FE Review Course – Fluid Mechanics

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Frank T.-C. Tsai, Ph.D., P.E.
Dept. of Civil & Environmental Engineering, LSU
ftsai@lsu.edu
(O) 578-4246

Get the handbook!!

- **SUPPLIED-REFERENCE HANDBOOK**
- 8th edition, 2nd revision, April 2011

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Visualize questions

- Draw diagrams
- Annotate diagrams with numbers, symbols, equations, etc.
- Find right equations from the reference handbook
- Skip questions if you cannot quickly find equations from the reference handbook

Contents:

1) Fluids properties: Density, specific volume, specific weight, and specific gravity
2) Stress, pressure, and viscosity
3) Surface tension and capillarity
4) The pressure field in a static liquid
5) Manometers
6) Forces on submerged surfaces and the center of pressure
7) Archimedes principle and buoyancy
8) One-dimensional flows
9) The field equation (Bernoulli equation)
10) Fluids measurements (Pitot tube, Venturi meter, and orifices)
11) Hydraulic Grade Line (HGL) and Energy Line (EL)
12) Reynolds number
13) Drag force on immersed bodies
14) Aerodynamics
15) Fluid flow (Pipe flow; Energy equation)
16) The impulse-momentum principle (Linear momentum equation)
17) Dimensional homogeneity and dimensional analysis and similitude
18) Open-channel flow

DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

$$\rho = \lim_{\Delta V \to 0} \Delta m / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} \Delta W / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} g \cdot \Delta m / \Delta V = \rho g$$

also $SG = \gamma/\gamma_w = \rho/\rho_w$, where

ρ = *density* (also called *mass density*),

Δm = mass of infinitesimal volume,

ΔV = volume of infinitesimal object considered,

γ = *specific weight,*
= ρg,

ΔW = weight of an infinitesimal volume,

SG = *specific gravity,*

ρ_w = density of water at standard conditions
= 1,000 kg/m^3 (62.43 lbm/ft^3), and

γ_ω = specific weight of water at standard conditions,
= 9,810 N/m^3 (62.4 lbf/ft^3), and
= 9,810 $kg/(m^2 \cdot s^2)$.

$$SG = \frac{\rho_{fluid}}{\rho_{H_2O@4^oC}} = \frac{\gamma_{fluid}}{\gamma_{H_2O@4^oC}}$$

$$E_v = -\frac{dp}{d\forall / \forall}$$

$$c = \sqrt{\frac{E_v}{\rho}}$$

SURFACE TENSION AND CAPILLARITY

Surface tension σ is the force per unit contact length

$\sigma = F/L$, where

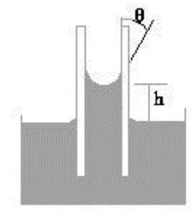
σ = surface tension, force/length,
F = surface force at the interface, and
L = length of interface.

The *capillary rise h* is approximated by

$h = (4\sigma \cos\beta)/(\gamma d)$, where

h = the height of the liquid in the vertical tube,
σ = the surface tension,
β = the angle made by the liquid with the wetted tube wall,
γ = specific weight of the liquid, and
d = the diameter of the capillary tube.

$$h = \frac{4\sigma\cos\theta}{\gamma d}$$



8. A clean glass tube is to be selected in the design of a manometer to measure the pressure of kerosene. Specific gravity of kerosene = 0.82 and surface tension of kerosene = 0.025 N/m. If the capillary rise is to be limited to 1 mm, the smallest diameter (cm) of the glass tube should be most nearly

★ A. 1.25
○ B. 1.50
○ C. 1.75
○ D. 2.00

$$\frac{1\text{ mm}}{1000} = \frac{4\times 0.025\times 1}{0.82\times 9810\times d}$$

(Page 62)

STRESS, PRESSURE, AND VISCOSITY

Stress is defined as

$$\tau(1) = \lim_{\Delta A \to 0} \Delta F/\Delta A, \text{ where}$$

$\tau(1)$ = surface stress vector at point 1,

ΔF = force acting on infinitesimal area ΔA, and

ΔA = infinitesimal area at point 1.

$$\tau_n = -P$$

$$\tau_t = \mu(dv/dy) \text{ (one-dimensional; i.e., } y\text{), where}$$

τ_n and τ_t = the normal and tangential stress components at point 1,

P = the pressure at point 1,

μ = *absolute dynamic viscosity* of the fluid $N{\bullet}s/m^2$ [lbm/(ft-sec)],

dv = differential velocity,

dy = differential distance, normal to boundary.

v = velocity at boundary condition, and

y = normal distance, measured from boundary.

v = *kinematic viscosity*; m^2/s (ft^2/sec) where $v = \mu/\rho$

For a thin Newtonian fluid film and a linear velocity profile,

$$v(y) = vy/\delta;\ dv/dy = v/\delta, \text{ where}$$

v = velocity of plate on film and

δ = thickness of fluid film.

For a power law (non-Newtonian) fluid

$$\tau_t = K\,(dv/dy)^n, \text{ where}$$

K = consistency index, and

n = power law index.

$n < 1 \equiv$ pseudo plastic

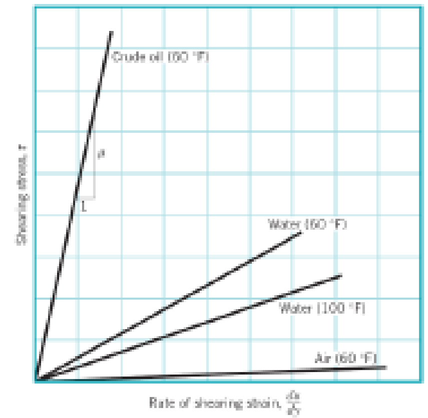
$n > 1 \equiv$ dilatant

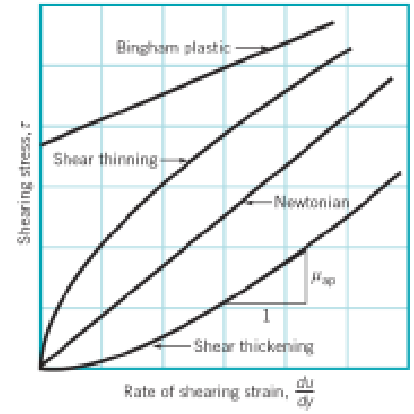
Newtonian vs. Non-Newtonian Fluids

Dilatant: $\tau \uparrow \quad du/dy \uparrow$

Newtonian: $\tau \propto \quad du/dy$

Pseudo plastic: $\tau \downarrow \quad du/dy \uparrow$





$$\tau \propto \frac{du}{dy}$$

μ = slope

$$\tau \propto \left(\frac{du}{dy}\right)^n$$

$n > 1$ slope increases with increasing τ (shear thickening)

$n < 1$ slope decreases with increasing τ (shear thinning)
Ex) blood, paint, liquid plastic

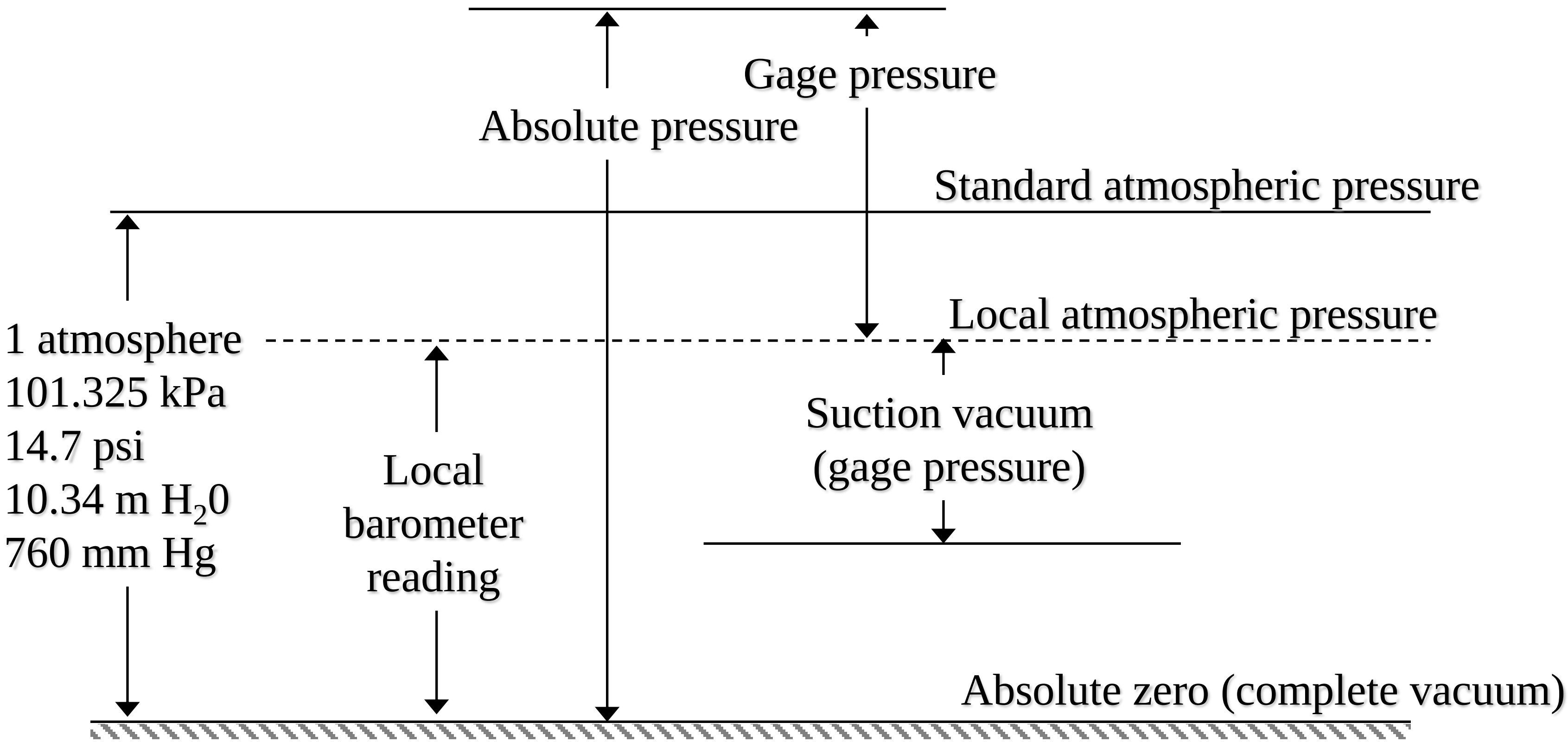
♦ **PROPERTIES OF WATER[f] (SI METRIC UNITS)**

Temperature °C	Specific Weight[a], γ, kN/m³	Density[a], ρ, kg/m³	Absolute Dynamic Viscosity[a], μ Pa•s	Kinematic Viscosity[a], υ m²/s	Vapor Pressure[e], p_v, kPa
0	9.805	999.8	0.001781	0.000001785	0.61
5	9.807	1000.0	0.001518	0.000001518	0.87
10	9.804	999.7	0.001307	0.000001306	1.23
15	9.798	999.1	0.001139	0.000001139	1.70
20	9.789	998.2	0.001002	0.000001003	2.34
25	9.777	997.0	0.000890	0.000000893	3.17
30	9.764	995.7	0.000798	0.000000800	4.24
40	9.730	992.2	0.000653	0.000000658	7.38
50	9.689	988.0	0.000547	0.000000553	12.33
60	9.642	983.2	0.000466	0.000000474	19.92
70	9.589	977.8	0.000404	0.000000413	31.16
80	9.530	971.8	0.000354	0.000000364	47.34
90	9.466	965.3	0.000315	0.000000326	70.10
100	9.399	958.4	0.000282	0.000000294	101.33

PROPERTIES OF WATER (ENGLISH UNITS)

Temperature (°F)	Specific Weight γ (lb/ft^3)	Mass Density ρ (lb • sec^2/ft^4)	Absolute Dynamic Viscosity μ ($\times 10^{-5}$ lb • sec/ft^2)	Kinematic Viscosity υ ($\times 10^{-5}$ ft^2/sec)	Vapor Pressure p_v (psi)
32	62.42	1.940	3.746	1.931	0.09
40	62.43	1.940	3.229	1.664	0.12
50	62.41	1.940	2.735	1.410	0.18
60	62.37	1.938	2.359	1.217	0.26
70	62.30	1.936	2.050	1.059	0.36
80	62.22	1.934	1.799	0.930	0.51
90	62.11	1.931	1.595	0.826	0.70
100	62.00	1.927	1.424	0.739	0.95
110	61.86	1.923	1.284	0.667	1.24
120	61.71	1.918	1.168	0.609	1.69
130	61.55	1.913	1.069	0.558	2.22
140	61.38	1.908	0.981	0.514	2.89
150	61.20	1.902	0.905	0476	3.72
160	61.00	1.896	0.838	0.442	4.74
170	60.80	1.890	0.780	0.413	5.99
180	60.58	1.883	0.726	0.385	7.51
190	60.36	1.876	0.678	0.362	9.34
200	60.12	1.868	0.637	0.341	11.52
212	59.83	1.860	0.593	0.319	14.70

Units and Scales of Pressure Measurement



6894.76 Pa/psi

$$p_{abs} = p_{gage} + p_{atm}$$

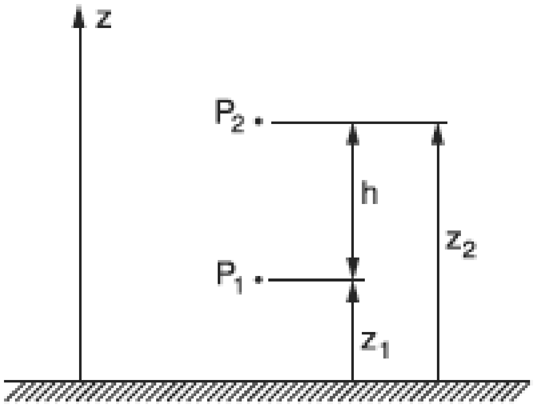
Absolute pressures are often indicated as *psia*, and gage pressure as *psig*.

