

# Concept Grounding and Knowledge of Set Theory

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**Abstract** C. S. Jenkins has recently proposed an account of arithmetical knowledge designed to be realist, empiricist, and apriorist: realist in that what's the case in arithmetic doesn't rely on us being any particular way; empiricist in that arithmetic knowledge crucially depends on the senses; and apriorist in that it accommodates the time-honored judgment that there is something special about arithmetical knowledge, something we have historically labeled with 'a priori'. I'm here concerned with the prospects for extending Jenkins's account beyond arithmetic—in particular, to set theory. After setting out the central elements of Jenkins's account and entertaining challenges to extending it to set theory, I conclude that a satisfactory such extension is unlikely.

**Keywords** Epistemology · Mathematics · Arithmetic · Set theory · Concepts

## Introduction

C. S. Jenkins (2005, 2008) proposes an account of arithmetical knowledge designed to be realist, empiricist, and apriorist: realist in that what's the case in arithmetic doesn't rely on us being any particular way; empiricist in that arithmetic knowledge crucially depends on the senses; and apriorist in that it accommodates the time-honored judgment that there is something special about arithmetical knowledge, something we have historically labeled with 'a priori'. I'm here concerned with the prospects for extending Jenkins's

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account beyond arithmetic. In particular, I'm interested in the possible extension of Jenkins's account to set theory.

Such an extension is important for at least three reasons. First, despite the independent desirability of an account of arithmetical knowledge, especially one which preserves the traditional status of arithmetical knowledge as *a priori*, such an account is of limited interest if it is specific to arithmetical knowledge.<sup>1</sup> Second, the view Jenkins offers appears to sit well with naturalism, as it purports to be empirical at bottom. Though the account is an account of what deserves to be called '*a priori*' according to much traditional usage of that term,<sup>2</sup> it only involves ways or sources of knowing that are explicable by the (non-mathematical) sciences. The potential of Jenkins's view to explain mathematical knowledge as both *a priori* and compatible with naturalism in the tradition of Quine enhances its interest. But naturalists can't settle for only arithmetic.<sup>3</sup> So if the account cannot be extended beyond arithmetic, especially to set theory, its use to naturalists will be severely limited.<sup>4</sup> Finally, Jenkins herself indicates that she sees her account extending to many types of *a priori* knowledge apart from arithmetic knowledge, including knowledge of set theory: "The kind of epistemology I develop here could, as far as I can tell, be extended to many—perhaps all—other kinds of *a priori* knowledge. Little is said about arithmetic that I would not also wish to say about (for instance) set theory and logic" (Jenkins 2008, p. 1).<sup>5</sup> So Jenkins herself sets extension of her account to set theory as a goal.

I begin ("[The Account](#)") with the central elements of Jenkins's account. I then ("[Extending the Account](#)") entertain two challenges to extending Jenkins's account to set theory. I finish ("[Concluding Remarks](#)") by considering a response open to Jenkins based on a strategy she advances for answering a challenge which is in certain respects similar to the chief difficulty I raise for her account in "[Extending the Account](#)". I conclude that a satisfactory extension of Jenkins's account of the epistemology of arithmetic to set theory is unlikely.

## The Account

The account of arithmetic knowledge offered by Jenkins is intended to integrate with her general analysis of knowledge. According to the latter, a subject

<sup>1</sup>In general, the more likely it is that arithmetic alone is within the scope of an account of mathematical epistemology, the more diminished the account's interest and importance. In this Jenkins's account is no different from any other in the philosophy of mathematics. Witness attempts to extend Frege's work beyond arithmetic (e.g., Fine 2002; Burgess 2005; Hale 2000; Shapiro 2000), similarly for Hellman's modal structuralism (1989) and Kitcher's naturalistic constructivism (1983, 1988).

<sup>2</sup>See Jenkins (2005, pp. 742–743).

<sup>3</sup>For more on this point, see Roland ([forthcoming](#)).

<sup>4</sup>Jenkins cites the accommodation of naturalistic concerns as a significant point in favor of her account of arithmetical knowledge. See, e.g., Jenkins (2008, p. 152).

<sup>5</sup>See also Jenkins (2008, p. 152).

S knows that  $p$  just in case  $[p]$  (the fact that  $p$ ) is a good explanation of S's believing that  $p$  for an "outsider," someone unfamiliar with the specifics of S's circumstances.<sup>6</sup> The details of this analysis need not detain us.<sup>7</sup> For present purposes, the important point is that to achieve the desired integration Jenkins must provide a story outlining how an arithmetic fact  $[p]$  can constitute a good explanation of a subject's believing that  $p$  for an outsider, a story which is at least plausible. That is, she must make it plausible that there is an explanatory link between arithmetic facts and arithmetic beliefs.

