SQRT((3.142*R(I)**2-B4)/3.142)-(XL(I)-XL(I-1))/2.0
      VOLNR=PPP*((RU50*RU50+RU50*RL50+RL50*RL50)/3.0-(RCBT(I)*RCBT(I)-
     1      RBT*RBT))
      FLPRN4=CVOLIN-VOLNR-FLPRN2
      CVOLRD=FLPRN2+FLPRN4
 1370 RCEFF=FLPRN4/FLINJ3
      CRCEFF=CVOLRD/CVOLIN
      FLPRG4=FLPRN4/CFGLCC
      CVLRDG=CVOLRD/CFGLCC
      WRITE(6,33000)FLPRG4,CVLRDG,I,RCEFF,CRCEFF,X,TILIFT,TILPFT
C
      GOTO20
C
C
C     PART 9 - FORMAT STATEMENTS
C     ------   -----------------
C
C
10000 FORMAT(6F12.0)
11000 FORMAT(5F12.0)
12000 FORMAT(4F12.0)
13000 FORMAT(4F12.0)
14000 FORMAT(3F12.0)
15000 FORMAT(2F12.0)
15200 FORMAT(1I3)
16000 FORMAT(1H1,35X,'DATA'/,36X,'----'///,6X,'POROUS MEDIUM'//,
     1  9X,'THICKNESS OF THE MEDIUM (FT)',30X,F10.6/,
     2  9X,'PERMEABILITY OF THE MEDIUM (MEINZERS)',20X,F11.6/,
     3  9X,'POROSITY OF THE MEDIUM (FRACTION)',27X,F8.6/,
     4  9X,'LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)',15X,F9.6/,
     5  9X,'COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)',14X,F8.6///
     6  )
17000 FORMAT(6X,'FLUID PROPERTIES'//,8X,'VISCOSITY OF THE FLUIDS (CP)'/,
     1  9X,'VISCOSITY OF THE INJECTED FLUID',29X,F8.6/,
     2  9X,'VISCOSITY OF THE NATIVE FLUID',31X,F8.6/,
     3  9X,'MEAN VISCOSITY OF THE TWO FLUIDS',28X,F8.6/)
18000 FORMAT(8X,'DENSITY OF THE FLUIDS (GM/CC)'/,
     1  9X,'DENSITY OF THE INJECTED FLUID',31X,F8.6/,
     2  9X,'DENSITY OF THE NATIVE FLUID',33X,F8.6/,
     3  9X,'DENSITY DIFF. BETWEEN THE FLUIDS',28X,F8.6///)
19000 FORMAT(6X,'OPERATING CONDITIONS'//,
     1  8X,'INJECTION AND PRODUCTION RATES (GAL/MIN)'/,
     2  9X,'INJECTION  RATE FOR FIRST  INJECTION  HALF-CYCLE',9X,F11.6/,
     3  9X,'PRODUCTION RATE FOR FIRST  PRODUCTION HALF-CYCLE',9X,F11.6/,
     4  9X,'INJECTION  RATE FOR SECOND INJECTION  HALF-CYCLE',9X,F11.6/,
     5  9X,'PRODUCTION RATE FOR SECOND PRODUCTION HALF-CYCLE',9X,F11.6/)
20000 FORMAT(8X,'VOLUME OF FLUID INJECTED OR PRODUCED (GALLONS)'/,
     1  9X,'FLUID INJECTED IN FIRST  INJECTION  HALF-CYCLE',4X,F12.0/,
     2  9X,'FLUID PRODUCED IN FIRST  PRODUCTION HALF-CYCLE',4X,F12.0/,
     3  9X,'FLUID INJECTED IN SECOND INJECTION  HALF-CYCLE',4X,F12.0/)
21000 FORMAT(8X,'TIME OF STATIC STORAGE (DAYS)'/,
     1  9X,'AT THE END OF FIRST  INJECTION HALF-CYCLE',16X,F11.6/,
     2  9X,'AT THE END OF SECOND INJECTION HALF-CYCLE',16X,F11.6)
32000 FORMAT(1H1,24X,'CALCULATION OF RECOVERY EFFICIENCY'/,
     1  25X,'----------------------------------'///)
```

```
33000 FORMAT(6X,'FLPRG4=',E13.6,6X,'CVLRDG=',E13.6,6X,'I=',I3/,7X,
     1  'RCEFF=',E13.6,6X,'CRCEFF=',E13.6,6X,'X=',E13.6/,6X,
     2  'TILIFT=',E13.6,6X,'TILPFT=',E13.6)
C
 1380 STOP
      END
```

DATA

POROUS MEDIUM

THICKNESS OF THE MEDIUM (FT)	100.000000
PERMEABILITY OF THE MEDIUM (MEINZERS)	400.000000
POROSITY OF THE MEDIUM (FRACTION)	0.300000
LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)	1.000000
COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)	0.000001

FLUID PROPERTIES

VISCOSITY OF THE FLUIDS (CP)

VISCOSITY OF THE INJECTED FLUID	1.000000
VISCOSITY OF THE NATIVE FLUID	1.000000
MEAN VISCOSITY OF THE TWO FLUIDS	1.000000

DENSITY OF THE FLUIDS (GM/CC)

DENSITY OF THE INJECTED FLUID	1.000000
DENSITY OF THE NATIVE FLUID	1.025000
DENSITY DIFF. BETWEEN THE FLUIDS	0.025000

OPERATING CONDITIONS

INJECTION AND PRODUCTION RATES (GAL/MIN)

INJECTION RATE FOR FIRST INJECTION HALF-CYCLE	1000.00000
PRODUCTION RATE FOR FIRST PRODUCTION HALF-CYCLE	1500.00000
INJECTION RATE FOR SECOND INJECTION HALF-CYCLE	1000.00000
PRODUCTION RATE FOR SECOND PRODUCTION HALF-CYCLE	1500.00000

VOLUME OF FLUID INJECTED OR PRODUCED (GALLONS)

FLUID INJECTED IN FIRST INJECTION HALF-CYCLE	250000000.
FLUID PRODUCED IN FIRST PRODUCTION HALF-CYCLE	166365200.
FLUID INJECTED IN SECOND INJECTION HALF-CYCLE	250000000.

TIME OF STATIC STORAGE (DAYS)

AT THE END OF FIRST INJECTION HALF-CYCLE	100.000000
AT THE END OF SECOND INJECTION HALF-CYCLE	100.000000

CALCULATION OF RECOVERY EFFICIENCY

```
FLPRG4= 0.198416E+09     CVLRDG= 0.364781E+09     I=146
 RCEFF= 0.793665E+00     CRCEFF= 0.729563E+00     X= 0.310674E+00
TILIFT= 0.100000E+02     TILPFT= 0.100000E+02
```

APPENDIX D

COMPUTER PROGRAM USED IN COMPUTING THE LONGITUDINAL DISPERSIVITY COEFFICIENT OF THE MINIAQUIFER

The computer program listed in the following pages is used to calculate the concentration of native fluid in the produced stream as a function of cumulative time since injection started. The program is for one cycle only and it is assumed that no density or viscosity differences exist between the injected and native fluids. The program is in FORTRAN IV language and is written for use on an IBM 360/65 system.