Fluid Statics

- Pressure vs. elevation
- Manometers
- Force over submerged plane and curved surfaces
- Buoyancy

THE PRESSURE FIELD IN A STATIC LIQUID

♦



The difference in pressure between two different points is

$$P_2 - P_1 = -\gamma (z_2 - z_1) = -\gamma h = -\rho g h$$

For a simple manometer,

$$P_o = P_2 + \gamma_2 z_2 - \gamma_1 z_1$$

Absolute pressure = atmospheric pressure + gage pressure reading

Absolute pressure = atmospheric pressure – vacuum gage pressure reading

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

Fluid Statics

- Pressure vs. elevation
- Manometers
- Force over submerged plane and curved surfaces
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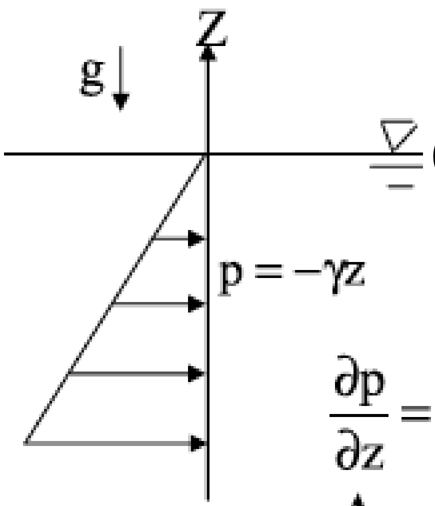
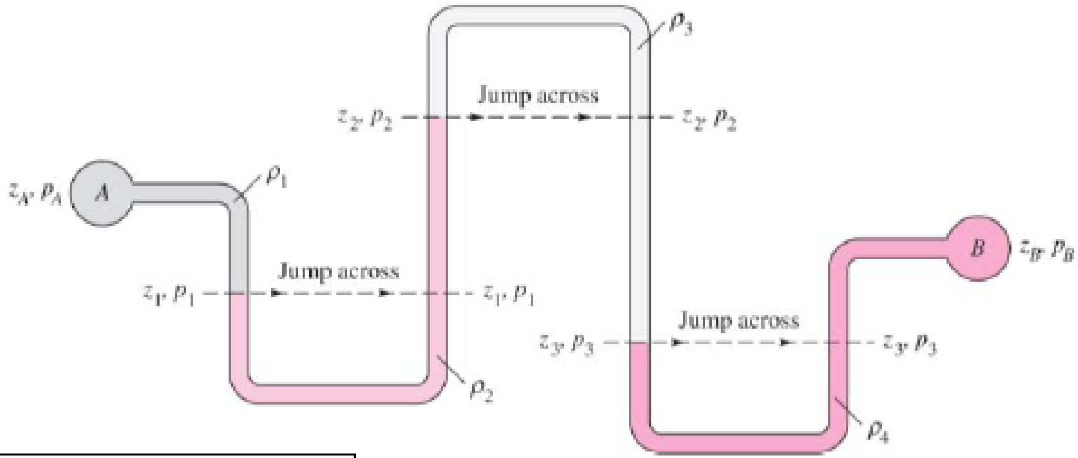


(z=0 at water surface and z is negative below surface)

$$\frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad \rho = \text{constant for liquid}$$

$$\Delta p = -\gamma \Delta z$$

$$p_2 - p_1 = -\gamma \left(z_2 - z_1 \right)$$



↓: add γh
Jump across: no change
↑: subtract γh

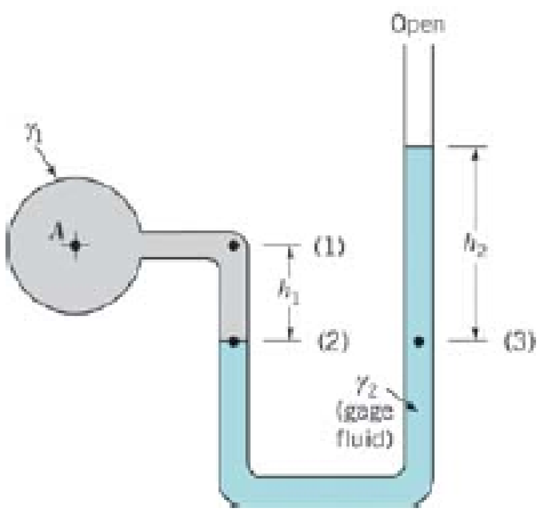


FIGURE 2.10 **Simple U-tube manometer.**

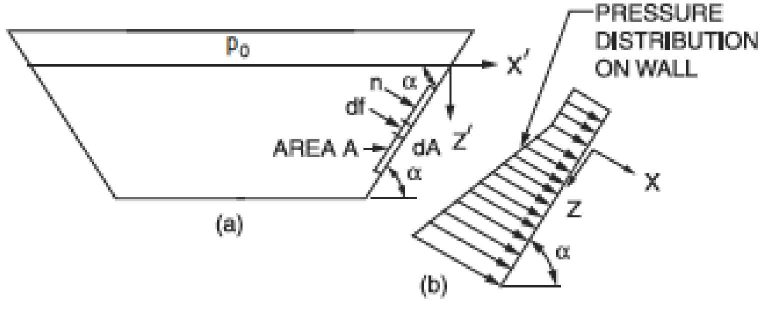
$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

$$\therefore p_A = \gamma_2 h_2 - \gamma_1 h_1$$

$$\rightarrow p_A \approx \gamma_2 h_2$$ (for gas)

FORCES ON SUBMERGED SURFACES AND THE CENTER OF PRESSURE

♦



Forces on a submerged plane wall. (a) Submerged plane surface. (b) Pressure distribution.

The pressure on a point at a distance Z' below the surface is

$$p = p_o + \gamma Z', \text{ for } Z' \ge 0$$

If the tank were open to the atmosphere, the effects of p_o could be ignored.

The coordinates of the *center of pressure* (*CP*) are

$$y^* = (\gamma I_{y_c z_c} \sin\alpha)/(p_c A) \text{ and}$$

$$z^* = (\gamma I_{y_c} \sin\alpha)/(p_c A), \text{ where}$$

y^* = the y-distance from the centroid (C) of area (A) to the center of pressure,

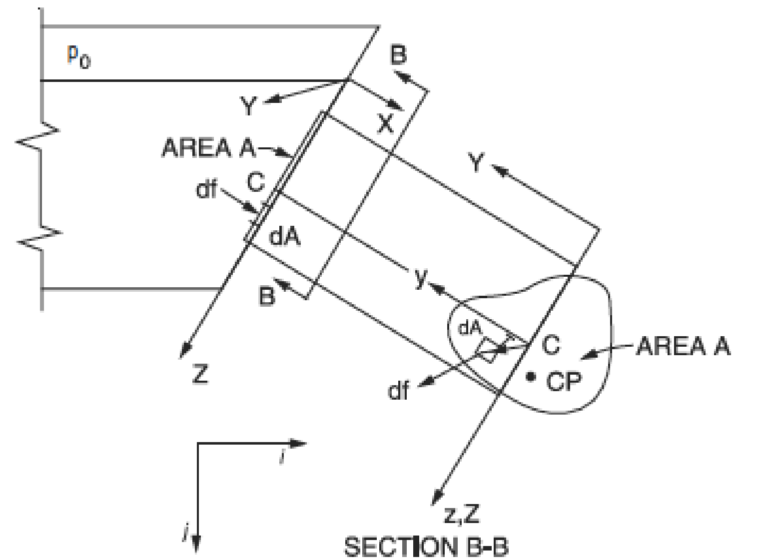
z^* = the z-distance from the centroid (C) of area (A) to the center of pressure,

I_{y_c} and $I_{y_c z_c}$ = the moment and product of inertia of the area,

p_c = the pressure at the centroid of area (A), and

Z_c = the slant distance from the water surface to the centroid (C) of area (A).

♦



If the free surface is open to the atmosphere, then $p_o = 0$ and $p_c = \gamma Z_c \sin\alpha$.

$$y^* = I_{y_c z_c}/(AZ_c) \text{ and } z^* = I_{y_c}/(AZ_c)$$

The force on a rectangular plate can be computed as

$$\boldsymbol{F} = [p_1 A_v + (p_2 - p_1) A_v/2]\mathbf{i} + V_f \gamma_f \mathbf{j}, \text{ where}$$

$\boldsymbol{F}$ = force on the plate,

p_1 = pressure at the top edge of the plate area,

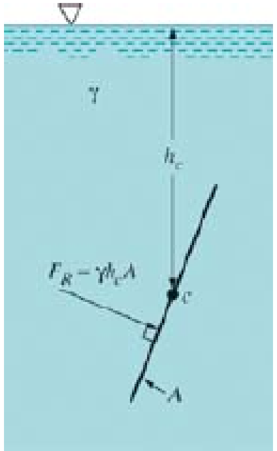
p_2 = pressure at the bottom edge of the plate area,

A_v = vertical projection of the plate area,

V_f = volume of column of fluid above plate, and

γ_f = specific weight of the fluid.

$$F_R = \gamma h_c A$$



$$F = \bar{p}A = \underbrace{\gamma \sin\alpha\, \bar{y}}\, A$$

$\bar{p}$ = pressure at centroid of A

Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface.

Center of Pressure

Center of pressure is in general below centroid since pressure increases with depth. Center of pressure is determined by equating the moments of the resultant and distributed forces about any arbitrary axis.

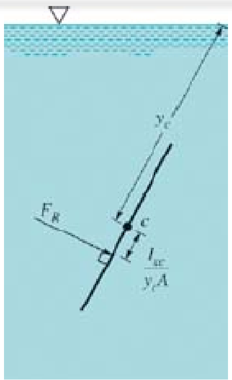
$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$

y_{cp} is below centroid by $\bar{I}/\bar{y}A$

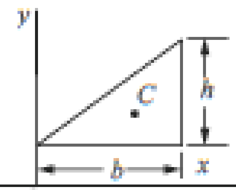
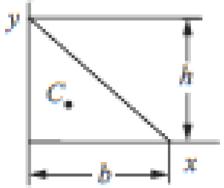
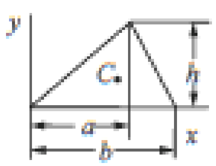
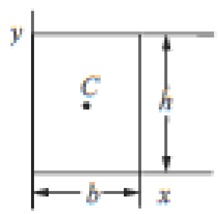
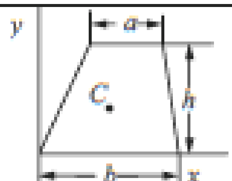
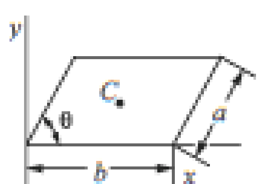
$$x_{cp} = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