The story Jenkins tells depends on arithmetic concepts being *grounded*, i.e., being such that they are systematically (non-accidentally) accurate, where an accurate concept is one that represents "some real feature of the world" (Jenkins 2005, p. 732).<sup>8</sup> Jenkins takes concept-grounding to be an empirical matter, in that "*the only data which could be relevant to concept justification and concept grounding are data obtained through the senses*" (Jenkins 2008, p. 137, original emphasis). She allows that a concept might be grounded in virtue of being composed of other grounded concepts. But if concept grounding is empirical, there must ultimately be "basic" concepts, concepts which have a direct connection (in the sense of not "passing through," via composition, other concepts) to the world. She contends that "there is a link, mediated by sensory input, between the world and [a subject's] mental [states], the existence of which makes it likely that the subject's basic arithmetical concepts accurately represent real features of the world" (Jenkins 2008, p. 141), and it is these sorts of links that give the relevant direct connection.<sup>9</sup> These links also form the basis of the explanatory links required by Jenkins's account of arithmetic knowledge: the arithmetic fact  $[p]$  can constitute a good explanation of a subject's belief that  $p$  if the concepts involved in the proposition that  $p$  are grounded. It will be important to better understand these links and how they might be established.

Let  $C$  be a basic arithmetic concept. Jenkins declines to offer a definitive view of how such a concept becomes grounded. She does, however, offer two possible ways this might happen. For present purposes, we can safely ignore the difference between these two possibilities.<sup>10</sup> On the picture of grounding basic concepts offered by Jenkins, we have internal mechanisms that enable a subject "to respond appropriately to certain features of [sensory] input" (Jenkins 2008, p. 138). Exercising these mechanisms eventually leads

<sup>6</sup>Jenkins sometimes treats 'p' as if it stands for a proposition and other times as if it stands for a fact (e.g., (2008, Chapter 4, especially §§4.1 and 4.2)). I will use 'p' to stand for a declarative sentence, '[p]' to stand for the fact that  $p$ , and 'that p' to stand for the proposition expressed by  $p$ .

<sup>7</sup>For those details, see Jenkins (2008, Chapter 3, especially §3.2).

<sup>8</sup>I will ignore some subtleties of Jenkins's definitions not relevant to my arguments. For details on her definitions, see Jenkins (2008, pp. 126–131, 269–271).

<sup>9</sup>The quotation in this sentence is specific to arithmetic, but Jenkins's clearly holds the same view for any concepts examination of which might yield a priori knowledge.

<sup>10</sup>See Jenkins (2008, §4.5) for details on both ways of directly grounding a concept Jenkins considers.

to possession of *C* by the subject. So the acquisition of *C* depends on having and deploying certain mechanisms. The relevant mechanisms attain a position in our cognitive architecture by being useful. The idea is that regardless of whether such mechanisms are acquired or innate, they would not be stable enough components of our cognitive architecture to give rise to the concepts they do unless they were useful. Now Jenkins deploys a version of the “no miracles” argument familiar from the debate over scientific realism.<sup>11</sup> What explains the usefulness of concept-producing mechanisms? The fact that such mechanisms track real features of the independent world. The usefulness of concept-producing mechanisms is “probably due to the fact that there exist real features of the world to which those [mechanisms enable a] subject to respond” and Jenkins assumes “that the usefulness of responding to a feature of sensory input like this would be miraculous, or inexplicable, unless that feature of sensory input were correlated with a feature of the independent world” (Jenkins 2008, p. 139).

On this view, we have *C* as a result of our possession and exercise of mechanisms which would not be in a position to produce *C* unless they were responding to, correlated with, real features of the world. So *C* is systematically correlated with real features of the world via the mechanisms which produced it. *C* is accurate, and non-accidentally so. And this is why, according to Jenkins, examining *C* (normally in concert with other appropriately produced concepts) can yield knowledge about the independent world.