A list of the required input data is presented at the beginning of the program. Following this is a complete list of all the variable names used in the program along with their definitions.

```
C
C
C      PROGRAM TO CALCULATE CONCENTRATION AT THE WELL BORE AS A FUNCTION
C       OF CUMULATIVE TIME SINCE INJECTION STARTED. THIS PROGRAM IS FOR
C       ONE CYCLE ONLY AND ASSUMES NO DENSITY OR VISCOSITY DIFFERENCES
C       BETWEEN INJECTED AND NATIVE FLUIDS.
C
C
C      PROGRAM 'CONCENT'
C
C
C      ********************************************************************
C
C      DATA TO BE READ IN
C
C       FIRST CARD - FORMAT(6F12.0)
C        QR1GM  = FLOW RATE FOR INJECTION HALF-CYCLE. (GPM)
C        QR2GM  = FLOW RATE FOR PRODUCTION HALF-CYCLE. (GPM)
C        TID    = TOTAL INJECTION TIME. (DAYS)
C        TSD    = TOTAL TIME FROM START OF INJECTION UNTIL
C                  CONCENTRATION CALCULATIONS ARE STARTED. (DAYS)
C        HFT    = AQUIFER THICKNESS. (FT)
C        PR     = AQUIFER POROSITY. (FRACTION)
C
C       SECOND CARD - FORMAT(4F12.0)
C        ALF    = LONGITUDINAL DISPERSIVITY COEFFICIENT. (CM)
C        DIFMOL = COEFFICIENT OF MOLECULAR DIFFUSION. ((SQ CM)/SEC)
C        DELTD  = TIME INCREMENT FOR CONCENTRATION CALCULATIONS.
C                  SHOULD BE SMALL ENOUGH SO THAT AT LEAST TEN POINTS
C                  ON THE CONCENTRATION PROFILE ARE OBTAINED. (DAYS)
C        RWFT   = WELL BORE RADIUS. (FT)
C
C      ********************************************************************
C
C      DEFINITION OF VARIABLE NAMES USED IN PROGRAM
C
C        CFFTCM = CONVERSION FACTOR. (CM/FT)
C        CGMCCS = CONVERSION FACTOR. ((CC/SEC)/(GAL/MIN))
C        CFDSEC = CONVERSION FACTOR. (SEC/DAY)
C        QR1    = FLOW RATE FOR INJECTION HALF-CYCLE. (CC/SEC)
C        QR2    = FLOW RATE FOR PRODUCTION HALF-CYCLE. (CC/SEC)
C        H      = AQUIFER THICKNESS. (CM)
C        DELT   = TIME INCREMENT FOR CONCENTRATION CALCULATIONS. (SEC)
C        RW     = WELL BORE RADIUS. (CM)
C        T1     = TOTAL INJECTION TIME. (SEC)
C        T2     = CUMULATIVE TIME AT WHICH CONCENTRATION IS
C                  COMPUTED. (SEC)
C        T2D    = CUMULATIVE TIME AT WHICH CONCENTRATION IS
C                  COMPUTED. (DAYS)
C        PPP    = PRODUCT OF PI, POROSITY, AND THICKNESS. (CM)
C        PPP1   = 2*PPP1. (CM)
C        QI1    = TWO DIMENSIONAL FLOW RATE FOR INJECTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        QP2    = TWO DIMENSIONAL FLOW RATE FOR PRODUCTION
C                  HALF-CYCLE. ((SQ CM)/SEC)
C        C11    = COMPUTED CONCENTRATION. (VOLUME FRACTION)
C        FP2    = SQUARE OF DENOMINATOR OF ARGUMENT OF COMPLEMENTARY
C                  ERROR FUNCTION.
C        XX     = ARGUMENT OF COMPLEMENTARY ERROR FUNCTION.
C
C      ********************************************************************
C
C
C
C
C      READING DATA
C      ------------
C
   10 READ(5,1000,END=30)QR1GM,QR2GM,TID,TSD,HFT,PR
      READ(5,2000)ALF,DIFMOL,DELTD,RWFT
C
C
C      PRINTING DATA
C      -------------
C
```

```
      WRITE(6,3000)ALF,DIFMOL,RWFT,TID
      WRITE(6,4000)
C
C
C     CONSTANTS AND CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ---------------------------------------------------------------
C
      CFDSEC=86400.0
      CFFTCM=30.4801
      CGMCCS=63.0906
      DELT=DELTD*CFDSEC
      H=HFT*CFFTCM
      PPP=3.1416*PR*H
      PPP1=2.0*PPP
      QR1=QR1GM*CGMCCS
      QI1=QR1/PPP1
      QR2=QR2GM*CGMCCS
      QP2=QR2/PPP1
      RW=RWFT*CFFTCM
      T1=TID*CFDSEC
C
C
C     CALCULATION OF CONCENTRATION VERSUS CUMULATIVE TIME
C     ---------------------------------------------------
C
      T2=TSD*CFDSEC
   20 FP2=1.333*ALF*((2.0*QP2*T2)**1.5-(2.0*QP2*T1)**1.5+(2.0*QI1*T1)**1
     1    .5)+DIFMOL*((2.0*QP2*T2)**2/QP2-(2.0*QP2*T1)**2/QP2+(2.0*QI1*T
     2    1)**2/QI1)