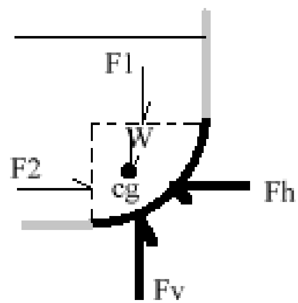


For plane surfaces with symmetry about an axis normal to 0-0, $\bar{I}_{xy} = 0$ and $x_{cp} = \bar{x}$.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = \left[bh\left(b^2 - ab + a^2\right)\right]/36$ $I_x = bh^3/12$ $I_y = \left[bh\left(b^2 + ab + a^2\right)\right]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = \left(b^2 - ab + a^2\right)/18$ $r_x^2 = h^2/6$ $r_y^2 = \left(b^2 + ab + a^2\right)/6$	$I_{x_c y_c} = \left[Ah(2a - b)\right]/36$ $= \left[bh^2(2a - b)\right]/72$ $I_{xy} = \left[Ah(2a + b)\right]/12$ $= \left[bh^2(2a + b)\right]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = \left[bh\left(b^2 + h^2\right)\right]/12$	$r_{x_c}^2 = h^2/12$ $r_{y_c}^2 = b^2/12$ $r_x^2 = h^2/3$ $r_y^2 = b^2/3$ $r_p^2 = \left(b^2 + h^2\right)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$
	$A = h(a+b)/2$ $y_c = \dfrac{h(2a+b)}{3(a+b)}$	$I_{x_c} = \dfrac{h^3\left(a^2 + 4ab + b^2\right)}{36(a+b)}$ $I_x = \dfrac{h^3(3a+b)}{12}$	$r_{x_c}^2 = \dfrac{h^2\left(a^2 + 4ab + b^2\right)}{18(a+b)}$ $r_x^2 = \dfrac{h^2(3a+b)}{6(a+b)}$	
	$A = ab\sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a\sin\theta)/2$	$I_{x_c} = \left(a^3b\sin^3\theta\right)/12$ $I_{y_c} = \left[ab\sin\theta\left(b^2 + a^2\cos^2\theta\right)\right]/12$ $I_x = \left(a^3b\sin^3\theta\right)/3$ $I_y = \left[ab\sin\theta(b + a\cos\theta)^2\right]/3$ $-\left(a^2b^2\sin\theta\cos\theta\right)/6$	$r_{x_c}^2 = (a\sin\theta)^2/12$ $r_{y_c}^2 = \left(b^2 + a^2\cos^2\theta\right)/12$ $r_x^2 = (a\sin\theta)^2/3$ $r_y^2 = (b + a\cos\theta)^2/3$ $-(ab\cos\theta)/6$	$I_{x_c y_c} = \left(a^3b\sin^2\theta\cos\theta\right)/12$

Housner, George W. & Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G. W. Housner & D.E. Hudson.

For **<u>curved surface</u>**, separate the pressure force into horizontal and vertical part. The horizontal part becomes plane surface and the vertical force becomes weight.

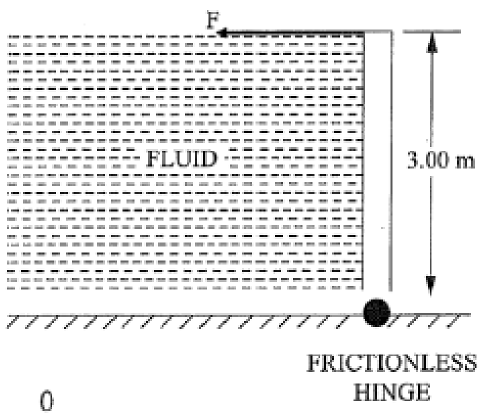


$$F_h = F_R = F_2 \text{ on the vertical projection, } F_v = \text{weight of fluid above} = \mathrm{W} + F_1$$

$$F_{buoyancy} = g_{fluid}{''}_{submerged}$$

If an object is submerged in several different fluids, must calculate the buoyancy in each of them, then add together. When using buoyancy in problems, FBD is often needed.

The rectangular homogeneous gate shown below is 3.00 m high × 1.00 m wide and has a frictionless hinge at the bottom. If the fluid on the left side of the gate has a density of 1,600 kg/m^3, the magnitude of the force **F** (kN) required to keep the gate closed is most nearly:



(A) 0
(B) 22
(C) 24
(D) 220

The mean pressure of the fluid acting on the gate is evaluated at the mean height, and the center of pressure is 2/3 of the height from the top; thus, the total force of the fluid is:

$$F_f = \rho g \frac{H}{2}(H) = 1,600(9.807)\frac{3}{2}(3) = 70,610 \text{ N}$$

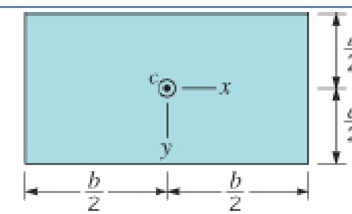
and its point of application is 1.00 m above the hinge. A moment balance about the hinge gives:

$$F(3) - F_f(1) = 0$$

$$F = \frac{F_f}{3} = \frac{70,610}{3} = 23,537 \text{ N}$$

THE CORRECT ANSWER IS: (C)

$$z^* = \frac{\frac{1\times 3^3}{12}}{3\times 1\times \frac{3}{2}} = 0.5$$ (page 63)



$A = ba$

$I_{xc} = \frac{1}{12}ba^3$

$I_{yc} = \frac{1}{12}ab^3$

$I_{xyc} = 0$

$$F_R = \gamma h_c A = \gamma\left(\frac{a}{2}\right)(ba) = \frac{1}{2}\gamma ba^2$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}ba^3}{\left(\frac{a}{2}\right)(ba)} + \frac{a}{2} = \frac{2}{3}a$$

$\Sigma M_O = 0$:

$$F\times a - F_R \times \left(\frac{a}{3}\right) = 0$$

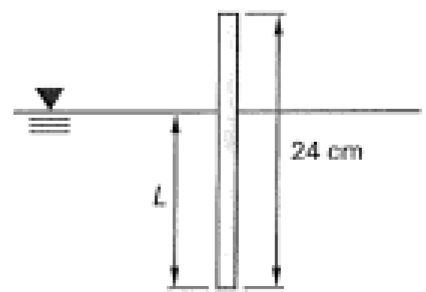
ARCHIMEDES PRINCIPLE AND BUOYANCY

1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the displaced fluid volume.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

96. A 24 cm long rod floats vertically in water. It has a 1 cm^2 cross section and a specific gravity of 0.6. Most nearly, what length, L, is submerged?



(A) 9.6 cm
(B) 14 cm
(C) 18 cm
(D) 19 cm

96. $\rho_{water}L = \rho_{rod}(24\text{ cm})$

$$L = \left(\frac{\rho_{rod}}{\rho_{water}}\right)(24\text{ cm}) = (\text{SG})(24\text{ cm})$$

$$= (0.6)(24\text{ cm})$$

$$= 14.4\text{ cm} \quad (14\text{ cm})$$

Answer is B.

ONE-DIMENSIONAL FLOWS

The Continuity Equation

So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1 v_1 = A_2 v_2$.

$$Q = Av$$

$$\dot{m} = \rho Q = \rho A v, \text{ where}$$

Q = volumetric flow rate,

$\dot{m}$ = mass flow rate,

A = cross section of area of flow,

v = average flow velocity, and

ρ = the fluid density.

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

Assuming a flow of 40 m^3/min, the velocity (m/s) through the pipe is most nearly:

The diameter of the pipe is 0.3 m.

(A) 9.4
(B) 2.4
(C) 1.4
(D) 0.047

Volume flow rate = area × velocity

$$Q = Av$$

$$v = \frac{Q}{A}$$

$$\left(\frac{40\ m^3}{min}\right)\left(\frac{min}{60\,s}\right)\left(\frac{1}{\pi(0.15)^2\ m^2}\right) = 9.4\ m/s$$

THE CORRECT ANSWER IS: (A)

The Field Equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \text{ or}$$

$$\frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g, \text{ where}$$

P_1, P_2 = pressure at sections 1 and 2,
v_1, v_2 = average velocity of the fluid at the sections,
z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),
γ = the specific weight of the fluid (ρg), and
g = the acceleration of gravity.

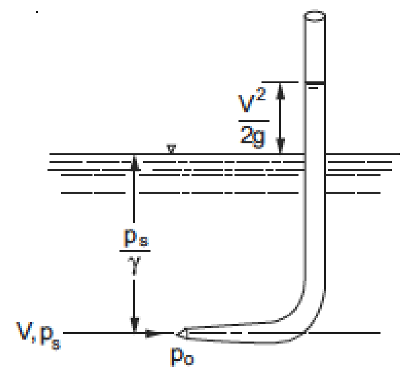
FLUID MEASUREMENTS

The Pitot Tube – From the stagnation pressure equation for an *incompressible fluid*,

$$v = \sqrt{(2/\rho)(p_0 - p_s)} = \sqrt{2g(p_0 - p_s)/\gamma}, \text{ where}$$

v = the velocity of the fluid,
p_0 = the stagnation pressure, and
p_s = the static pressure of the fluid at the elevation where the measurement is taken.

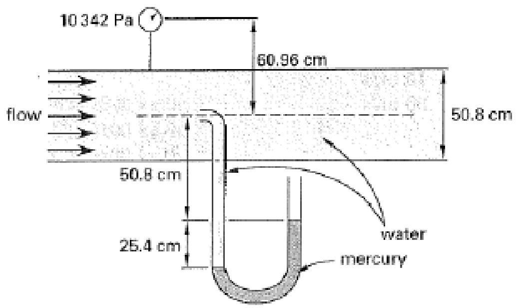
•



For a *compressible fluid*, use the above incompressible fluid equation if the Mach number ≤ 0.3.

• Vennard, J.K., *Elementary Fluid Mechanics*, 6th ed., J.K. Vennard, 1954.

39. A static pressure gauge and mercury manometer are connected to a 50.8 cm pipeline flowing full of water. One cubic centimeter of mercury has a mass of 0.1336 N. What is most nearly the velocity at the center of the pipeline?



(A) 0.66 m/s
(B) 0.79 m/s
(C) 4.5 m/s
(D) 5.7 m/s

39. The static pressure is

$$p_s = (60.96\text{ cm})\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) + 10\,342\text{ Pa}$$
$$= 16\,322\text{ Pa}$$

The stagnation pressure is

$$p_0 = \left(0.1336\ \frac{\text{N}}{\text{cm}^3}\right)(25.4\text{ cm})\left(100\ \frac{\text{cm}}{\text{m}}\right)^2 - \left(9810\ \frac{\text{N}}{\text{m}^3}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)(50.8\text{ cm} + 25.4\text{ cm})$$
$$= 26\,459\text{ Pa}$$

$$\text{v} = \sqrt{\frac{(2)(p_0 - p_s)}{\rho}} = \sqrt{\frac{(2)(26\,459\text{ Pa} - 16\,322\text{ Pa})}{1000\ \frac{\text{kg}}{\text{m}^3}}} = 4.5\text{ m/s}$$

Answer is C.

38. A perfect venturi with a throat diameter of 1.8 cm is placed horizontally in a pipe with a 5 cm inside diameter. Eight kg of water flow through the pipe each second. What is most nearly the difference between the pipe and venturi throat static pressures?

(A) 30 kPa
(B) 490 kPa
(C) 640 kPa
(D) 970 kPa

38. $$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi\left((5\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\right)^2}{4}$$
$$= 0.001963\text{ m}^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi\left((1.8\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\right)^2}{4}$$
$$= 2.545\times 10^{-4}\text{ m}^2$$

$$\mathrm{v}_1 = \frac{\dot{m}}{\rho A_1} = \frac{8.0\ \frac{\text{kg}}{\text{s}}}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)(0.001963\text{ m}^2)}$$
$$= 4.07\text{ m/s}$$

$$\mathrm{v}_2 = \frac{\dot{m}}{\rho A_2} = \frac{8.0\ \frac{\text{kg}}{\text{s}}}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)(2.545\times 10^{-4}\text{ m}^2)}$$
$$= 31.43\text{ m/s}$$

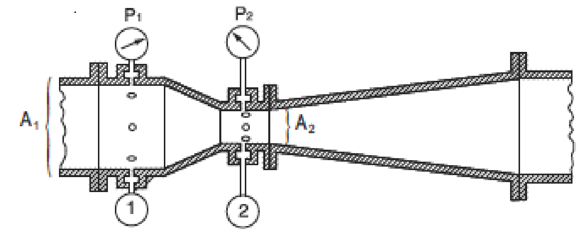
Venturi Meters

$$Q = \frac{C_v A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1}{\gamma}+z_1-\frac{p_2}{\gamma}-z_2\right)},\text{ where}$$

C_v = the coefficient of velocity, and
γ = ρg.

The above equation is for *incompressible fluids*.