Consider, for example, our knowledge that  $7 + 5 = 12$ . According to Jenkins, knowing that  $7 + 5 = 12$  results from the following (regardless of whether or not a putative knower realizes it):

- (1) A correctly conducted investigation of our concepts of 7, +, 5, =, and 12 leads us to believe that  $7 + 5 = 12$ .
- (2) Our concepts of 7, +, 5, =, and 12 are grounded.
- (3) The proposition that  $7 + 5 = 12$  is known.<sup>12</sup>

According to the account of grounding just rehearsed, sensory experience plays an essential role in (2). Our concepts of 7, +, 5, =, and 12 (or perhaps their basic constituents) are produced by mechanisms which are appropriately sensitive to sensory input, and “no miracles” considerations deliver the non-accidental accuracy (groundedness) of those concepts, which provides an explanatory link sufficient for knowing on Jenkins’s general analysis of knowledge. Owing to the essential use of the senses in knowing that  $7 + 5 = 12$ , our knowledge that  $7 + 5 = 12$  counts as empirical on Jenkins’s account. But that  $7 + 5 = 12$  is also known a priori on this view, as Jenkins holds that believing truly that *p* on the basis of introspective investigation of the concepts

<sup>11</sup>See, e.g., Boyd (1980, pp. 617–618) and Putnam (1975, p. 73).

<sup>12</sup>See (i)–(iii) on p. 742 of Jenkins (2005). Cf. (1)–(3) on p. 148 of Jenkins (2008).

occurring in the proposition that  $p$  is a way of knowing that  $p$  independent of empirical evidence (i.e., a priori), provided those concepts are grounded.<sup>13</sup>

Notice that this is not an account of apriority in terms of analyticity, since for Jenkins the proposition that  $p$  isn't true owing to its constituent concepts. It's true owing to the way the (non-conceptual) world is; one merely comes to appreciate that the proposition that  $p$  is true by examining the relevant concepts.<sup>14</sup> Notice also that knowing that  $p$  (e.g., that  $7 + 5 = 12$ ) crucially relies on being able to acquire the relevant concepts (e.g., the concepts of 7, +, 5, =, and 12), as well as the groundedness of those concepts. This point might seem too obvious to mention, but it's important because it means that if we're unable to acquire certain concepts or if we're unable to acquire them in such a way that they're grounded, then we're also unable to know any proposition belief of which relies on possessing those concepts.

This last point is significant for the first of two challenges I'll raise for Jenkins's account in the next section, to wit, that sensory input is insufficient to give us a concept of *set* consonant with our established set-beliefs. The second challenge assumes a partial solution to the first. According to it, even if sensory input is sufficient to give us a (grounded) concept  $C$  which we use as if it were the concept *set*, it's indeterminate whether or not  $C$  is actually the concept of *set* or the more general concept of *class*. This indeterminacy introduces an unacceptable element of luck where the truth values of  $C$ -beliefs are concerned, which undermines the extension of Jenkins's account to set theory.

## Extending the Account

Granting that arithmetic knowledge might be secured on this account, it's not at all clear that all mathematical knowledge—set-theoretic knowledge, in particular—can be similarly secured. Grounded concepts correspond to mind- and language-independent features of the world. They accurately (and non-accidentally) represent those features to us owing to the responsiveness of our conceptual scheme, via our cognitive faculties, to sensory input. Thus is a systematic, explanatory link forged between our concepts (conceptual scheme) and the world such that correct investigation of the former may yield knowledge about the latter.<sup>15</sup> Jenkins intends this story to apply to arithmetic. The idea is that we are sensitive to arithmetic features of the world, and that sensory input can (and does) provide us with concepts answering to those features, arithmetic concepts. It's not hard to see how this might work for concepts like those of 7, +, 5, = and 12. We observe a collection of seven objects

<sup>13</sup>Thanks to an anonymous referee for catching a subtle, but important, error in a previous version of this passage.

<sup>14</sup>See, e.g., Jenkins (2008, p. 121).

<sup>15</sup>See Jenkins (2008, Chapter 4) and Jenkins (2005, especially §§2 and 3).

and a collection of five objects and that putting the two collections together gives us a collection of twelve objects.<sup>16</sup>

Jenkins claims that her account “can be extended to all of arithmetic” (Jenkins 2005, p. 744). We grant this for the sake of argument, accepting that sensory input provides us with concepts sufficient for the whole of arithmetic. What sensory input is likely to provide us concepts of large cardinals?<sup>17</sup> Or even of the least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$ , to use an example due to George Boolos?<sup>18</sup> What features of the observable world are encoded into our concepts of such numbers? If the answer is “none,” as it seems to be, then why think that our concepts of such (transfinite) numbers are even apt for acquisition in response to sensory input, let alone actually grounded? That is, why think that Jenkins’s account extends to our *prima facie* knowledge of set theory?<sup>19</sup>