      XX=(-QP2*(T2-T1)-RW**2/2.0+QI1*T1)/FP2**0.5
      C11=ERFC(XX)/2.0
      T2D=T2/CFDSEC
C
C
C     PRINTING COMPUTED VALUES OF CONCENTRATION VERSUS CUMULATIVE TIME
C     ----------------------------------------------------------------
C
      WRITE(6,5000)T2D,C11
C
      IF(C11.GT.0.99)GOTO10
      T2=T2+DELT
      GOTO20
C
C
C     FORMAT STATEMENTS
C     -----------------
 1000 FORMAT(6F12.0)
 2000 FORMAT(4F12.0)
 3000 FORMAT(1H1,9X,'LONGITUDINAL DISPERSIVITY COEFFICIENT (CM)',8X,F13.
     1  6/,10X,'COEFFICIENT OF MOLECULAR DIFFUSION ((SQ CM)/SEC)',2X,F13
     2  .6/,10X,'WELL BORE RADIUS (FT)',29X,F13.6/,10X,
     3  'INJECTION TIME (DAYS)',29X,F13.6///)
 4000 FORMAT(32X,'CONCENTRATION PROFILE'//,29X,'TIME',12X,
     1  'CONCENTRATION'/,28X,'(DAYS)',9X,'(VOLUME FRACTION)'//)
 5000 FORMAT(27X,F7.3,16X,F5.3)
C
   30 STOP
      END
```

```
      WRITE(6,3000)ALF,DIFMOL,RWFT,TID
      WRITE(6,4000)
C
C
C     CONSTANTS AND CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ---------------------------------------------------------------
C
      CFDSEC=86400.0
      CFFTCM=30.4801
      CGMCCS=63.0906
      DELT=DELTD*CFDSEC
      H=HFT*CFFTCM
      PPP=3.1416*PR*H
      PPP1=2.0*PPP
      QR1=QR1GM*CGMCCS
      QI1=QR1/PPP1
      QR2=QR2GM*CGMCCS
      QP2=QR2/PPP1
      RW=RWFT*CFFTCM
      T1=TID*CFDSEC
C
C
C     CALCULATION OF CONCENTRATION VERSUS CUMULATIVE TIME
C     ---------------------------------------------------
C
      T2=TSD*CFDSEC
   20 FP2=1.333*ALF*((2.0*QP2*T2)**1.5-(2.0*QP2*T1)**1.5+(2.0*QI1*T1)**1
     1     .5)+DIFMOL*((2.0*QP2*T2)**2/QP2-(2.0*QP2*T1)**2/QP2+(2.0*QI1*T
     2     1)**2/QI1)
      XX=(-QP2*(T2-T1)-RW**2/2.0+QI1*T1)/FP2**0.5
      C11=ERFC(XX)/2.0
      T2D=T2/CFDSEC
C
C
C     PRINTING COMPUTED VALUES OF CONCENTRATION VERSUS CUMULATIVE TIME
C     ----------------------------------------------------------------
C
      WRITE(6,5000)T2D,C11
C
      IF(C11.GT.0.99)GOTO10
      T2=T2+DELT
      GOTO20
C
C
C     FORMAT STATEMENTS
C     -----------------
 1000 FORMAT(6F12.0)
 2000 FORMAT(4F12.0)
 3000 FORMAT(1H1,9X,'LONGITUDINAL DISPERSIVITY COEFFICIENT (CM)',8X,F13.
     1  6/,10X,'COEFFICIENT OF MOLECULAR DIFFUSION ((SQ CM)/SEC)',2X,F13
     2  .6/,10X,'WELL BORE RADIUS (FT)',29X,F13.6/,10X,
     3  'INJECTION TIME (DAYS)',29X,F13.6///)
 4000 FORMAT(32X,'CONCENTRATION PROFILE'//,29X,'TIME',12X,
     1  'CONCENTRATION'/,28X,'(DAYS)',9X,'(VOLUME FRACTION)'//)
 5000 FORMAT(27X,F7.3,16X,F5.3)
C
   30 STOP
      END
```

LONGITUDINAL DISPERSIVITY COEFFICIENT (CM)	1.000000
COEFFICIENT OF MOLECULAR DIFFUSION ((SQ CM)/SEC)	0.000001
WELL BORE RADIUS (FT)	1.000000
INJECTION TIME (DAYS)	1.000000

CONCENTRATION PROFILE

TIME (DAYS)	CONCENTRATION (VOLUME FRACTION)
1.800	0.007
1.820	0.014
1.840	0.027
1.860	0.048
1.880	0.078
1.900	0.121
1.920	0.177
1.940	0.246
1.960	0.326
1.980	0.414
2.000	0.504
2.020	0.593
2.040	0.677
2.060	0.751
2.080	0.814
2.100	0.866
2.120	0.906
2.140	0.936
2.160	0.958
2.180	0.973
2.200	0.983
2.220	0.990
2.240	0.994

APPENDIX E

DETAILS OF EXPERIMENTAL VERIFICATION OF MATHEMATICAL MODEL

TABLE E-I

DATA FOR

SINGLE-WELL INJECTION AND PRODUCTION

	RUN NUMBER									
	1	2	3	6	8	11	12	14	16	17
	Run Type									
	1A	1C	1B	1C	1C	1A	1C	1C	1A	1A
FLUID PROPERTIES										
Visc. of Inj. Fluid (cp)	0.942	0.966	0.967	0.900	0.950	0.924	0.924	0.923	0.917	0.917
Visc. of Nat. Fluid (cp)	0.954	0.967	0.961	0.901	0.956	0.926	0.926	0.925	0.919	0.919
Mean Viscosity (cp)	0.948	0.967	0.964	0.901	0.953	0.925	0.925	0.924	0.918	0.918
Density of Inj. Fluid (gm/cc)	0.783	0.783	0.784	0.854	0.791	0.861	0.861	0.859	0.854	0.854
Density of Nat. Fluid (gm/cc)	0.781	0.781	0.782	0.776	0.782	0.785	0.785	0.782	0.778	0.778
Density Difference (gm/cc)	0.002	0.002	0.002	0.077	0.009	0.076	0.076	0.077	0.076	0.076
FIRST CYCLE										
Injection Rate (cc/min)	24.138	24.138	6.705	17.880	20.115	20.115	20.115	20.115	1.609	0.366
Volume Injected (cc)	5170.	5170.	5174.	4300.	5029.	5029.	5029.	5029.	5064.	5029.
Static Storage (min)	0.	0.	0.	0.	0.	0.	0.	250.	0.	0.
Production Rate (cc/min)	24.138	24.138	6.705	17.880	20.115	20.115	20.115	20.115	1.609	0.366
Volume Produced (cc)	4797.	4867.	4707.	3194.	4687.	3860.	3887.	3436.	1991.	852.
Cycle Recovery Eff. (%)										
Experimental	93	94	91	74	93	77	77	68	39	17
Computed	85	86	77	61	78	64	64	58	20	0
Cum. Recovery Eff. (%)										
Experimental	93	94	91	74	93	77	77	68	39	17
Computed	85	86	77	61	78	64	64	58	20	0
Value of D'less Group @ B.T.	0.074	0.063	0.284	0.970	0.241	0.964	0.964	1.537	9.927	--
SECOND CYCLE										
Injection Rate (cc/min)		24.138	6.705	17.880	20.115		20.115	20.115		
Volume Injected (cc)		5170.	5174.	4800.	5029.		5029.	5029.		
Static Storage (min)		0.	0.	0.	0.		0.	250.		
Production Rate (cc/min)		24.138	6.705	17.880	20.115		20.115	20.115		
Volume Produced (cc)		4932.	4866.	3760.	4747.		4484.	4194.		