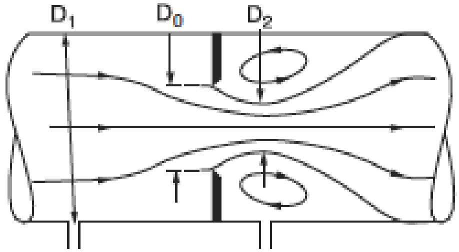
•



$$p_1 - p_2 = \left(\frac{\rho}{2}\right)(\mathrm{v}_2^2 - \mathrm{v}_1^2)$$
$$= \left(\frac{1000\ \frac{\text{kg}}{\text{m}^3}}{2}\right)\left(\left(31.43\ \frac{\text{m}}{\text{s}}\right)^2 - \left(4.075\ \frac{\text{m}}{\text{s}}\right)^2\right)$$
$$\times\left(\frac{1\text{ kPa}}{1000\text{ Pa}}\right)$$
$$= 486\text{ kPa}\quad(490\text{ kPa})$$

Answer is B.

Orifices The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by $C_c A$.

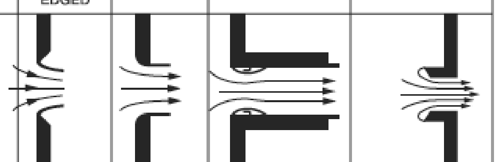
- 

$$Q = CA_0\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

where C, the *coefficient of the meter* (*orifice coefficient*), is given by

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2\left(A_0/A_1\right)^2}}$$

- ORIFICES AND THEIR NOMINAL COEFFICIENTS

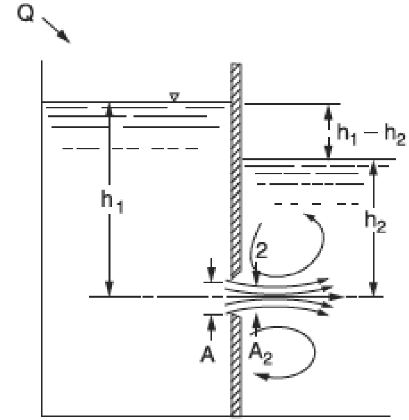


	SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
C	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98

For incompressible flow through a horizontal orifice meter installation

$$Q = CA_0\sqrt{\frac{2}{\rho}\left(p_1 - p_2\right)}$$

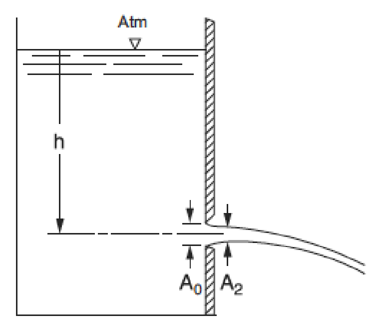
Submerged Orifice operating under steady-flow conditions:

- 

$$Q = A_2 v_2 = C_c C_v A\sqrt{2g\left(h_1 - h_2\right)}$$
$$= CA\sqrt{2g\left(h_1 - h_2\right)}$$

in which the product of C_c and C_v is defined as the *coefficient of discharge* of the orifice.

Orifice Discharging Freely into Atmosphere

- 

$$Q = CA_0\sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

ENERGY LINE (BERNOULLI EQUATION)
The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the "total head line" above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $v^2/2g$ term.

HYDRAULIC GRADIENT (GRADE LINE)
The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

$$\text{EGL} = \frac{p}{g} + z + a\frac{V^2}{2g}$$

$$\text{HGL} = \frac{p}{g} + z$$

Pressure head (w.r.t. reference pressure)

Elevation head (w.r.t. datum)

velocity head

Piezometric head

EGL: line connecting

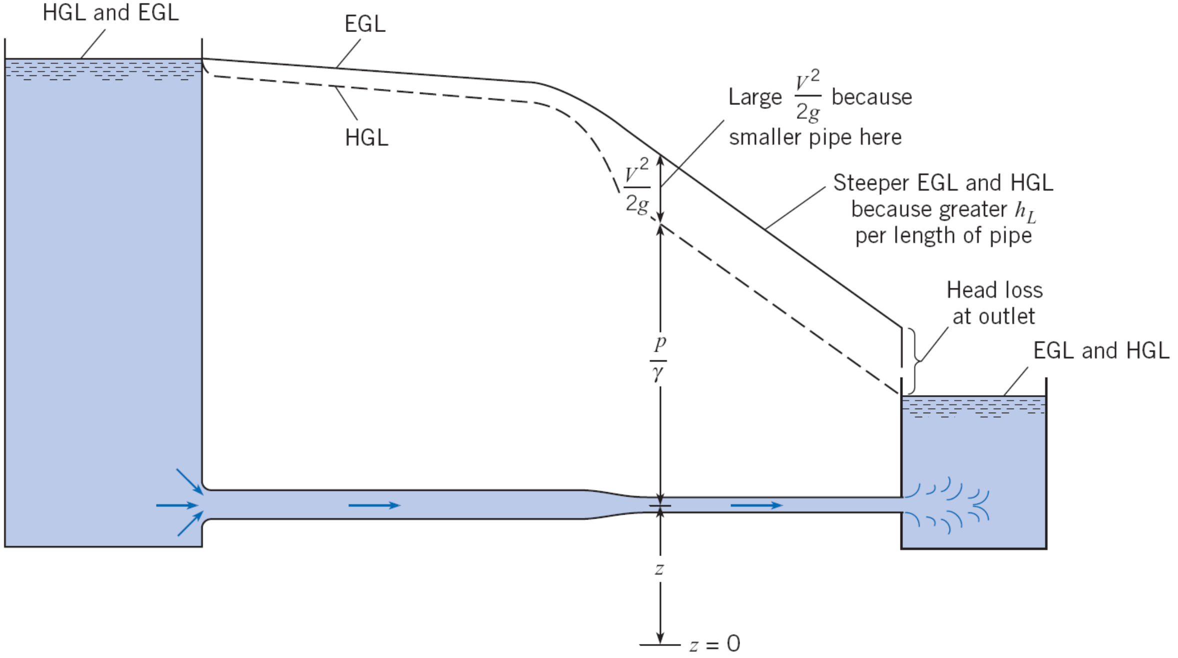
$$\frac{p}{\gamma}+\frac{v^2}{2g}+z$$

(total head line)

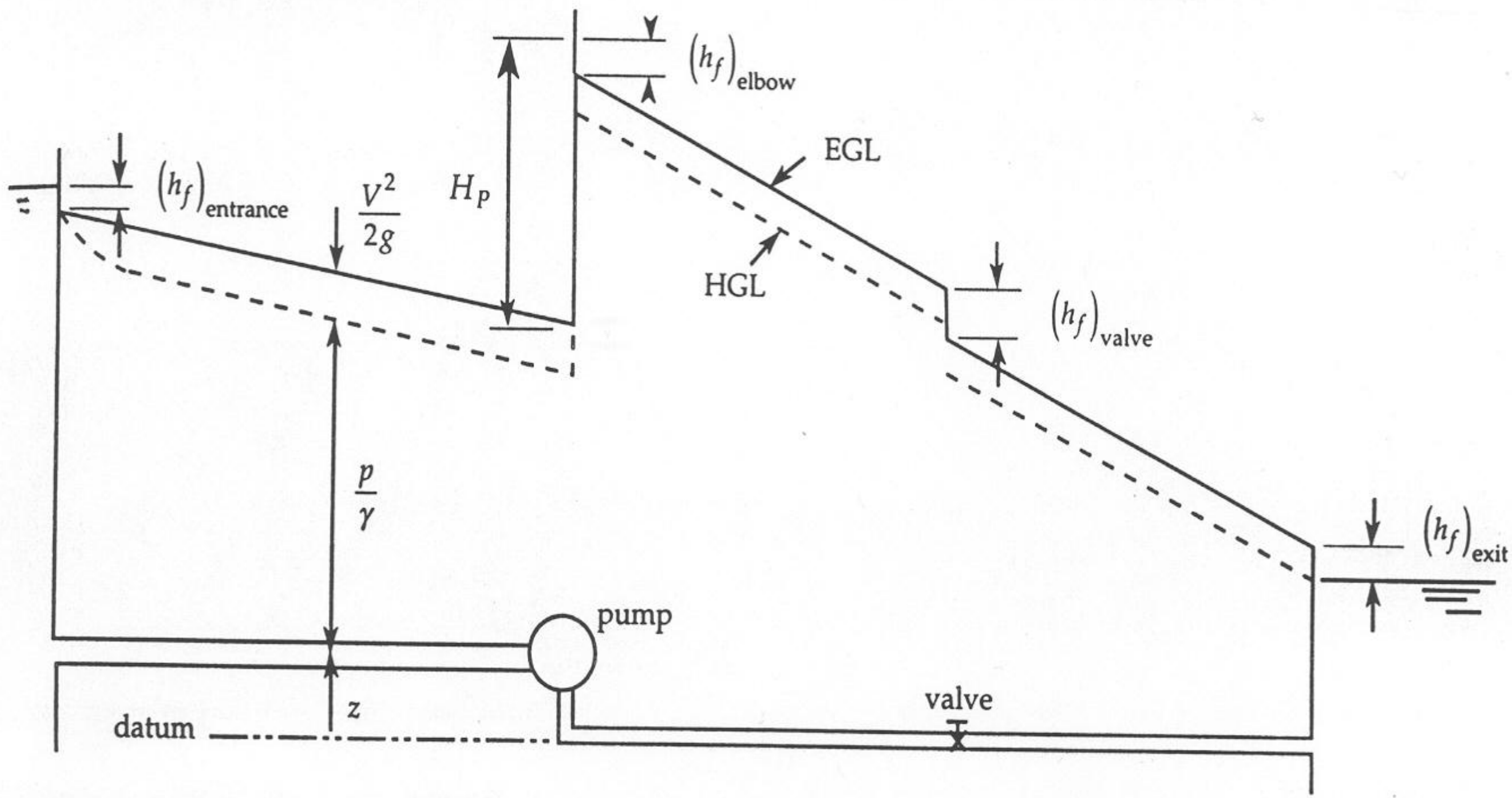
HGL: line connecting

$$\frac{p}{\gamma}+z$$

(piezometric head line)



Example EGL & HGL



The *drag force* F_D on **objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid** is

$$F_D = \frac{C_D \rho v^2 A}{2}, \text{ where}$$

C_D = the *drag coefficient*,

v = the velocity (m/s) of the flowing fluid or moving object, and

A = the *projected area* (m^2) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

For **flat plates placed parallel with the flow**

$C_D = 1.33/\mathrm{Re}^{0.5}$ $(10^4 < \mathrm{Re} < 5 \times 10^5)$

$C_D = 0.031/\mathrm{Re}^{1/7}$ $(10^6 < \mathrm{Re} < 10^9)$

The characteristic length in the Reynolds Number (Re) is the length of the plate parallel with the flow. For blunt objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) which is perpendicular to the flow.

67. The drag coefficient for a car with a frontal area of 28 ft^2 is 0.32. Assuming the density of air to be 2.4×10^{-3} slugs/ft^3, the drag force (lb) on this car when driven at 60 mph against a head wind of 20 mph is most nearly

- ○ **A.** 37
- ○ **B.** 83
- ○ **C.** 148
- ○ **D.** 185

(page 64)

Hint: Drag force = $\frac{C_D \rho A U^2}{2}$ **where, C_D is the coefficient of drag; ρ is the density of air; A is the frontal area; and U is the relative velocity.**