One might respond to this worry using Jenkins’s contention that composites of grounded concepts are grounded.<sup>20</sup> The idea is that possessing the central concepts deployed in the axioms of set theory—viz., the concepts of *set* and *membership* ( $\in$ ) along with the relevant logical concepts—will enable us to acquire the problematic cardinality concepts in question; the latter are just composites of the former. We can specify the conditions that must be satisfied in order to be a large cardinal or the least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$  in terms of *set* and  $\in$ . For example,

$$x = \bigcup \{f(n) \mid n \in \mathbb{N}\}, \tag{\dagger}$$

where  $f(n)$  is defined recursively by  $f(0) = \aleph_0$  and  $f(n + 1) = \aleph_{f(n)}$ , defines the condition  $\phi_{\dagger}$  such that a thing satisfies  $\phi_{\dagger}$  just in case it is the least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$ , and conditions on being a union of sets and being a function are definable from *set* and  $\in$  in the usual way. Assuming (i) that we have a grounded concept of recursive definition, (ii) that understanding  $\phi_{\dagger}$  implies possessing the concept *least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$* , and (iii) that this connection between understanding definable conditions and possessing concepts generalizes, we can acquire possession of the problematic cardinality concepts given possession of the concepts *set* and  $\in$ .

To the extent that assumptions (i)–(iii) can be made out, this approach shows promise. However, there are problems getting it off the ground, problems concerning the likelihood of acquiring the concept of *set* via sensory input. Possessing the concept of *set* is necessary for the strategy just rehearsed to work. I’m willing to grant that we can acquire the concept *class* via sensory

<sup>16</sup>See, e.g., Jenkins (2005, §3) for more sophisticated ways this might go.

<sup>17</sup>For details on large cardinals, see Kanamori (1994).

<sup>18</sup>See Boolos (1998, p. 120). The least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$  is the union of  $\{\aleph_0, \aleph_{\aleph_0}, \aleph_{\aleph_{\aleph_0}}, \dots\}$ .

<sup>19</sup>Simply rejecting large cardinals won’t help here, since the existence of the least cardinal  $\lambda$  such that  $\lambda = \aleph_\lambda$  is provable in Zermelo-Fraenkel set theory with the Axiom of Choice (ZFC).

<sup>20</sup>See, e.g., Jenkins (2008, pp. 127 and 136–137) and Jenkins (2005, §2).

input, say by extensionally distinguishing different collections. This collection of five oranges is not the same as that collections of five oranges because, though they're both collections of five oranges, they're collections of five *different* oranges. I'm willing to grant that we can generalize on this idea to acquire the concept of *infinite class*. But I find it difficult to accept that we can acquire the concept *set*, the concept under which fall those classes which aren't proper classes, via sensory input. Sensory experience is too coarse-grained to make the distinction between the concepts of *set* and *class*. Any sensory experience that might be thought to give us the concept *set* is just as likely to actually give us the concept *class*. After all, these concepts are coextensive in the (finite) world of our observations.<sup>21</sup>

Jenkins advocates using our best science as a guide to which concepts we should take to be justified (and so to which concepts we should take as genuine candidates for being grounded).

[C]oncepts which are indispensable for understanding our sensory input are probably... ones it is rationally respectable to rely upon as accurate representations of—and guides to—the independent world. [...] In order to find out which concepts are indispensable for the relevant purposes, we would, I think, do well to seek out those concepts which are indispensably involved in our best scientific theories of the world. Jenkins (2008, pp. 144–145)

Given this, we might think that simply consulting our best science would give us what we need. However, appeal to our best science and concepts indispensable to it won't help, since every set the use of which is essential to our best theory of the world is a class. (All sets are classes.) So it's at best contentious that we can acquire the concept of *set* in this way.

One might reply to this that just because every set indispensable to our best science is a class, it doesn't follow that our best science can get along without the concept of *set* by using the concept of *class* in its stead. After all, the reply continues, it doesn't follow from the fact that all dogs are mammals that our best biological theories can get along without the concept of *dog* by using the concept of *mammal* in its stead.<sup>22</sup> These cases, however, are quite different. First, note that if we include (all of) mathematics in our best science, then the question of the indispensability of the concept of *set* to our best science is a non-issue. However, all indications are that Jenkins's use of 'best science' does not include (all of) mathematics, but is rather limited to our best natural, and perhaps social, scientific theories. (See, e.g., the block quotation above.)

<sup>21</sup>Notice that there is a certain affinity with the inconsistency of Frege's logicism and the chief response to it here. One way of thinking about Russell's paradox is as showing that Frege erred by not distinguishing proper classes from sets. Contemporary set theory avoids this error by restricting Comprehension. The point I'm making in the text can be taken as highlighting that the distinction between sets and proper classes which leads to this restriction cannot be acquired purely via sensory input. Rather, its source is (at least in part) theoretical in nature.