Cycle Recovery Eff. (%)										
Experimental		95	94	87	94		89	83		
Computed		90	85	77	86		85	79		
Cum. Recovery Eff. (%)										
Experimental		95	93	81	94		83	76		
Computed		88	81	74	82		75	68		
Value of D'less Group @ B.T.		0.103	0.482	1.646	0.405		1.711	2.770		
THIRD CYCLE										
Injection Rate (cc/min)		24.138		17.880	20.115		20.115	20.115		
Volume Injected (cc)		5170.		4300.	5029.		5029.	5029.		
Static Storage (min)		0.		0.	0.		0.	250.		
Production Rate (cc/min)		24.138		17.880	20.115		20.115	20.115		
Volume Produced (cc)		5036.		3937.	4823.		4643.	4472.		
Cycle Recovery Eff. (%)										
Experimental		97		92	96		92	89		
Computed		92		85	88		87	81		
Cum. Recovery Eff. (%)										
Experimental		96		84	95		86	80		
Computed		89		78	84		79	72		
Value of D'less Group @ B.T.		0.141		2.316	0.550		2.355	3.827		

TABLE E-2

DATA FOR

SINGLE-WELL INJECTION AND MULTI-WELL PRODUCTION

	RUN NUMBER 4 5 Run Type 2A	2C
FLUID PROPERTIES		
Visc. of Inj. Fluid (cp)	0.979	0.990
Visc. of Nat. Fluid (cp)	0.965	0.980
Mean Viscosity (cp)	0.972	0.985
Density of Inj. Fluid (gm/cc)	0.783	0.792
Density of Nat. Fluid (gm/cc)	0.781	0.781
Density Difference (gm/cc)	0.002	0.011
FIRST CYCLE		
Injection Rate (cc/min)	18.285	18.285
Volume Injected (cc)	4754.	5181.
Static Storage (min)	9.750	8.583
First Production Rate (cc/min)	18.285	18.285
Volume Produced (cc)	3134.	3390.
Step Recovery Eff. (%)		
Experimental	66	65
Computed	67	62
Cum. Recovery Eff. (%)		
Experimental	66	65
Computed	67	62
Value of D'less Group @ B.T.	0.098	0.294
Second Production Rate (cc/min)	3.657	3.657
Volume Produced (cc)	948.	1285.
Step Recovery Eff. (%)		
Experimental	20	25
Computed	15	13
Cycle Recovery Eff. (%)		
Experimental	86	90
Computed	82	76
Cum. Recovery Eff. (%)		
Experimental	86	90
Computed	82	76
Value of D'less Group @ B.T.	0.104	0.309
SECOND CYCLE		
Injection Rate (cc/min)		18.285
Volume Injected (cc)		5181.
Static Storage (min)		5.417
First Production Rate (cc/min)		18.285
Volume Produced (cc)		3590.
Step Recovery Eff. (%)		
Experimental		69
Computed		70

	RUN NUMBER 4 5 Run Type 2A	2C
SECOND CYCLE (continued)		
Cum. Recovery Eff. (%)		
Experimental		80
Computed		73
Value of D'less Group @ B.T.		0.568
Second Production Rate (cc/min)		18.285
Volume Produced (cc)		1082.
Step Recovery Eff. (%)		
Experimental		21
Computed		13
Cycle Recovery Eff. (%)		
Experimental		90
Computed		83
Cum. Recovery Eff. (%)		
Experimental		90
Computed		80
Value of D'less Group @ B.T.		0.631
THIRD CYCLE		
Injection Rate (cc/min)		18.285
Volume Injected (cc)		5181.
Static Storage (min)		7.000
First Production Rate (cc/min)		18.285
Volume Produced (cc)		3680.
Step Recovery Eff. (%)		
Experimental		71
Computed		74
Cum. Recovery Eff. (%)		
Experimental		84
Computed		78
Value of D'less Group @ B.T.		0.799
Second Production Rate (cc/min)		18.285
Volume Produced (cc)		1403.
Step Recovery Eff. (%)		
Experimental		27
Computed		14
Cycle Recovery Eff. (%)		
Experimental		98
Computed		88
Cum. Recovery Eff. (%)		
Experimental		92
Computed		82
Value of D'less Group @ B.T.		0.852

TABLE E-3

DATA FOR MULTI-WELL INJECTION AND PRODUCTION

RUN NUMBER 9

Run Type 3C

FLUID PROPERTIES	
Visc. of Inj. Fluid (cp)	0.944
Visc. of Nat. Fluid (cp)	0.946
Mean Viscosity (cp)	0.945
Density of Inj. Fluid (gm/cc)	0.792
Density of Nat. Fluid (gm/cc)	0.783
Density Difference (gm/cc)	0.009
FIRST CYCLE	
First Injection Rate (cc/min)	4.023
Volume Injected (cc)	1006.
Second Injection Rate (cc/min)	20.115
Volume Injected (cc)	4023.
Static Storage (min)	0.
First Production Rate (cc/min)	20.115
Volume Produced (cc)	3755.
Step Recovery Eff. (%)	
Experimental	75
Computed	63
Cum. Recovery Eff. (%)	
Experimental	75
Computed	63
Value of D'less Group @ B.T.	0.398
Second Production Rate (cc/min)	4.023
Volume Produced (cc)	768.
Step Recovery Eff. (%)	
Experimental	15
Computed	12
Cycle Recovery Eff. (%)	
Experimental	90
Computed	76
Cum. Recovery Eff. (%)	
Experimental	90
Computed	76
Value of D'less Group @ B.T.	0.472
SECOND CYCLE	
First Injection Rate (cc/min)	4.023
Volume Injected (cc)	1006.
Second Injection Rate (cc/min)	20.115
Volume Injected (cc)	4023.
Static Storage (min)	0.
First Production Rate (cc/min)	20.115
Volume Produced (cc)	3984.
Step Recovery Eff. (%)	
Experimental	79
Computed	71

SECOND CYCLE (continued)	
Cum. Recovery Eff. (%)	
Experimental	85
Computed	74
Value of D'less Group @ B.T.	0.716
Second Production Rate (cc/min)	4.023
Volume Produced (cc)	698.
Step Recovery Eff. (%)	
Experimental	14
Computed	14
Cycle Recovery Eff. (%)	
Experimental	93
Computed	85
Cum. Recovery Eff. (%)	
Experimental	92
Computed	81
Value of D'less Group @ B.T.	0.769
THIRD CYCLE	
First Injection Rate (cc/min)	4.023
Volume Injected (cc)	1006.
Second Injection Rate (cc/min)	20.115
Volume Injected (cc)	4023.
Static Storage (min)	0.
First Production Rate (cc/min)	20.115
Volume Produced (cc)	4036.
Step Recovery Eff. (%)	
Experimental	80
Computed	75
Cum. Recovery Eff. (%)	
Experimental	88
Computed	79
Value of D'less Group @ B.T.	0.973
Second Production Rate (cc/min)	4.023
Volume Produced (cc)	973.