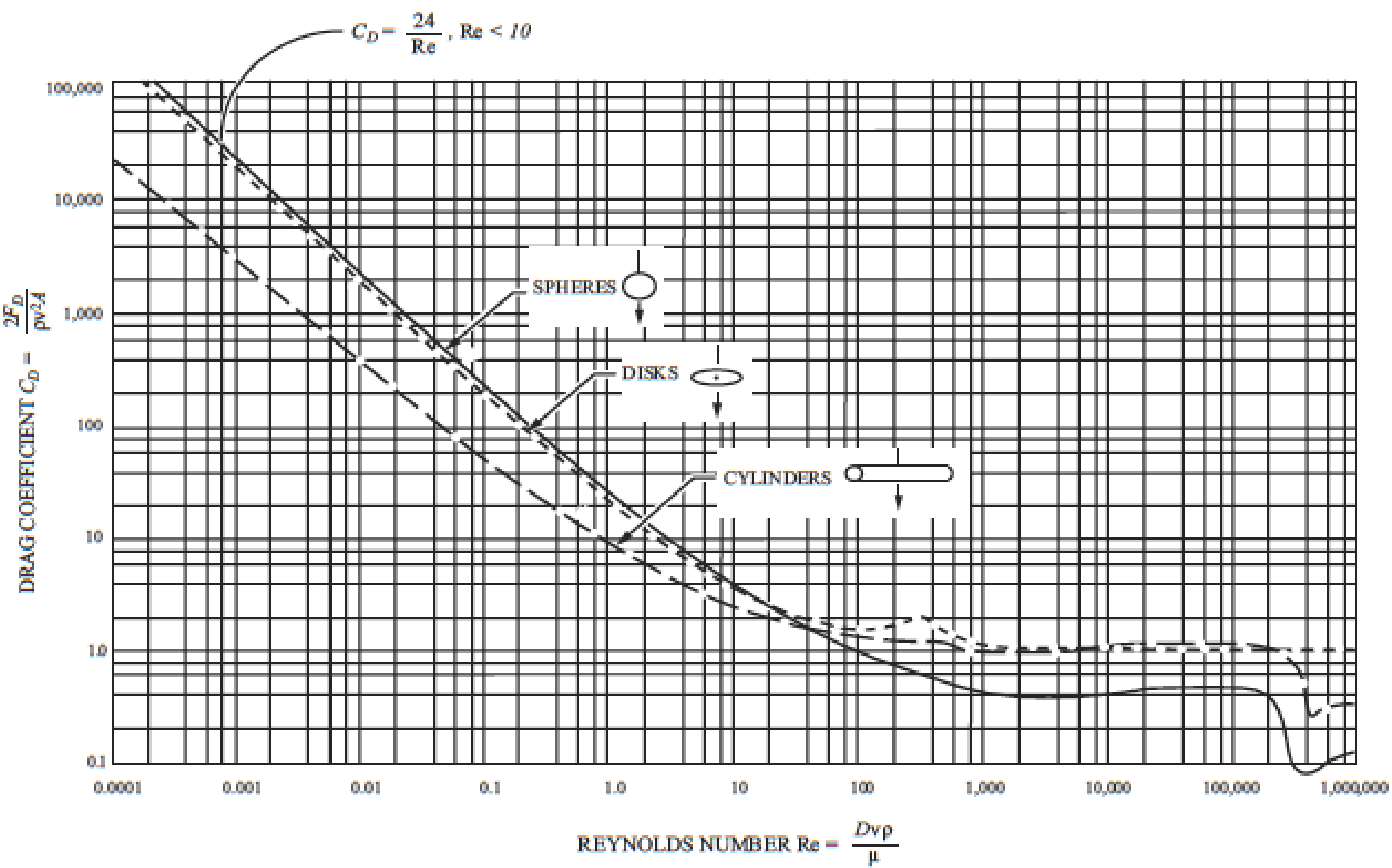
Solution: Relative v ... **ft/s**

Hence drag force= $\frac{(0.32)(2.4\times10^{-3})(27\ ft^2)(117.3\ ft/s)^2}{2}$ **= 148 lb.**

Therefore, the key is (C).

60+20 mph=80mph=117.3 ft/s

DRAG COEFFICIENT FOR SPHERES, DISKS, AND CYLINDERS



Note: Intermediate divisions are 2, 4, 6, and 8

STEADY, INCOMPRESSIBLE FLOW IN CONDUITS AND PIPES

The energy equation for incompressible flow is

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f \text{ or}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f$$

h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $v_1 = v_2$.

The pressure drop $p_1 - p_2$ is given by the following:

$$p_1 - p_2 = \gamma h_f = \rho g h_f$$

The *Darcy-Weisbach equation* is

$$h_f = f\frac{L}{D}\frac{v^2}{2g}, \text{ where}$$

f = $f(\text{Re}, e/D)$, the Moody or Darcy friction factor,

D = diameter of the pipe,

L = length over which the pressure drop occurs,

e = roughness factor for the pipe, and all other symbols are defined as before.

An alternative formulation employed by chemical engineers is

$$h_f = \left(4 f_{\text{Fanning}}\right)\frac{Lv^2}{D2g} = \frac{2 f_{\text{Fanning}} Lv^2}{Dg}$$

$$\text{Fanning friction factor, } f_{\text{Fanning}} = \frac{f}{4}$$

A chart that gives f versus Re for various values of e/D, known as a *Moody* or *Stanton diagram*, is available at the end of this section.

Friction Factor for Laminar Flow

The equation for Q in terms of the pressure drop Δp_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region.

$$Q = \frac{\pi R^4 \Delta p_f}{8\mu L} = \frac{\pi D^4 \Delta p_f}{128\mu L}$$

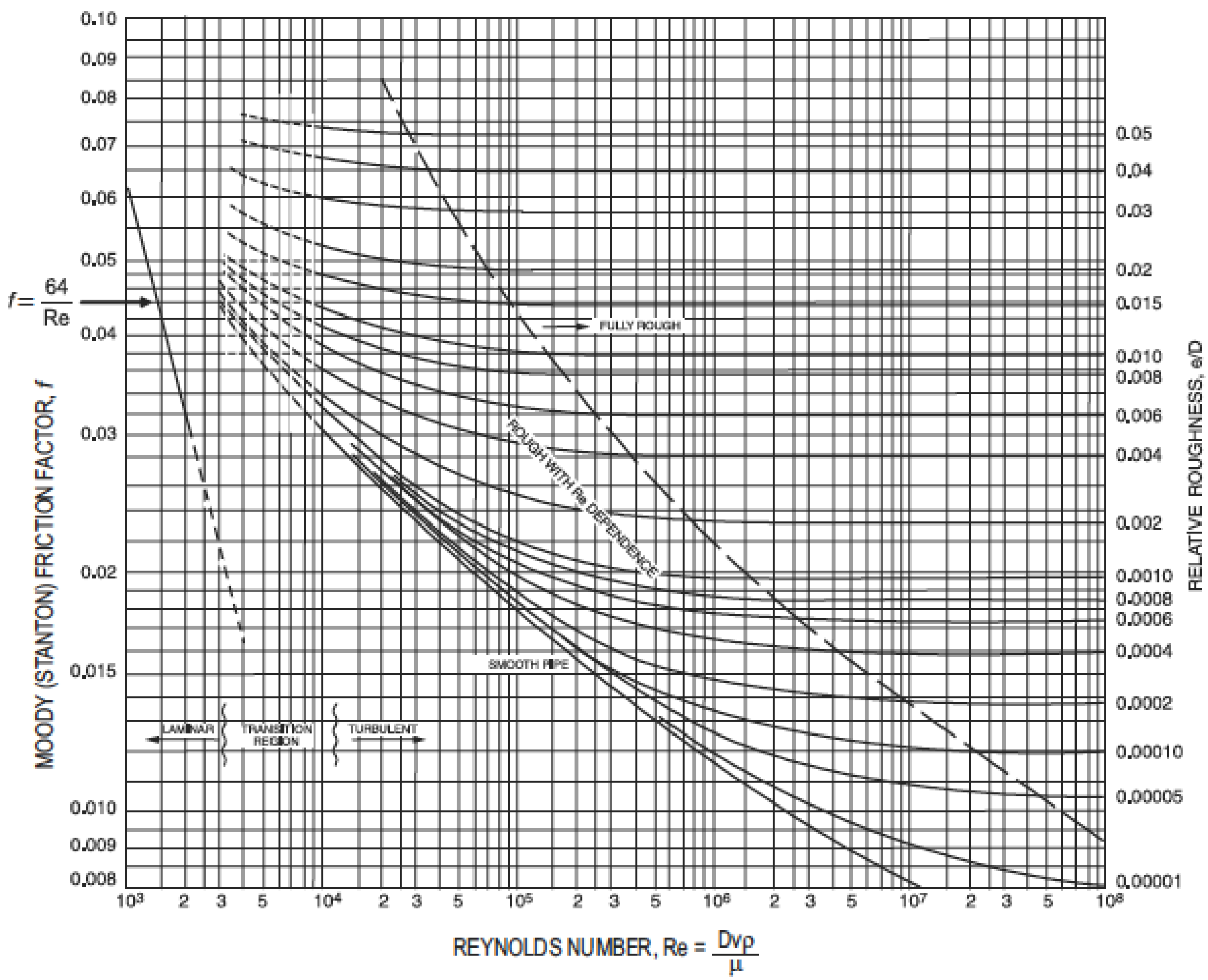
Flow in Noncircular Conduits

Analysis of flow in conduits having a noncircular cross section uses the *hydraulic radius* R_H, or the *hydraulic diameter* D_H, as follows

$$R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4}$$

MOODY (STANTON) DIAGRAM

Material	e (ft)	e (mm)
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015



Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{ fitting}}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{ fitting}}, \text{ where}$$

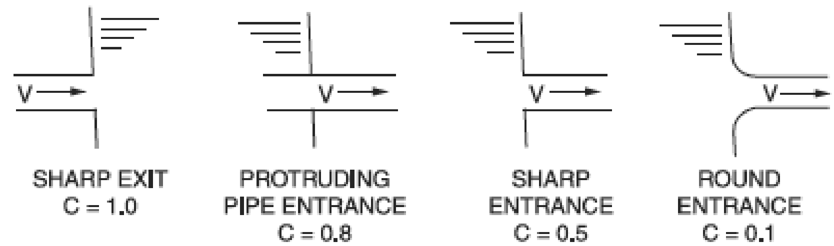
$$h_{f,\text{ fitting}} = C\frac{v^2}{2g}, \text{ and } \frac{v^2}{2g} = 1 \text{ velocity head}$$

Specific fittings have characteristic values of C, which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\text{ fitting}} = 0.04\, v^2/2g$$

The *head loss* at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f,\text{ fitting}}$ equation. Values for C for various cases are shown as follows.

◆



PUMP POWER EQUATION

$$\dot{W} = Q\gamma h/\eta = Q\rho g h/\eta, \text{ where}$$

Q = volumetric flow (m^3/s or cfs),

h = head (m or ft) the fluid has to be lifted,

η = efficiency, and

$\dot{W}$ = power (watts or ft-lbf/sec).

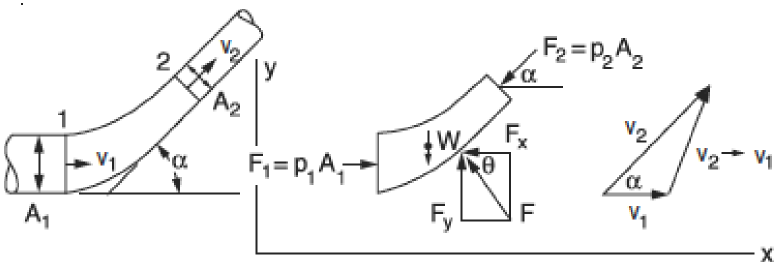
Turbine :

$$\dot{W} = Q\gamma h\eta$$

Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipe line may be computed using the impulse-momentum principle.

•



$$p_1A_1 - p_2A_2\cos\alpha - \boldsymbol{F}_x = Q\rho\,(v_2\cos\alpha - v_1)$$

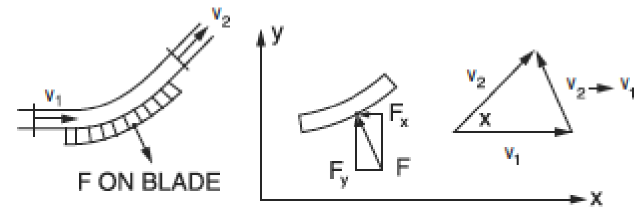
$$\boldsymbol{F}_y - W - p_2A_2\sin\alpha = Q\rho\,(v_2\sin\alpha - 0),\text{ where}$$

$\boldsymbol{F}$ = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign), $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are the x-component and y-component of the force,

Deflectors and Blades

Fixed Blade

•

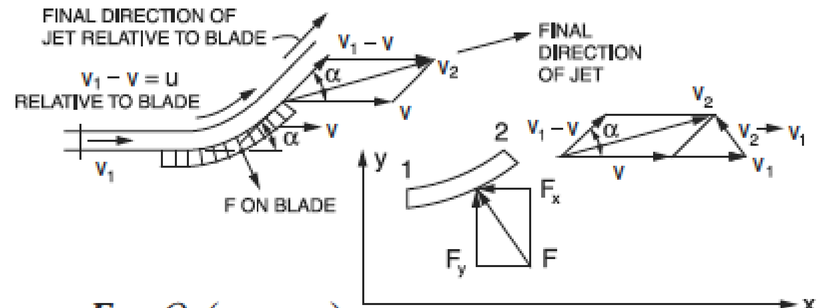


$$-\boldsymbol{F}_x = Q\rho(v_2\cos\alpha - v_1)$$

$$\boldsymbol{F}_y = Q\rho(v_2\sin\alpha - 0)$$

Moving Blade

•



$$-\boldsymbol{F}_x = Q\rho(v_{2x} - v_{1x})$$

$$= -\,Q\rho(v_1 - v)(1 - \cos\alpha)$$

$$\boldsymbol{F}_y = Q\rho(v_{2y} - v_{1y})$$

$$= +\,Q\rho(v_1 - v)\sin\alpha,\text{ where}$$

v = the velocity of the blade.