<sup>22</sup>Thanks to an anonymous referee for raising this issue.

Thus, the question of the indispensability of the concept of *set* to our best science is live. My point at the end of the previous paragraph is simply that nothing in our experience of the world is such that accounting for it requires a proper class (a set which is not a class); every occurrence of the word ‘set’ in our best science, including the mathematics there applied, could be replaced with the word ‘class’ without changing the distribution of truth values over the relevant claims. It’s hard to see why this doesn’t yield that the concept ‘set’ is dispensable to our best science.<sup>23</sup> It’s another story with the concepts of *mammal* and *dog*, precisely because there are aspects of our experience of the world the (currently) best account of which requires the concept of *dog* as well as that of *mammal* (e.g., experience that leads us to ask why not all mammals bark—not all mammals are dogs). Substituting ‘mammal’ for ‘dog’ in our best science would yield radical changes in the distribution of truth values over the relevant claims because unlike the concepts of *set* and *class* sensory experience is fine-grained enough to distinguish between the concepts of *mammal* and *dog*. Our sensible experience includes mammals which are dogs and mammals which aren’t dogs; it does not include classes which aren’t sets (and neither does our best account of that experience, our best science).

Suppose now that there is a (grounded) concept C acquirable on the basis of sensory experience which we deploy in doing science and mathematics, often expressing it using the term ‘set’. The above considerations can be interpreted as arguing that it is indeterminate whether C is in fact the concept of *set* or the concept of *class* (rather than as arguing that the concept of *set* is simply unacquirable on Jenkins’s view). Hence, on the operative assumption the problem I’ve identified for Jenkins’s account is a sort of indeterminacy problem. This presents a challenge to Jenkins’s view because, whether or not C is grounded, this indeterminacy undermines our ability to know propositions involving C. Given the indeterminacy in question, our C-beliefs (apparent set-beliefs) may indeed be set-beliefs; however, they may be class-beliefs. And in light of how we acquired C, it’s a matter of luck as to which sorts of beliefs we actually have: the exact same sensory experience might have yielded possession of the concept *set* or of the concept of *class*. Which type of beliefs C-beliefs are makes a difference to the truth value of some of those beliefs.<sup>24</sup> But then whether or not at least some of our C-beliefs are known is a matter of luck on this view.

This problem is reminiscent of the famous Ginet–Goldman fake barn case.<sup>25</sup> Recall that in this case the subject, Henry, sees (in good light, etc.) what appears to be a barn and on this basis forms the belief that there is a barn in front of him. As a matter of fact there is a barn in front of him, but he fails

<sup>23</sup>I’m not arguing that our best science can do without *any* concept functioning as the concept of *set* now does. Some such concept is arguably indispensable to our best science. Rather, I’m arguing that the concept of *class* could serve the function that *set* now does in our best science every bit as well as *set*.

<sup>24</sup>I give an example below.

<sup>25</sup>See Goldman (1976).



to know that there is a barn in front of him because, unknown to Henry, the area he's in is loaded with fake barn façades. Despite Henry's belief being true, it might easily have been false. Henry's belief is what has come to be called *unsafe*.<sup>26</sup> So it's an accident, a matter of luck, that Henry has a true belief. We can schematically present this sort of case as follows:

- (I) A subject S believes that p on the basis of the world being represented to S as if p.
- (II) p, but it might easily have been that not-p despite the world being represented to S as if p

(I) and (II) yield that S's belief that p is unsafe; so it's just lucky that S's belief that p is true.<sup>27</sup> Henry believes that there is a barn in front of him on the basis of the world being represented to him (via visual perception) as if there is a barn in front of him. There is a barn in front of him, but this might easily have not been the case despite the world being represented to him as if there is a barn in front of him. Henry's true belief is a lucky belief. Similarly, in the case of C-beliefs the world is represented to us as if certain things are the case with C's. But (at least some) true C-beliefs might easily have been false, despite the world being represented to us as if they are true. So (at least some of) our C-beliefs are lucky beliefs, and so fail to be knowledge.<sup>28,29</sup>

My contention that it's indeterminate whether experience gives us the concept *set* or the more general concept *class*, and that as a result at least some apparent set-beliefs are only true (for Jenkins) by luck and so cannot be knowledge, might invite the reply that since it doesn't matter to our best science which of these concepts we have, it doesn't matter at all whether we have the concept *class* rather than the concept *set*. Naturalistically inclined philosophers might be especially keen to offer such a reply. After all, naturalists are chiefly

<sup>26</sup>See Pritchard (2005).