Step Recovery Eff. (%)	
Experimental	19
Computed	13
Cycle Recovery Eff. (%)	
Experimental	99
Computed	89
Cum. Recovery Eff. (%)	
Experimental	94
Computed	84
Value of D'less Group @ B.T.	1.015

TABLE E-4

DATA FOR MULTI-WELL INJECTION AND PRODUCTION

	RUN NUMBER 10	13	15
	Run Type 4C	4C	4C
FLUID PROPERTIES			
Visc. of Inj. Fluid (cp)	0.949	0.922	0.926
Visc. of Nat. Fluid (cp)	0.942	0.925	0.927
Mean Viscosity (cp)	0.945	0.923	0.927
Density of Inj. Fluid (gm/cc)	0.793	0.860	0.860
Density of Nat. Fluid (gm/cc)	0.783	0.784	0.784
Density Difference (gm/cc)	0.010	0.076	0.076
FIRST CYCLE			
First Injection Rate (cc/min)	4.023	4.023	4.023
Volume Injected (cc)	1006.	1341.	1341.
Second Injection Rate (cc/min)	20.115	20.115	20.115
Volume Injected (cc)	5029.	5029.	5029.
Static Storage (min)	0.	0.	250.
Production Rate (cc/min)	20.115	20.115	20.115
Volume Produced (cc)	4600.	4020.	3566.
Cycle Recovery Eff. (%)			
Experimental	91	80	71
Computed	78	78	72
*Cum. Recovery Eff. (%)			
Experimental	76	63	56
Computed	65	62	57
Value of D'less Group @ B.T.	0.463	2.045	2.592
SECOND CYCLE			
Injection Rate (cc/min)	20.115	20.115	20.115
Volume Injected (cc)	5029.	5029.	5029.
Static Storage (min)	0.	0.	250.
Production Rate (cc/min)	20.115	20.115	20.115
Volume Produced (cc)	4861.	4429.	3913.
Cycle Recovery Eff. (%)			
Experimental	97	88	78
Computed	86	85	78
*Cum. Recovery Eff. (%)			
Experimental	86	74	66
Computed	75	72	66
Value of D'less Group @ B.T.	0.645	2.766	3.769
THIRD CYCLE			
Injection Rate (cc/min)	20.115	20.115	20.155
Volume Injected (cc)	5029.	5029.	5029.
Static Storage (min)	0.	0.	250.
Production Rate (cc/min)	20.115	20.115	20.115
Volume Produced (cc)	5015.	4568.	4476.
Cycle Recovery Eff. (%)			
Experimental	100	91	89
Computed	89	86	80
*Cum. Recovery Eff. (%)			
Experimental	90	79	73
Computed	79	76	70
Value of D'less Group @ B.T.	0.810	3.404	4.806

*All cumulative recovery efficiencies were computed with the "cushion water" volume included.

TABLE E-5

DATA FOR SINGLE WELL OPERATION

(Kumar, 1968)

	RUN NUMBER 2	3	6
	Run Type 1A	1A	1A
FLUID PROPERTIES			
Visc. of Inj. Fluid (cp)	0.833	0.801	0.656
Visc. of Nat. Fluid (cp)	0.781	0.647	0.571
Mean Viscosity (cp)	0.807	0.724	0.614
Density of Inj. Fluid (gm/cc)	0.765	0.765	0.751
Density of Nat. Fluid (gm/cc)	1.129	0.957	0.823
Density Difference (gm/cc)	0.364	0.192	0.072
FIRST CYCLE			
Injection Rate (cc/min)	74.843	22.857	6.000
Volume Injected (cc)	4528.	3200.	1974.
Static Storage (min)	14.500	13.333	121.000
Production Rate (cc/min)	95.124	50.455	6.000
Volume Produced (cc)	2933.	1850.	168.
Cycle Recovery Eff. (%)			
Experimental	65	58	9
Computed	62	52	31
Cum. Recovery Eff. (%)			
Experimental	65	58	9
Computed	62	52	31
Value of D'less Group @ B.T.	0.388	0.526	0.844

TABLE E-6

DATA FOR SINGLE WELL OPERATION

(Kumar, 1968)

	RUN NUMBER 4	5
	Run Type 5B	5B
FLUID PROPERTIES		
Visc. of Inj. Fluid (cp)	0.799	0.729
Visc. of Nat. Fluid (cp)	0.656	0.592
Mean Viscosity (cp)	0.728	0.661
Density of Inj. Fluid (gm/cc)	0.764	0.761
Density of Nat. Fluid (gm/cc)	0.870	0.805
Density Difference (gm/cc)	0.106	0.044
FIRST CYCLE		
Injection Rate (cc/min)	63.000	89.714
Volume Injected (cc)	3150.	3140.
Static Storage (min)	20.167	20.000
Production Rate (cc/min)	66.250	70.667
Volume Produced (cc)	1325.	2120.
SECOND CYCLE		
Injection Rate (cc/min)	72.422	86.333
Volume Injected (cc)	1545.	2590.
Static Storage (min)	20.167	13.333
Production Rate (cc/min)	73.489	78.912
Volume Produced (cc)	2250.	2900.
Cum. Recovery Eff. (%)		
Experimental	76	88
Computed	70	78
Value of D'less Group @ B.T.	0.253	0.139

APPENDIX F

COMPUTER PROGRAM FOR PREDICTING THE FRONTAL POSITION DURING THE FIRST INJECTION HALF CYCLE

The computer program listed on the following pages is used to calculate the leading and lagging edges of the mixed zone when a fluid is injected into a saline aquifer. The program is for a single well only and assumes a constant injection rate. The frontal positions on the roof and floor of the aquifer are printed out at the time that the specified volume has been injected. The program is in FORTRAN IV language and is written for use on an IBM 360/65 system.

A list of required input data is presented at the beginning of the program. Following this is a complete list of all the variable names used in the program along with their definitions.

```
C
C
C     PROGRAM TO CALCULATE THE POSITIONS OF THE LEADING AND LAGGING
C      EDGES OF THE MIXED ZONE WHEN A FLUID IS INJECTED INTO A SALINE
C      AQUIFER.
C
C
C     PROGRAM 'FRONT' (SINGLE WELL)
C
C
C     ***********************************************************************
C
C     DATA TO BE READ IN
C
C      FIRST CARD - FORMAT(3F12.0)
C       TILIFT = INTERVAL LENGTH FOR CALCULATIONS DURING INJECTION
C                A GOOD STARTING VALUE FOR THIS VARIABLE IS TEN FEET.