THE IMPULSE-MOMENTUM PRINCIPLE

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

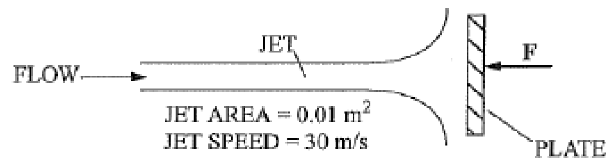
$$\Sigma F = Q_2\rho_2 v_2 - Q_1\rho_1 v_1, \text{ where}$$

ΣF = the resultant of all external forces acting on the control volume,

$Q_1\rho_1 v_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and

$Q_2\rho_2 v_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.

A horizontal jet of water (density = 1,000 kg/m^3) is deflected perpendicularly to the original jet stream by a plate as shown below.



The magnitude of force **F** (kN) required to hold the plate in place is most nearly:

(A) 4.5
(B) 9.0
(C) 45.0
(D) 90.0

$$Q = A_1V_1 = (0.01\ \text{m}^2)(30\ \text{m/s})$$

$$= 0.3\ \text{m}^3/\text{s}$$

Since the water jet is deflected perpendicularly, the force F must deflect the total horizontal momentum of the water.

$$F = \rho QV = (1{,}000\ \text{kg/m}^3)\ (0.3\ \text{m}^3/\text{s})\ (30\ \text{m/s}) = 9{,}000\ \text{N} = 9.0\ \text{kN}$$

THE CORRECT ANSWER IS: (B)

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve n variables is equal to the number $(n - \bar{r})$, where $\bar{r}$ is the number of basic dimensions (i.e., M, L, T) needed to express the variables dimensionally.

- Dimensional equation:

$$D = f(d, V, \rho, \mu)$$

- Buckingham's Pi Theorem:

$$D \doteq MLT^{-1},\ d \doteq L,\ V \doteq LT^{-1},\ \rho \doteq ML^{-3},\ \mu \doteq ML^{-1}T^{-1}$$

Thus,

$n = 5\ (D, d, V, \rho, \mu)$

$r = 3\ (M, L, T)$

$\therefore k = n - r = 2$ Pi parameters

$$\Pi_1 = \frac{D}{\rho U^2 D^2} \left(\text{or } \frac{D}{\frac{1}{2}\rho U^2 A}\right) = C_D$$

$$\Pi_2 = \frac{\rho U D}{\mu} = Re$$

Dimensionless parameters

SIMILITUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically*, *kinematically*, and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_p}\right]_p = \left[\frac{F_I}{F_p}\right]_m = \left[\frac{\rho v^2}{p}\right]_p = \left[\frac{\rho v^2}{p}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{v l \rho}{\mu}\right]_p = \left[\frac{v l \rho}{\mu}\right]_m = [\mathrm{Re}]_p = [\mathrm{Re}]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{v^2}{lg}\right]_p = \left[\frac{v^2}{lg}\right]_m = [\mathrm{Fr}]_p = [\mathrm{Fr}]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho v^2}{E_v}\right]_p = \left[\frac{\rho v^2}{E_v}\right]_m = [\mathrm{Ca}]_p = [\mathrm{Ca}]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l v^2}{\sigma}\right]_p = \left[\frac{\rho l v^2}{\sigma}\right]_m = [\mathrm{We}]_p = [\mathrm{We}]_m$$

F_I = inertia force,
F_P = pressure force,
F_V = viscous force,
F_G = gravity force,
F_E = elastic force,
F_T = surface tension force,
Re = Reynolds number,
We = Weber number,
Ca = Cauchy number,
Fr = Froude number,
l = characteristic length,
v = velocity,
ρ = density,
σ = surface tension,
E_v = bulk modulus,
μ = dynamic viscosity,
p = pressure, and
g = acceleration of gravity.

Example

If a flow rate of $0.2m^3/s$ is measured over a 9 to1 scale model of a weir, what flow rate can be expected on the prototype?

Flow over a weir is an openchannel flow. Use Froude number for modeling.

$$\frac{v_m^2}{l_m g} = \frac{v_p^2}{l_p g} \Rightarrow \frac{v_p}{v_m} = \sqrt{\frac{l_p}{l_m}} = 3 \Rightarrow \frac{Q_p}{Q_m} = \frac{v_p l_p^2}{v_m l_m^2} = 243$$

If the model force is at 1000N, what will be the force on the prototype?

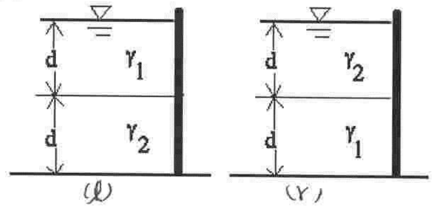
$$\left(\frac{F}{\rho v^2 l^2}\right)_m = \left(\frac{F}{\rho v^2 l^2}\right)_p \Rightarrow \frac{F_p}{F_m} = \frac{(\rho v^2 l^2)_p}{(\rho v^2 l^2)_m} = 3^2 9^2 = 729$$

(use drag force)

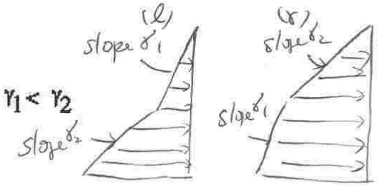
Part A

1. In the following two cases, which case has higher total force acting on the side wall? (The two fluids are separated somehow, so that the heavier fluid can remain stably on top in the right side case.)

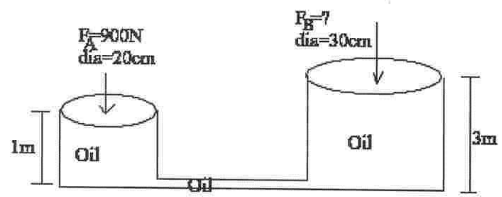
   (a) right side case (b) left side case (c) the same in both cases.



$\gamma_1 < \gamma_2$



2. What is the force at B if the tanks contain stationary oil at SG=0.7?



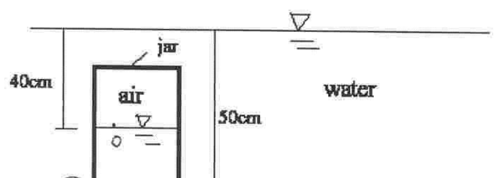
$P_A - \gamma_{oil} \times 2 = P_B$

$\frac{F_A}{A_A} - \gamma_{oil} \times 2 = \frac{F_B}{A_B}$

$\frac{900}{\frac{\pi}{4}(0.2)^2} - 0.7 \times 9800 \times 2 = \frac{F_B}{\frac{\pi}{4}(0.3)^2}$

(a) 2025 N (b) 1050N (c) 668 N (d) 580 N

3. What is the gage pressure in the inverted jar?

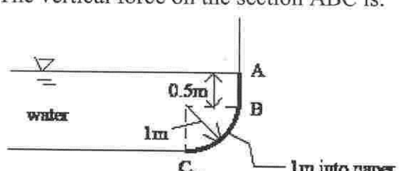


$P_o - \gamma_{H_2O}(0.4) = P_{air} = 0$

$P_o = 9800 \times 0.4$

(a) 3920Pa (b) 980Pa (c) 4900Pa (d) It is a negative pressure

4. The vertical force on the section ABC is:



0.5
1m

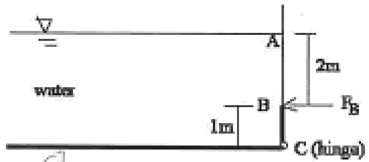
$F_R = \gamma h_c A = 9800 \times (\frac{1.5}{2}) \times 1^2 = 9800 \times 0.75$

$F_V = \gamma \forall_{above} = 9800(0.5 \times 1 \times 1 + \frac{\pi}{4} 1^2 \times 1)$

(a) 9.8kN (b) 12.6kN (c) 14.7kN (d) 4.9kN (e) 7.7kN

5. Find the horizontal force on section BC of the gate shown in the previous problem.
(a) 14.7kN (b) 12.6kN (c) 11kN (d) 7.35kN (e) 6.3kN

6. What is the force F_B per 1m into paper required to hold the gate (1m into paper)?



$F_R = 9800 \times 2.5 \times 1^2$

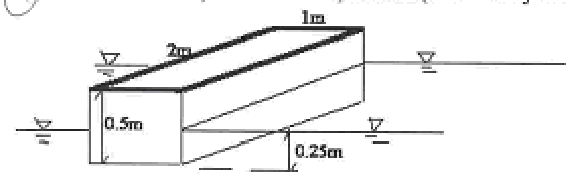
$y_R = y_c + \frac{I_{xc}}{y_c A} = 2.5 + \frac{\frac{1}{12} 1^3 \times 1}{2.5 \times 1^2}$

$F_R (3 - y_R) = F \times 1$

(a) 11.5kN (b) 24.5kN (c) 2.3kN

7. A block-shape canoe has a 0.25m draft as shown when empty. If a person of 150kg mass is to sit inside, will the canoe

a) float b) sink c) neutral (water will just reach the brim)?



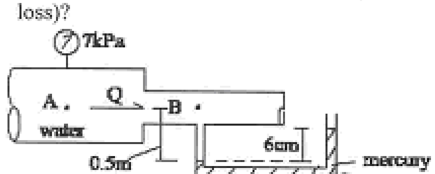
$W_T = W_{canoe} + 150 \times 9.8$

$W_{canoe} = 9800 \times 0.25 \times 1 \times 2$

$\therefore W_T = 6370$

Max buoyancy $= 9800 \times 0.5 \times 2 \times 1$
$= 9800 > 6370$

8. The velocity at A is $1 m/s$. What is v_B if the flow is incompressible and frictionless (no energy loss)?

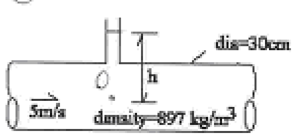


$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g}$ $P_B + \gamma_{H_2O} 0.5 - \gamma_{Hg} (0.06) = 0$

$\frac{-7000}{9800} + \frac{1}{2 \times 9.8} = \frac{\gamma_{Hg} (0.06) - \gamma_{H_2O} 0.5}{\gamma_{H_2O}} + \frac{V_B^2}{2 \times 9.8}$

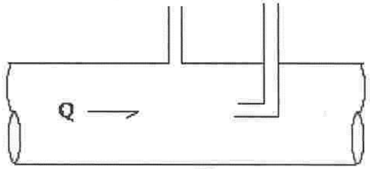
(a) $1.5 m/s$ (b) $11.6 m/s$ (c) $2.97 m/s$ (d) $2.03 m/s$

9. The pipe centerline pressure just below the manometer is 19kPa (gage). How high will the liquid rise in the manometer tube?
(a) $2.16 m$ (b) $1.94 m$ (c) $3.44 m$ (d) $3.22 m$ (e) none of the above



$\frac{P_0}{\gamma} = \frac{19000}{897 \times 9.8}$

10. A Pitot/static tube inserted in a water flow as shown reads a static pressure 50mm(Hg) and a stagnation pressure 55mm(Hg). What is the water velocity?

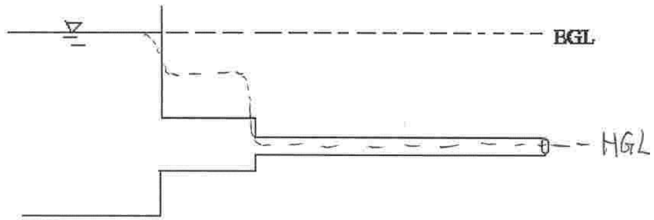


$$\frac{P_{stagnation}}{\gamma} = \frac{P_{static}}{\gamma} + \frac{V^2}{2g}$$

$$\frac{0.055\times13.6\times9800}{9800} = \frac{0.05\times13.6\times9800}{9800} + \frac{V^2}{2g}$$

(a) 5*mm/s* (b) 1.15*m/s* (c) 0.68*m/s* (d) none of the above

11. Assume no head loss, hence the EGL is horizontal. Sketch HGL along the pipeline.



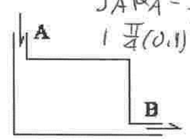
HGL must go through the pipe exit.

12. Calculate the density at B if the flow is steady state.

At A, diameter=10cm At B, diameter=18cm
Velocity=15m/s velocity=6m/s
Density=1kg/m^3 density=_________(kg/m^3).

$$\rho_A Q_A = \rho_B Q_B$$

$$1\ \frac{\pi}{4}(0.1)^2(0.15) = \rho_B \frac{\pi}{4}(0.18)^2(0.06)$$



(a) 1.3 (b) 0.4 (c) 2.5 (d) 0.77

13. Calculate the frictional head loss per 10m of 30cm-diameter concrete pipe (ε=0.5mm). The fluid is standard air at 15°C and velocity 4m/s.