<sup>27</sup>Notice that I'm not equating a true belief's being unsafe with its being lucky. Rather, I'm using the fact that any true unsafe belief is lucky.

<sup>28</sup>It's worth noting that Jenkins endorses the view that lucky beliefs aren't knowledge. Indeed, the non-accidental accuracy required for groundedness of a concept is intended to mirror the non-accidental truth required for knowledge. See Jenkins (2008, p. 128.)

<sup>29</sup>The indeterminacy worry I've raised might bring to mind referential indeterminacy of the modest (i.e., non-Quinean) sort addressed in Field (1973) and Wilson (1982), suggesting that a partial denotation-style solution to referential indeterminacy might yield a solution to the problem at hand. Assuming a simple correlation of general terms (predicates) with concepts and identifying concepts with their extensions, we might try to frame the problem at hand in terms of 'set' partially denoting the class of sets and partially denoting the class of classes and say that the denotation of 'set' was refined to just the class of sets. But since the class of sets is not disjoint from the class of classes (the former is a subclass of the latter) this doesn't conform to the Field–Wilson partial denotation strategy of dealing with referential indeterminacy. Moreover, even if we ignore the restriction to pairwise disjoint partial extensions operative in that strategy, there is little reason to think that the reference of 'set' was refined purely (or even mostly) in response to sensory input. So, despite a superficial similarity, the type of referential indeterminacy which is our present concern substantially differs from that of concern to Field and Wilson. Accordingly, a partial denotation-style solution to our indeterminacy problem is implausible.

concerned with the epistemology of our best science, in particular with giving an account of the epistemology of our best science within our best science.<sup>30</sup> So if we have what we need to do this, that's enough for naturalists.

The first problem with this reply is that, while naturalism is chiefly concerned with the epistemology of our best science, various naturalists have become increasingly interested in extending naturalism to mathematics, and not only to the mathematics traditionally thought to fall within the scope of the Quine–Putnam indispensability arguments<sup>31</sup> but to all of contemporary mathematics.<sup>32</sup> As I argue elsewhere, this project requires a certain internal integrity of mathematics to be respected, the upshot of which is that certain types of revision are not legitimately advocated from outside mathematics.<sup>33</sup> Advocating revising our understanding of set theory as being about sets to its actually being about classes generally in order to secure the a priori status of arithmetic for naturalism would violate this requirement. This won't seem like much of a problem to anyone who isn't interested in the project of extending naturalism to mathematics as a whole. However, much of what's really exciting about Jenkins's proposal is its potential use to this project.<sup>34</sup> Giving up on this potential doesn't make the proposal uninteresting or unimportant, but it does diminish its interest and importance.<sup>35</sup>

In any case, a decisive problem with this reply is that if ZFC is really a theory of classes generally, then some of its theorems (and hence its axioms as well) are false. This should trouble us regardless of whether we're concerned with naturalizing the whole of mathematics. As one simple example, consider

$$\forall x \exists y (x \in y). \quad (\ddagger)$$

Informally, interpreted in the standard way as a sentence of ZFC,  $(\ddagger)$  says that every set is a member of some set. This is, of course, true (e.g., every set is a

<sup>30</sup>I here understand naturalism in accordance with the slogan “The epistemology of empirical science is an empirical science” (Boyd 1990, p. 366). There are, of course, other ways to understand naturalism. However, since Jenkins locates the naturalistic credentials of her view in “the only source of knowledge [being] one that relies in an essential way on the use of our familiar sensory apparatus” (Jenkins 2008, p. 152), said apparatus being as described by our best (empirical) science, my understanding is consonant with Jenkins's own.

<sup>31</sup>See, e.g., Quine (1948, 1954) and Putnam (1971, 1975)

<sup>32</sup>See, e.g., Colyvan (2001) and Kitcher (1983, 1988).

<sup>33</sup>See Roland (forthcoming).

<sup>34</sup>See n. 4.