C                THE FRONTAL POSITIONS COMPUTED BY THE PROGRAM ARE
C                SENSITIVE TO THE TOTAL NUMBER OF COMPUTATION
C                INTERVALS. HENCE THE VALUE OF THIS VARIABLE SHOULD BE
C                DECREASED UNTIL THE COMPUTED POSITIONS OF THE FRONT
C                DO NOT CHANGE SIGNIFICANTLY. (FT)
C       RINCFT = LENGTH OF INCREMENT FOR CALCULATION OF MIXED ZONE
C                LENGTHS. THIS VARIABLE SHOULD NOT EXCEED ONE TENTH
C                THE VALUE USED FOR INTERVAL LENGTH FOR CALCULATIONS
C                DURING INJECTION. (FT)
C       TINCFT = INCREMENT BY WHICH TILIFT IS INCREASED IF MIXED ZONE
C                INTERSECTS WELL DURING CALCULATIONS FOR FIRST
C                INTERVAL. THIS VARIABLE SHOULD NOT EXCEED ONE TENTH
C                THE VALUE USED FOR INTERVAL LENGTH FOR CALCULATIONS
C                DURING INJECTION. (FT)
C
C      SECOND CARD - FORMAT(5F12.0)
C       HFT    = AQUIFER THICKNESS. (FT)
C       PLYMEI = AQUIFER PERMEABILITY. (MEINZERS)
C       PR     = POROSITY. (FRACTION)
C       ALF    = LONGITUDINAL DISPERSIVITY COEFFICIENT. VALUES IN EXCESS
C                OF TEN CENTIMETERS SHOULD NOT BE USED. (CM)
C       DIFMOL = COEFFICIENT OF MOLECULAR DIFFUSION. ((SQ CM)/SEC)
C
C      THIRD CARD - FORMAT(4F12.0)
C       VISCP1 = VISCOSITY OF THE INJECTED FLUID. (CP)
C       VISCP2 = VISCOSITY OF THE NATIVE FLUID. (CP)
C       DENS1  = DENSITY OF THE INJECTED FLUID. (GM/CC)
C       DENS2  = DENSITY OF THE NATIVE FLUID. (GM/CC)
C
C      FOURTH CARD - FORMAT(1F12.0)
C       QR1GM  = INJECTION RATE. (GPM)
C
C      FIFTH CARD - FORMAT(1F12.0)
C       FLING1 = VOLUME OF FLUID INJECTED. (GAL)
C
C     ***********************************************************************
C
C     DEFINITION OF VARIABLE NAMES USED IN PROGRAM
C
C       A1     = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF
C                RU50. (SQ CM)
C       B1     = INTERMEDIATE VALUE USED IN COMPUTING VALUES OF
C                RL50. (SQ CM)
C       CFCPP  = CONVERSION FACTOR. (POISE/CENTIPOISE)
C       CFFTCM = CONVERSION FACTOR. (CM/FT)
C       CFGLCC = CONVERSION FACTOR. (CC/GAL)
C       CFMSCM = CONVERSION FACTOR. ((SQ CM)/MEINZER)
C       CGMCCS = CONVERSION FACTOR. ((CC/SEC)/(GAL/MIN))
C       C11    = COMPUTED CONCENTRATION AT THE RADIUS AND AT THE TIME
C                BEING CONSIDERED. (VOLUME FRACTION)
C       DENDF  = DENS1 - DENS2. (GM/CC)
C       DG     = DENSITY GRADIENT. ((GM/CC)/CM)
C       DM1    = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                PARAMETER GIVEN BY EQUATION 3.7A.
C       DM2    = DIMENSIONLESS GROUP. SECOND GROUP ON RIGHT SIDE OF
C                EQUATION 3.7A.
C       DM2A   = A CONSTANT USED IN THE COMPUTATION OF THE DIMENSIONLESS
C                PARAMETER GIVEN BY EQUATION 3.7A.
```

```
C       DNOMI1 = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                IN EQUATION 3.5 FOR FIRST INJECTION HALF-CYCLE.
C       DSDF   = DENSITY DIFFERENCE BETWEEN INJECTED AND NATIVE
C                FLUIDS. (GM/CC)
C       FLINJ1 = VOLUME OF FLUID INJECTED. (CC)
C       H      = AQUIFER THICKNESS. (CM)
C       I      = SUBSCRIPT DESIGNATING COMPUTATION INTERVAL.
C       ICOUNT = DUMMY VARIABLE USED AS A COUNTER.
C       NINT1  = TOTAL NUMBER OF COMPUTATION INTERVALS.
C       PLY    = AQUIFER PERMEABILITY. (SQ CM)
C       PPP    = PRODUCT OF PI, POROSITY, AND THICKNESS. (CM)
C       PPP1   = 2*PPP. (CM)
C       QI1    = TWO DIMENSIONAL FLOW RATE DURING
C                INJECTION. ((SQ CM)/SEC)
C       QR1    = FLOW RATE DURING INJECTION. (CC/SEC)
C       QSI    = TWO DIMENSIONAL PSEUDO FLOW RATE DURING
C                INJECTION. ((SQ CM/SEC)
C       R(I)   = RADIUS OF INJECTED FLUID AT THE ITH COMPUTATION
C                INTERVAL ASSUMING NO MIXING OR GRAVITATIONAL
C                SEGREGATION. (CM)
C       RCHECK = THE VALUE OF RL50 OR RU50 FOR THE LAST COMPUTATION
C                INTERVAL. (CM)
C       RCFT   = RADIUS TO LEADING OR LAGGING EDGE OF MIXED ZONE. (FT)
C       RINC   = LENGTH OF INCREMENT USED FOR CALCULATION OF MIXED
C                ZONE LENGTHS. (CM)
C       RINJ1  = RADIUS OF INJECTED FLUID AT THE END OF INJECTION
C                ASSUMING NO MIXING AND NO GRAVITATIONAL
C                SEGREGATION. (CM)
C       RL50   = MINIMUM RADIUS TO 50 PERCENT CONCENTRATION LINE. (CM)
C       RU50   = MAXIMUM RADIUS TO 50 PERCENT CONCENTRATION LINE. (CM)
C       R1     = RADIUS AT WHICH CONCENTRATION IS BEING COMPUTED. ALSO
C                INNER RADIUS OF MIXED ZONE ASSUMING NO GRAVITATIONAL
C                SEGREGATION. (CM)
C       R2     = OUTER RADIUS OF MIXED ZONE ASSUMING NO GRAVITATIONAL
C                SEGREGATION. (CM)
C       R3     = LENGTH OF MIXED ZONE ASSUMING NO GRAVITATIONAL
C                SEGREGATION. (CM)
C       R4     = TILI/2. (CM)
C       R5     = RADIUS TO THE MIDPOINT BETWEEN R(I) AND R(I-1). (CM)
C       R5SQ   = R5 SQUARED. (SQ CM)
C       R50SQ  = EITHER RL50 SQUARED OR RU50 SQUARED. (SQ CM)
C       SDENOM = DENOMINATOR OF ARGUMENT OF COMPLEMENTARY ERROR FUNCTION
C                IN EQUATION 3.5 USING PSEUDO INJECTION RATE.