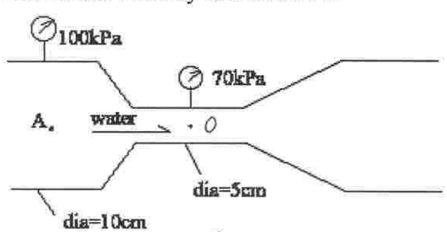
(a) 0.4*m* (b) 0.98*m* (c) 0.67*m* (d) 0.5*m*

$$Re = \frac{4\times0.3}{1.46\times10^{-5}} = 0.82\times10^5 \quad \frac{\varepsilon}{D} = 0.0017$$

$$f = 0.024$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = 0.024\frac{10}{0.3}\frac{4^2}{2\times9.8}$$

14. A venturi meter is used to measure flow velocity. Given the manometer reading as shown below, what is the velocity at section A?



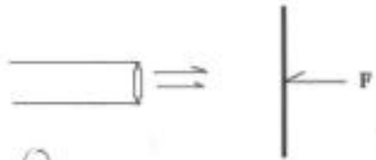
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} = \frac{P_O}{\gamma} + \frac{V_O^2}{2g}$$

$V_O = 4V_A$ (continuity eq.)

$$\frac{100\times10^3}{9800} + \frac{V_A^2}{2\times9.8} = \frac{70\times10^3}{9800} + \frac{16V_A^2}{2\times9.8}$$

(a) 10*m/s* (b) 2*m/s* (c) 8*m/s* (d) 1*m/s* (e) none of the above

15. A horizontal nozzle sends out a water jet at 10m/s towards the vertical plate as shown. If the nozzle diameter is 1cm, find the force F required to hold the plate stationary.

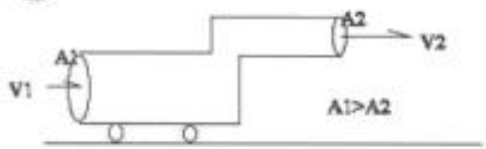


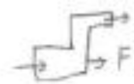
$\rho Q V = 1000 \times \frac{\pi}{4}(0.01)^2 10 \times 10$

(a) 7.85N (b)8kN (c) 78.5N (d) 0.8N

16. The cart is originally locked. Incompressible airflow passes through the fixture as shown. Which way will the cart go if the wheels are released?

a) to the left b) to the right c) motionless





$\vec{F} = \rho Q \vec{V}_2 - \rho Q \vec{V}_1$

$F = \rho Q V_2 - \rho Q V_1$

$V_2 > V_1 \quad \therefore F = +$

$\therefore$ F on air is →

F on cart is ←

17. Ignore the mass and friction of the sprinklers, which one spins faster? The sprinkler nozzle diameters are identical, the velocity of the jet are also identical.

a) the one on left b) the one on right c) the same.

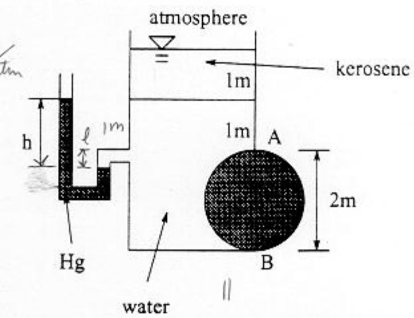
$\vec{T}_{orque} = (\rho Q \vec{r} \times \vec{v})_{out}$

$= \rho Q (1 \times V + 2 \times V)\hat{e}$

for both cases

Part B. (5 pts each)

1. Determine
   (a) the manometer reading h.
   (b) total pressure force on the cylindrical plug AB. The plug has a length of $2m$ into the paper.
   (c) the point of application of the total pressure force on AB relative to B.



a) $\cancel{p_{atm}} + \rho_K g 1 + \rho_{H_2O} g 1 + \rho_{H_2O} g l - \rho_{Hg} g h = \cancel{p_{atm}}$

$$h = \frac{\rho_K g 1 + \rho_{H_2O} g (1+l)}{\rho_{Hg} g}$$

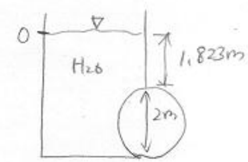
$$= \frac{0.823}{13.6} + \frac{1+l}{13.6}$$

b) $F_h = p_c A_v = [\rho_K g 1 + \rho_{H_2O} g (2)]\, 2 \times 2$

$= [0.823 + 2](1000)(9.8) \times 2 \times 2$

$= 110662 (N)$

$F_v = \rho g_{H_2O} \forall_{sub} = 9800(\frac{1}{2}\pi \times 2) = 30772 (N)$

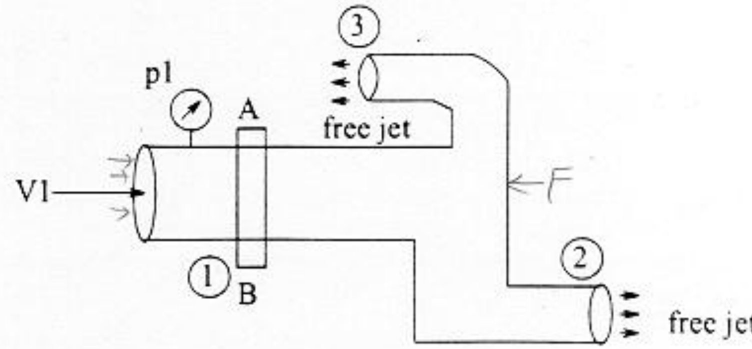


c) $z_r = z_c + \frac{I_{xc}}{z_c A_v} = z_c + \frac{\frac{1}{12} 2^3 \times 2}{z_c \times 2 \times 2} = 2.823 + \frac{\frac{1}{12} 2^3 \times 2}{2.823 \times 4} = 2.94 (m)$ below 0

2. Water flows at a steady rate through the horizontal device shown. The following data apply: p_1 = 120kPa (gauge), V_1 = 3 m/s, D_1 = 0.5 m, V_2 = 6 m/s, D_2 = 0.2 m, D_3 = 0.1 m.

(a) What is V_3 ?

(b) What is the horizontal thrust on the flange AB ?



a) $V_1 \frac{\pi}{4} D_1^2 = V_2 \frac{\pi}{4} D_2^2 + V_3 \frac{\pi}{4} D_3^2$

$3(0.5)^2 = 6(0.2)^2 + V_3(0.1)^2$

$V_3 = 51$ m/s

b) $\rho Q_3(-V_3) + \rho Q_2(V_2) - \rho Q_1(V_1) = -F + P_1 \frac{\pi}{4} D_1^2$ $\rightarrow x$

$(1000)(51)\frac{\pi}{4}(0.1)^2(-51) + (1000)\, 6\, \frac{\pi}{4}(0.2)^2(6) - (1000)\, 3\, \frac{\pi}{4}(0.5)^2(3) = -F + 120000\frac{\pi}{4}(0.5)^2$

$-20417.85 + 1130.4 + 1766.25 = -F + 23550$

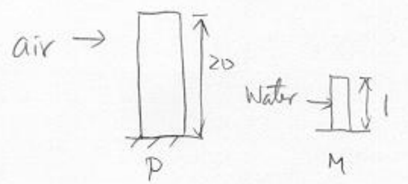
$F = 41071$ (N)

3. We wish to determine the wind force on a water tower when a wind normal to the centerline of the water tower is 60 km/h. To do this we examine in a water tunnel a geometrically similar model reduced by $\frac{1}{20}$ scale.

(a) What should the water tunnel velocity be if Reynolds number is used for dynamic similarity?

(b) If the force on the model is measured at 100 N. what is the projected force on the prototype ?

(c) What is the expected ratio of torque about the base of the tower ? i.e. prototype torque/model torque ?



a) $$\left.\frac{\rho V D}{\mu}\right|_P = \left.\frac{\rho V D}{\mu}\right|_M$$

$$\frac{1.19 \times 60\,\mathrm{km/h} \times 20}{18 \times 10^{-6}} = \frac{1000 \times V \times 1}{0.89 \times 10^{-3}}$$

$$V = 70.6\ \mathrm{km/h} = 19.61\ \mathrm{m/sec}$$

b) $$\left.\frac{F}{\rho V^2 L^2}\right|_P = \left.\frac{F}{\rho V^2 L^2}\right|_M$$

$$F = \rho V Q = \rho V^2 A = \rho V^2 \ell^2$$

$$\frac{F_P}{1.19 \times 60^2 \times 20^2} = \frac{100}{1000 \times 70.6^2 \times 1^2}$$

$$F_P = 34.4\ N$$

c) $$\frac{F_P\, 20}{F_m\, 1} = \frac{34.4 \times 20}{100 \times 1} = 6.88$$

$$\tau = F\ell$$

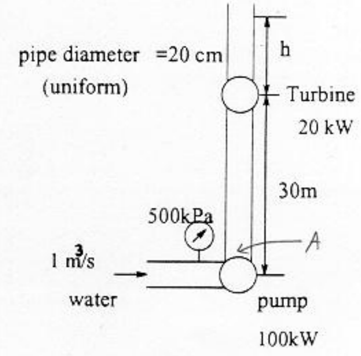
4. A pump delivers 100 kW of power on a vertical flow of water for a skyscraper. The pipe is made of commerical steel. At 30 m above the pump, a turbine draws off 20 kW of power. The volume flow rate is 1 m^3/s. Take $\rho_{H_2O} = 1000\ kg/m^3$ and $\nu_{H_2O} = 10^{-6} m^2/s$ (Note: this is the kinematic viscosity and $\mu = \rho\nu$.)

(a) What is the Reynolds number of the flow?

(b) What is the friction factor?

(c) What is the head of water right after the pump ? (Ignore the size of the pump.)

(d) How high h is when the gauge pressure in the flow drops to 10^4 Pa ?



a) $Re = \frac{VD}{\nu}$ $\quad V = \frac{1}{\frac{\pi}{4}(0.2)^2} = 31.8\ m/s$

$= \frac{\frac{1}{\frac{\pi}{4}(0.2)^2}\ 0.2}{10^{-6}} = 6.37\times10^6$

b) $\frac{\varepsilon}{D} = \frac{0.046}{200} = 2.3\times10^{-4}$

$f \cong 0.014$

c) $h = \frac{p}{\rho g} + \frac{V^2}{2g} + z$

$$\dot{W} = Q\gamma h \Rightarrow h = \frac{\dot{W}}{Q\gamma}$$

$h_{before} - h_{after} = -\frac{100\times10^3}{\rho g Q}$

$h_{before} = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_{before} = \frac{500\times10^3}{10^3\times9.8} + \frac{31.8^2}{2\times9.8} + 0 = 102.61$

$$h_{after} = h_{\text{before}} + h_{\text{Pump}}$$

$h_{after} = \frac{100\times10^3}{10^3\times9.8\times1} + 102.61 = 112.82(m) = h_A$

$$h_{after} = h_{\text{point}} + h_{L,\text{friction}} + h_{\text{Turbine}}$$

d) @ h height

$h_h = \frac{10^4}{\rho g} + \frac{31.8^2}{2g} + h + 30$

$h_A - h_h = f\frac{L}{D}\frac{V^2}{2g} + \frac{20\times10^3}{\rho g Q} = 0.014\ \frac{30+h}{0.2}\ \frac{31.8^2}{2\times9.8} + \frac{20\times10^3}{10^3\times9.8\times1}$

$112.82 - \left(\frac{10^4}{10^3\times9.8} + \frac{31.8^2}{2\times9.8} + h + 30\right) = 0.014\ \frac{30+h}{0.2}\ \frac{31.8^2}{2\times9.8} + \frac{20\times10^3}{10^3\times9.8\times1}$

$30.2 - h = 3.61\ (30+h) + 2.04 \qquad h = -17.4\ m$

Hence water pressure $< 10^4$ Pa before the turbine

15. A river has a continuous water flow of 10 m^3/s between two bridges that are 1000 m apart. At bridge A, upstream, the river has a cross-sectional area of 150 m^2, while at bridge B, downstream, the river has a cross-sectional area of 100 m^2. The increase in water velocity between the two bridges is most nearly

✓(A) 0.033 m/s
(B) 0.067 m/s
(C) 0.075 m/s
(D) 0.130 m/s

$$V_A - V_B = \frac{Q_A}{A_A} - \frac{Q_B}{A_B} = \frac{10}{150} - \frac{10}{100} = -0.033$$

9. Water flows at 20°C through 10 m of 8 mm inside diameter smooth glass pipe at 2.0 m/s. The friction factor for glass is 0.0180. The head loss caused by friction is most nearly

Page 65: $h_f = f \frac{v^2}{2g}\frac{L}{D} = 0.018 \frac{2^2}{2 \times 9.81} \times \frac{10}{\frac{8}{1000}}$

$= 4.59$ m

10. A centrifugal pump lifts groundwater 100 m vertically to a surface storage tank at a rate of 0.25 m^3/s. The pump has a 75% efficiency. The power required to drive this pump is most nearly

✗(A) 330 kW
(B) 350 kW
(C) 480 kW
(D) 500 kW

Page 66: $\dot{w} = \frac{Q \gamma h}{\eta} = \frac{0.25 \times 9810 \times 100}{0.75}$

$= 327$ kW

13. A capillary tube 3.8 mm in diameter is placed in a beaker of 40°C distilled water. The surface tension is 0.0696 N/m, and the angle made by the water with the wetted tube wall is negligible. The specific weight of water at this temperature is 9.730 kN/m^3. The height to which the water will rise in the tube is most nearly

(A) 1.2 mm
(B) 3.6 mm
✗(C) 7.5 mm
(D) 9.2 mm

Page 62: $h = \frac{4\sigma \cos\beta}{\gamma d} = \frac{4 \times 0.0696 \times \cos 0^\circ}{9730 \times \frac{3.8}{1000}}$

$= 0.0075$ m

$= 7.5$ mm

9. A reservoir with a water surface at an elevation of 200 m drains through a 1 m inside diameter pipe with the outlet at an elevation of 180 m. The pipe outlet empties to atmospheric pressure. The total head losses in the pipe and fittings are 18 m. Assume a steady, incompressible flow of 4.92 m^3/s.