<sup>35</sup>One might worry that the clash between the internal integrity of mathematics and revising set theory in the way just described depends on an equivocation on ‘naturalism’ that renders this problem moot. The naturalistic slogan extended to encompass mathematics is “The epistemology of science, mathematical as well as non-mathematical, is an empirical science,” and I might here appear to be sliding between understanding naturalism on one hand according to this slogan and on the other as opposition to what John Burgess and Gideon Rosen call ‘alienated’ epistemology (1997, p. 33). I actually think that these two conceptions of naturalism are closely related, so that the equivocation is merely apparent, but to directly address this issue would take us too far afield. However, given the decisiveness of the second problem, whether or not the equivocation is genuine is of little importance to the main thrust of the paper. Thanks to an anonymous referee for raising this issue.

member of its own power set and every set has a power set). But interpreted as a sentence of ZFC considered as a theory of classes generally, ( $\ddagger$ ) is false; one of the central differences between sets and proper classes is that no proper class is a member of any class. It follows that the axioms of ZFC are not true of classes generally. This is known; I'm not saying anything novel. However, it shows that whether the concept of *set* or the concept of *class* is in play in ZFC matters.<sup>36</sup>

At this point, we might be tempted to revisit my claim that while we may be able to acquire the concept of *class* via sensory input it's dubious that we can likewise acquire the concept of *set*. Perhaps I was too quick. A set is a class that can be a member of another class. Can't we acquire this concept directly by observation? Suppose we have two oranges. The collection of these two oranges is a class. Now we put these two oranges together with another three oranges, making sure to keep the original two together (maybe we set them off from the others, spatially or by drawing a mental line around them). Now we have a collection of five oranges, but we also have a collection of three individual oranges and one collection of two oranges. Can't we see this latter collection as having the original collection of two oranges as a member, giving us the concept of a class being a member of another class? And then can't we just stipulate that sets are collections (classes) for which this sort of thing is possible? I think that this suggestion fails to give us what we need.<sup>37</sup>

So far we've been concerned only with the basic set-theoretic concept of *set*, not contesting the possibility of acquiring the other basic set-theoretic concept, that of *membership*, via sensory input. But for the strategy just outlined to work, this possibility needs to be an actuality. The relation holding between the class of two oranges and the class of three oranges plus that class of two oranges needs to be  $\in$  for the above to yield the concept of *set*. However, why think that the relation in question is  $\in$  rather than part-whole? Indeed, it seems that the concept of *part-whole* is at least as likely as that of *membership* to be acquired by the exercise with oranges just outlined. (Think about the type of confusion commonly encountered when trying to explain membership to someone beginning to learn set theory.) And if the concept acquired is the concept of *part-whole*, then the above suggestion for how we acquire the concept of *set*, rather than that of *class* generally, via sensory input doesn't work.<sup>38</sup>

<sup>36</sup>We might try to alleviate this difficulty by introducing a distinguished predicate  $S$  into the language of set theory such that ' $Sx$ ' informally says that  $x$  is a set. Relativizing the axioms of ZFC to  $S$  would then do the trick; the relativization of ( $\ddagger$ ) to  $S$  would then be a theorem, not ( $\ddagger$ ) itself. The problem, of course, is that in order to help with the problem at hand the concept associated with this new predicate must be the concept of *set*, and it's precisely for want of that very concept that we've proposed introducing the new predicate.

<sup>37</sup>In addition to the problem I'm about to discuss, there might also be a problem of indeterminacy between membership and subset. I leave this issue aside.

<sup>38</sup>A Lewis-style reduction of set theory to part-whole theory à la (1991) won't help here, since such a reduction requires a primitive singleton-set operation and so assumes already concepts of both *set* and *membership*.

## Concluding Remarks

Nothing I've said implies that we don't have the concept of *set*, or even that sensory input plays no role in our acquisition of that concept. For instance, we might acquire the concept of *class* by experience with collections of physical objects and then come to have the concept of *set* by a process of refinement and systematization, axiomatization being the most obvious example of such a process. Less apocryphally, it's quite plausible to think that we acquired the concept of *set* as a result of Cantor's investigations into infinity and continuity stemming from his work on representing Fourier series<sup>39</sup> and the development of those investigations, notably Zermelo's (1908/1967) axiomatization. The questions regarding Fourier series Cantor addressed (likely) ultimately trace back to questions about how to organize and explain sensible experience,<sup>40</sup> and in this way sensory input gets into the picture. But isolating and grasping, i.e., acquiring, the concept *set* additionally requires theoretical input, typically manifested in contemporary settings by axiomatization. Sensory input alone isn't enough to determinately give us the concept of *set* rather than that of *class*. This being the case, on Jenkins's view (at least some) true apparent set-beliefs are lucky, and so fail to be knowledge.