C       TILI   = INTERVAL LENGTH FOR COMPUTATIONS DURING INJECTION. (CM)
C       TRT    = CUMULATIVE TRAVEL TIME OF INJECTED FLUID-NATIVE FLUID
C                INTERFACE. (SEC)
C       TSP    = PSEUDO TIME USED IN GRAVITATIONAL SEGREGATION
C                CALCULATIONS. (SEC)
C       TT     = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL PLUS THE
C                PSEUDO TIME FOR THAT INTERVAL. (SEC)
C       T1     = TIME AT WHICH CONCENTRATION IS BEING COMPUTED DURING
C                INJECTION. (SEC)
C       T11    = TIME OF TRAVEL ACROSS ANY COMPUTATION INTERVAL. (SEC)
C       VIS    = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (POISE)
C       VISCP  = MEAN VISCOSITY OF INJECTED AND NATIVE FLUIDS. (CP)
C       X      = VALUE OF DIMENSIONLESS PARAMETER GIVEN BY
C                EQUATION 3.7A.
C       XL(I)  = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                LINE FOR LINEAR GEOMETRY. (CM)
C       XR(I)  = HORIZONTAL PROJECTION OF 50 PERCENT CONCENTRATION
C                LINE FOR RADIAL GEOMETRY. (CM)
C       XX     = ARGUMENT OF COMPLEMENTARY ERROR FUNCTION FOR
C                EQUATION 3.5.
C       YL(I)  = RATIO OF HORIZONTAL PROJECTION OF 50 PERCENT
C                CONCENTRATION LINE TO AQUIFER THICKNESS FOR LINEAR
C                GEOMETRY.
C
C     ***********************************************************************
C
C
C
```

```
      DIMENSION R(1000),YL(1000),XL(1000),XR(1000),RCBT(1000)
C
C
C     PART 1 - READING DATA
C     ------   ------------
C
C
   20 READ(5,10000,END=1380)TILIFT,RINCFT,TINCFT
      READ(5,11000)HFT,PLYMEI,PR,ALF,DIFMOL
      READ(5,12000)VISCP1,VISCP2,DENS1,DENS2
   21 READ(5,13000)QR1GM
      READ(5,14000)FLING1
C
C     CALCULATION OF DENSITY DIFFERENCE AND MEAN VISCOSITY OF FLUIDS
C
      DENDF=DENS1-DENS2
      DSDF=ABS(DENS1-DENS2)
      VISCP=(VISCP1+VISCP2)/2.0
C
C
C     PART 2 - PRINTING DATA
C     ------   -------------
C
C
      WRITE(6,16000)HFT,PLYMEI,PR,ALF,DIFMOL
      WRITE(6,17000)VISCP1,VISCP2,VISCP
      WRITE(6,18000)DENS1,DENS2,DSDF
   24 WRITE(6,19000)QR1GM
      WRITE(6,20000)FLING1
C
      GOTO50
   30 TILIFT=TILIFT+TINCFT
      GOTO60
C
C
C     PART 3 - CONSTANTS AND CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ------   ----------------------------------------------------------------
C
C
   50 CFFTCM=30.4801
      CFMSCM=0.4842154E-9*VISCP2
      CFGLCC=3785.434
      CGMCCS=63.0906
      CFCPP=0.01
      RINC=RINCFT*CFFTCM
      H=HFT*CFFTCM
      PPP=3.1416*PR*H
      PPP1=2.0*PPP
      PLY=PLYMEI*CFMSCM
      FLINJ1=FLING1*CFGLCC
   51 QR1=QR1GM*CGMCCS
      VIS=VISCP*CFCPP
      DM1=(PLY*981.*DSDF)/(PR*VIS*H)
      DM2A=VIS**0.6667/(DSDF**1.6667*981.**0.3333)
   60 TILI=TILIFT*CFFTCM
      R4=TILI/2.0
C
C
C     PART 4 - CALCULATIONS FOR INJECTION OF FLUID
C     ------   -----------------------------------
C
C
C     CALCULATIONS OF INTERVALS AND RADIUS OF INJECTION
C
      RINJ1=SQRT(FLINJ1/PPP)
      NINT1=RINJ1/TILI
C
C     CALCULATION OF MIXED ZONE DUE TO DIFFUSION AND DISPERSION
C
      I=1
      TRT=0.0
      R(I)=TILI
   61 R5=R(I)-R4
      R5SQ=R5*R5
      R1=R5-RINC
      T1=PPP*R5SQ/QR1
```

```
      QI1=QR1/PPP1
      DNOMI1=2.0*SQRT(1.333*ALF*(2.0*QI1*T1)**1.5+DIFMOL*(2.0*QI1*T1)**2
     1       /QI1)
   70 XX=(R5SQ-R1*R1)/DNOMI1
      C11=ERFC(XX)/2.0
      IF(C11-0.5)80,80,120
   80 IF(C11-0.03)90,90,100
   90 R2=R1
      GOTO110
  100 R1=R1-RINC
      IF(R1)30,30,70
  110 R1=R5+RINC
      GOTO70
  120 IF(C11-0.97)130,140,140
  130 R1=R1+RINC
      GOTO70
  140 R3=R1-R2
C
C     CALCULATION OF INTERFACE PROJECTION DUE TO GRAVITY SEGREGATION
C
      IF(I-1)150,150,190
  150 DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*R(I)**2/QR1
  151 TRT=TRT+T11
      X=DM1*T11*DM2**0.5
      IF(X-0.1)160,160,165
  160 YL(I)=20.0*X
      GOTO180
  165 IF(X-1.0)170,175,175
  170 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO180
  175 YL(I)=6.5+2.0*X
  180 XL(I)=YL(I)*H
      GOTO290
  190 DG=DSDF/R3
      DM2=DM2A*DG
      T11=PPP*ABS(R(I)**2-R(I-1)**2)/QR1
  191 TRT=TRT+T11
      IF(YL(I-1)-2.0)195,195,200
  195 X=YL(I-1)/20.0
      GOTO215
  200 IF(YL(I-1)-8.5)205,210,210
  205 X=1.2993-SQRT(172.1873-19.2784*YL(I-1))/9.6392
      GOTO215
  210 X=(YL(I-1)-6.5)/2.0
  215 TSP=X/(DM1*DM2**0.5)
  220 TT=T11+TSP
      IF(I-NINT1)240,240,330