A turbine is installed at the pipe outlet. The chosen turbine has an efficiency of 85% and does not add any head loss to the system. The expected power output of the turbine is most nearly

✓ (A) 82 kW
(B) 96 kW
(C) 100 kW
(D) 120 kW

page 65:

$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_{turbine} + h_f$

($P_1 = 0$, $z_1 = 200$ m, $V_1 = 0$, $P_2 = 0$, $z_2 = 180$ m, $V_2 = 0$, $h_f = 18$ m)

$h_{turbine} = 2$ m

$Q\gamma h \eta = 4.92 \times 9810 \times 2 \times 0.85 = 82051$ W

Problems 10 and 11 are based on the following information.

A circular sewer with a 1.5 m inside diameter is designed for a flow rate of 15 m^3/s when flowing full. Assume that the Manning roughness coefficient and Darcy friction factor are constant with depth of flow.

10. The flow rate when the depth of flow is 0.50 m is most nearly

(A) 1.7 m^3/s
✓(B) 3.4 m^3/s
(C) 5.0 m^3/s
(D) 7.5 m^3/s

Page 160: $\frac{d}{D} = \frac{0.5}{1.5} = \frac{1}{3}$

$\rightarrow \frac{Q}{Q_{full}} = 0.23 \rightarrow Q = 0.23 \times 15 = 3.45$

11. The velocity of flow when the depth of flow is 0.50 m is most nearly

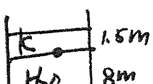
(A) 2.8 m/s
(B) 4.2 m/s
(C) 4.8 m/s
✓(D) 6.6 m/s

Page 160:

$\frac{V}{V_f} = 0.8 \rightarrow V = 0.8 \times \frac{15}{\frac{\pi}{4}(1.5)^2} = 6.8$

29. An open tank contains 8.0 m of water beneath 1.5 m of kerosene. Kerosene has a specific weight of 8.0 kN/m^3. The pressure at the kerosene/water interface is most nearly

(A) 3.5 kPa
(B) 5.0 kPa
(C) 8.0 kPa
✗ (D) 12 kPa



$P = \gamma_k h = 8000 \times 1.5$ m $= 12000$ Pa $= 12$ kPa

14. Water flows through a 30.0 cm inside diameter pipe at an initial velocity of 1.9 m/min. The pipe diameter subsequently reduces to 15.0 cm before discharging into an open channel. The discharge velocity is most nearly

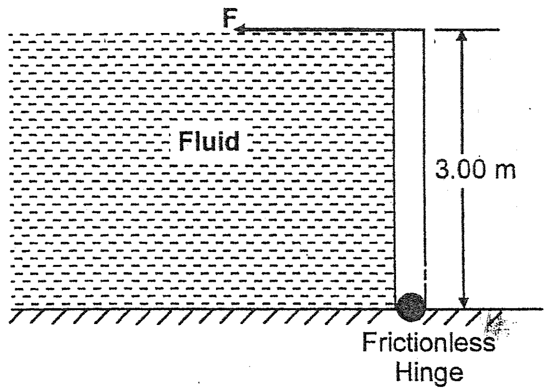
(A) 3.8 m/min
✗(B) 7.5 m/min
(C) 8.6 m/min
(D) 9.3 m/min

Page: 63 $A_1V_1 = A_2V_2$

$\frac{\pi}{4}(30)^2 \times 1.9 = \frac{\pi}{4}(15)^2 \times V_2$

$\rightarrow V_2 = 7.6$ m/min

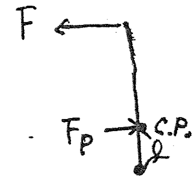
47.



The rectangular homogeneous gate shown above is 3.00 meters high and has a frictionless hinge at the bottom. If the fluid on the left side of the gate has a mass of 1,600 kilograms per cubic meter, the magnitude of the force **F** required per meter of width to keep the gate closed is most nearly

(A) 0 kN/m
(B) 22 kN/m
(C) 24 kN/m
(D) 220 kN/m

Concept:



$F \times 3 = F_p \times \ell$

(1) Page 63: $P_c = \gamma z_c \sin\alpha$

$= 1600 \times 9.81 \times 1.5 \times \sin 90^\circ$

$= 23544\ N/m^2$

$F_p = P_c A = 23544 \times 3 \times 1$

$= 70632\ N$

$z^* = \dfrac{\gamma I_{yc} \sin\alpha}{P_c A}$ (see Page 51 for I_{yc})

$= \dfrac{1600 \times 9.81 \times \frac{1 \times 3^3}{12} \times \sin 90}{70632}$

$= 0.5\ m$

$\ell = 1.5 - 0.5 = 1\ m$

$F \times 3 = 70632 \times 1 \rightarrow F = 23544$

$= 23.5\ kN$ A

48. Which of the following statements is true of viscosity?

(A) It is the ratio of inertial to viscous force.

(B) It always has a large effect on the value of the friction factor.

(C) It is the ratio of the shear stress to the rate of shear deformation.

(D) It is usually low when turbulent forces predominate.

Page 62: $\tau_t = \mu \dfrac{dV}{dy}$

$\rightarrow \mu = \dfrac{\tau_t}{\frac{dV}{dy}} = \dfrac{\text{shear stress}}{\text{rate of shear deformation}}$

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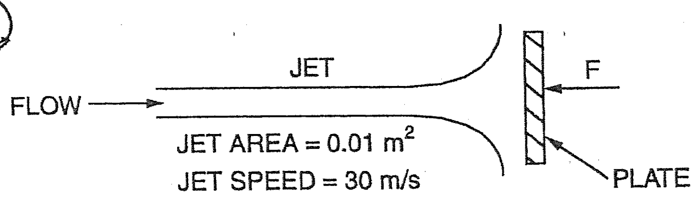
(49)



A horizontal jet of water (density = 1,000 kilograms per cubic meter) is deflected perpendicularly to the original jet stream by a plate with an area of 0.500 square meter as shown above. The magnitude of force **F** required to hold the plate is most nearly

(A) 4.5 kN
✓ (B) 8.8 kN
(C) 45.0 kN
(D) 88.0 kN

Page 66:

$-F_x = Q\rho(V_2 \cos\alpha - V_1)$

$= 30 \times 0.01 \times 1000 \times (0 - 30)$

$= -9000\ N = -9\ kN$ (on fluid)

$F = -F_x = 9\ kN$ on the plate.

(17) A concrete sanitary sewer is 400 feet long and 30 inches in diameter. It flows full without surcharge between a manhole (invert elevation 101.00) and a lift station (invert elevation 100.00). If the Manning roughness coefficient is 0.013 and is assumed to be constant with depth of flow, the capacity of the sewer is most nearly

(A) 4.2 cfs
(B) 9.8 cfs
✓(C) 20.5 cfs
(D) 32.6 cfs

101.00′ 400′ d = 30″ 100′

$R = \frac{A}{P} = \frac{\frac{1}{4}\pi D^2}{\pi D} = \frac{D}{4}$

$Q = VA = \frac{1.49}{n} R^{2/3} S^{1/2} A$ (Page 67)

$= \frac{1.49}{0.013}\left(\frac{30/12}{4}\right)^{2/3}\left(\frac{1}{400}\right)^{1/2}\pi\left(\frac{1}{2}\cdot\frac{30}{12}\right)^2 = 20.56$ cfs

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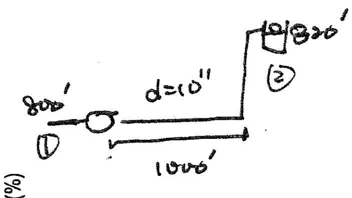
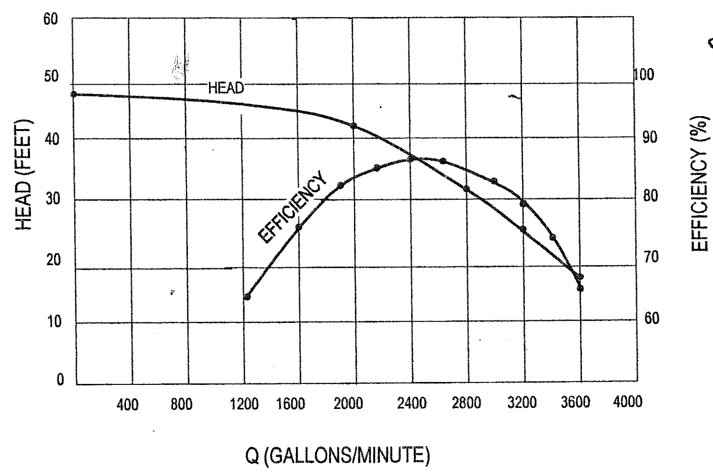
Civil Afternoon Questions

Questions 15–16

A water supply system draws water from a river at an elevation of 800 feet and delivers it to a holding reservoir at an elevation of 820 feet. The pipeline that delivers water to the reservoir is 1,000 feet long and is 10-inch-diameter cast iron. Minor losses and entrance/exit losses are negligible. A single pump is used. Pump characteristics are shown in the figure below.





15. If friction losses are calculated using the Darcy equation with a friction factor f = 0.02, the head loss in the 1,000-foot force main for a flow rate of 1,500 gpm is most nearly

(A) 4.15 feet
(B) 11.63 feet
✓(C) 13.96 feet
(D) 20.00 feet

Darcy equation $h_L = f\frac{L}{D}\frac{V^2}{2g}$ (Page 65)

$V = \frac{Q}{A} = \frac{1500\text{ gpm}}{\pi(\frac{1}{2}\frac{10}{12})^2 ft^2} = \frac{1500 \times 0.134 \frac{ft^3}{60\,s}}{\pi(\frac{1}{2}\frac{10}{12})^2 ft^2} = 6.142\ ft/s$

$h_L = 0.02 \times \frac{1000\ ft}{\frac{10}{12} ft}\frac{(6.142\ ft/s)^2}{2 \times 32.2\ ft/s^2} = 14\ ft$

(Page 20 for unit conversion)

16.

Pumping Rate, gpm	System Friction Loss, ft	Pump Head, ft
1,000 4.09	6.2 = 0.373V²	47
1,500	14.0	45
2,000	24.9	44
2,500	39.0	34
3,000	52.6	28

If friction losses are calculated using the Darcy equation with a friction factor f = 0.02, the pumping rate of the pump is most nearly

(A) 1,500 gpm
✓(B) 2,000 gpm
(C) 2,500 gpm
(D) 3,000 gpm

Deliver water to the reservoir:

$TDH = 20\ ft + 0.02 \times \frac{1000}{\frac{10}{12}}\frac{V^2}{2 \times 32.2}$ + system friction loss

$= 20 + 0.373V^2$

Page 65: $\frac{P_1}{\gamma_1} + z_1 + \frac{V_1^2}{2g} + h_{pump} = \frac{P_2}{\gamma_2} + z_2 + \frac{V_2^2}{2g} + 0.02\frac{1000}{\frac{10}{12}}\frac{V^2}{2 \times 32.2}$

($P_1/\gamma_1 = 0$, $z_1 = 800$, $V_1^2/2g = 0$, $P_2/\gamma_2 = 0$, $z_2 = 820$, $V_2^2/2g = 0$)

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