We've been granting all along that Jenkins's account works for the whole of arithmetic. Note, though, that the same kind of indeterminacy problem I've identified for the concepts of *set* and *class* might also arise for some arithmetic concepts. For instance, the concept we often express using '+' might be indeterminate between the concept of *plus* and that of Kripke's *quus*.<sup>41</sup> In fact, Jenkins considers this possibility (or something very much like it) and one might think her response to it will also work for the indeterminacy problem I've been pressing against her view. Her response has two parts, one dismissive and the other more substantive.<sup>42</sup>

The dismissive consists of claiming that the result of taking this problem seriously is arithmetical scepticism, and that as such it is no more threatening to her view than any form of scepticism is a threat to any epistemology of the relevant kind. As she says, "Those who doubt that epistemology is in the business of defeating scepticism may not yet see any problem here at all" (Jenkins 2008, p. 227). Jenkins obviously doesn't think that epistemology is in the business of defeating scepticism. However, as a reply to the indeterminacy problem this seems to me to have things backwards. Assuming we have some (lots) of mathematical knowledge, the task of an epistemology for mathematics is to tell a plausible story about how we come by that knowledge. A central test for any epistemology of mathematics is that it accommodates the mathematics

<sup>39</sup>See Dauben (1990).

<sup>40</sup>See Grattan-Guinness (1997, especially Chapter 2).

<sup>41</sup>See Kripke (1982).

<sup>42</sup>See Jenkins (2008, pp. 227–230).

we already think we know (or at least most of the less contentious bits). But this is precisely the test Jenkins's account fails, at the very least with respect to set theory. I take it as a datum that every set is the member of some set. Jenkins's account does not accommodate this datum. So the correct consequence of the indeterminacy problem I've raised is not scepticism about set theory; it's that the story Jenkins tells about the epistemology of arithmetic is inadequate as a story about (at least) the epistemology of set theory.

The substantive response offered by Jenkins is modeled on a common reply to well-known underdetermination worries in the philosophy of science, worries premised on the fact that no finite body of observational data is sufficient to determine a single theory to account for that data. The common reply is to invoke so-called *theoretical virtues* such as explanatory power, simplicity, unificatory power, and fruitfulness. The idea is that given empirically equivalent<sup>43</sup> theories  $T_1, \dots, T_k$ , we use theoretical virtues to choose one of the theories over the others. After all, we can't choose among them on the basis of empirical data since they're empirically equivalent. Jenkins suggests modifying this response to apply to concepts: "For instance, we can say that, if two concepts have an equally good fit with our sensory input but one of them is simpler than the other, we should (epistemically, and not merely pragmatically) prefer the simpler concept" (Jenkins 2008, p. 228). She is somewhat less committal about how other theoretical virtues might apply to concepts, and also about whether there are conceptual virtues that have no theoretical counterparts, but considering how the strategy she indicates with simplicity might apply to *set* and *class* is instructive.

The suggestion is that we should prefer the simpler concept between *set* and *class*. I must confess it's not at clear to me how to decide which of these concepts is simpler than the other.<sup>44</sup> Indeed, I suspect that the strategy under consideration isn't apt for the case at hand. Jenkins's strategy seems to implicitly rely on the two concepts with equally good fit to sensory input being rivals; accepting one requires discarding the other. This isn't the situation with *set* and *class*. If we accept *class*, extant mathematics will lead us to find a way to distinguish non-proper classes from proper classes. The former are, of course, just sets. So we still want the concept of *set*. On the other hand, if we accept *set*, the problems that originally led us to distinguish proper classes from sets (i.e., the set-theoretic paradoxes) will again lead us to do the same. That is, we still want the concept of *class* if we accept the concept of *set*. Either way, we want (need) both concepts. It's not a matter of preferring one over the other in the same way it is when we're talking about empirically equivalent theories. And it seems to me that this generalizes to any theoretical virtue modified to apply

<sup>43</sup>Theories  $T$  and  $T'$  are empirically equivalent just in case they have all and only the same observational consequences (when paired with the same set of auxiliary hypotheses). Intuitively, empirically equivalent theories are indistinguishable by experience—past, present, or future.

<sup>44</sup>Cf. Kripke on simplicity and the content of hypotheses (1982, p. 38).

to *set* and *class*, simply because the aim is different when applying theoretical virtues to empirically equivalent theories versus applying modified theoretical virtues to *set* and *class*. In the former, the aim is to eliminate rivals; in the latter, it's not.

So I take it that neither way of answering the indeterminacy worry concerning *plus* and *quus* rehearsed by Jenkins works for the indeterminacy problem I've raised for her view. This being the case, I conclude that, however promising her account for the epistemology of arithmetic, it does not extend to set theory.

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