  240 X=DM1*TT*DM2**0.5
      IF(X-0.1)250,250,255
  250 YL(I)=20.0*X
      GOTO270
  255 IF(X-1.0)260,265,265
  260 YL(I)=0.7958+12.5238*X-4.8196*X**2
      GOTO270
  265 YL(I)=6.5+2.0*X
  270 XL(I)=YL(I)*H
      GOTO300
C
C     APPROXIMATION TO RADIAL GEOMETRY
C
  290 RU50=R(I)+XL(I)/2.0
      RL50=R(I)-XL(I)/2.0
      IF(RL50)295,310,310
  295 WRITE(6,21000)
      GOTO20
  300 A1=3.142*((R(I-1)+XR(I-1)/2.0)**2-R(I-1)**2)
      B1=3.142*(R(I-1)**2-(R(I-1)-XR(I-1)/2.0)**2)
      RU50=SQRT((3.142*R(I)**2+A1)/3.142)+(XL(I)-XL(I-1))/2.0
      RL50=SQRT((3.142*R(I)**2-B1)/3.142)-(XL(I)-XL(I-1))/2.0
  310 XR(I)=RU50-RL50
      I=I+1
      IF(I-NINT1)320,320,330
  320 R(I)=R(I-1)+TILI
      GOTO61
```

```
C
C      CALCULATION OF LEADING AND LAGGING EDGES OF MIXED ZONE
C
  330 WRITE(6,22000)
C
      ICOUNT=0
      IF(DENDF)340,340,350
  340 WRITE(6,23000)
      GOTO390
  350 WRITE(6,23000)
      GOTO400
  360 IF(DENDF)370,370,380
  370 WRITE(6,24000)
      GOTO400
  380 WRITE(6,24000)
      GOTO390
  390 R50SQ=RL50*RL50
      RCHECK=RL50
      GOTO410
  400 R50SQ=RU50*RU50
      RCHECK=RL50
  410 QSI=R50SQ/(2.0*TRT)
      SDENOM=2.0*SQRT(1.333*ALF*(R50SQ)**1.5+DIFMOL*(R50SQ)**2/QSI)
      RC=RCHECK-RINC
  420 XX=(R50SQ-RC*RC)/SDENOM
      C11=ERFC(XX)/2.0
      IF(C11-0.5)430,430,490
  430 IF(C11-0.03)440,440,460
  440 RCFT=RC/CFFTCM
  450 WRITE(6,25000)RCFT
      GOTO480
  460 RC=RC-RINC
      IF(RC)470,470,420
  470 RCFT=0.0
      GOTO450
  480 RC=RCHECK+RINC
      GOTO420
  490 IF(C11-0.97)500,510,510
  500 RC=RC+RINC
      GOTO420
  510 RCFT=RC/CFFTCM
      WRITE(6,26000)RCFT
      IF(ICOUNT-1)520,530,530
  520 ICOUNT=ICOUNT+1
      GOTO360
  530 WRITE(6,27000)TILIFT
C
      GOTO20
C
C
C      PART 5 - FORMAT STATEMENTS
C      ------   -----------------
C
C
10000 FORMAT(3F12.0)
11000 FORMAT(5F12.0)
12000 FORMAT(4F12.0)
13000 FORMAT(1F12.0)
14000 FORMAT(1F12.0)
16000 FORMAT(1H1,35X,'DATA'/,36X,'----'////,6X,'POROUS MEDIUM'//,
     1  9X,'THICKNESS OF THE MEDIUM (FT)',30X,F10.6/,
     2  9X,'PERMEABILITY OF THE MEDIUM (MEINZERS)',20X,F11.6/,
     3  9X,'POROSITY OF THE MEDIUM (FRACTION)',27X,F8.6/,
     4  9X,'LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)',15X,F9.6/,
     5  9X,'COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)',14X,F8.6///
     6  )
17000 FORMAT(6X,'FLUID PROPERTIES'//,8X,'VISCOSITY OF THE FLUIDS (CP)'/,
     1  9X,'VISCOSITY OF THE INJECTED FLUID',29X,F8.6/,
     2  9X,'VISCOSITY OF THE NATIVE FLUID',31X,F8.6/,
     3  9X,'MEAN VISCOSITY OF THE TWO FLUIDS',28X,F8.6/)
18000 FORMAT(8X,'DENSITY OF THE FLUIDS (GM/CC)'/,
     1  9X,'DENSITY OF THE INJECTED FLUID',31X,F8.6/,
     2  9X,'DENSITY OF THE NATIVE FLUID',33X,F8.6/,
     3  9X,'DENSITY DIFF. BETWEEN THE FLUIDS',28X,F8.6///)
19000 FORMAT(6X,'INJECTION RATE (GAL/MIN)',35X,F12.6///)
20000 FORMAT(6X,'VOLUME OF FLUID INJECTED (GALLONS)',17X,F20.6)
```

```
21000 FORMAT(1H1,'INJECTION RATE TOO SMALL TO OVERCOME GRAVITY LAYDOWN')
22000 FORMAT(1H1,9X,'POSITION OF FRONT AT THE END OF INJECTION'/,10X,
     1  '----------------------------------------'////)
23000 FORMAT(8X,'RADII ON FLOOR OF AQUIFER (FEET)'/)
24000 FORMAT(8X,'RADII ON ROOF OF AQUIFER (FEET)'/)
25000 FORMAT(10X,'LAGGING EDGE OF MIXED ZONE',13X,F8.2)
26000 FORMAT(10X,'LEADING EDGE OF MIXED ZONE',13X,F8.2///)
27000 FORMAT(8X,'COMPUTATION INTERVAL LENGTH (FEET)',10X,F5.2)
C
 1380 STOP
      END
```

DATA

POROUS MEDIUM

THICKNESS OF THE MEDIUM (FT)	100.000000
PERMEABILITY OF THE MEDIUM (MEINZERS)	400.000000
POROSITY OF THE MEDIUM (FRACTION)	0.300000
LONGITUDINAL DISPERSIVITY OF THE MEDIUM (CM)	1.000000
COEFFICIENT OF MOLECULAR DIFFUSION (SQ CM/SEC)	0.000001

FLUID PROPERTIES

VISCOSITY OF THE FLUIDS (CP)	
VISCOSITY OF THE INJECTED FLUID	1.000000
VISCOSITY OF THE NATIVE FLUID	1.000000
MEAN VISCOSITY OF THE TWO FLUIDS	1.000000
DENSITY OF THE FLUIDS (GM/CC)	
DENSITY OF THE INJECTED FLUID	1.000000
DENSITY OF THE NATIVE FLUID	1.025000
DENSITY DIFF. BETWEEN THE FLUIDS	0.025000

INJECTION RATE (GAL/MIN)	500.000000
VOLUME OF FLUID INJECTED (GALLONS)	100000000.

POSITION OF FRONT AT THE END OF INJECTION

RADII ON FLOOR OF AQUIFER (FEET)

LAGGING EDGE OF MIXED ZONE	310.73
LEADING EDGE OF MIXED ZONE	320.73

RADII ON ROOF OF AQUIFER (FEET)

LAGGING EDGE OF MIXED ZONE	418.66
LEADING EDGE OF MIXED ZONE	430.66

COMPUTATION INTERVAL LENGTH (FEET)	10.00