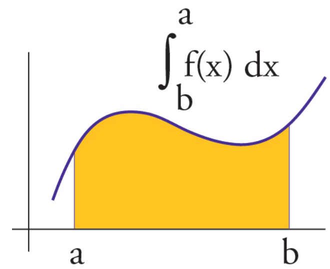
Shell Tutorial Center

Integration Techniques: U-sub



When doing differentiation we had some basic functions, such as polynomial functions, trig functions, exponential function, etc., whose derivative functions have been determined and rules to tell what to do when these basic functions are involved in a multiplication, division, or composition operation. However with integration, except in the case functions which are derivatives of the basic functions, we are looking at simplified versions of the functions that were generated using the product, quotient or chain rules and thus may lose sight of the underlying structure of rule which would lead us back to the original function. The various integration techniques are designed to back-track simplified forms to functions which produced the given derivative. **We may pick the wrong path (technique) and need to restart the process.**

This process generally takes us back through the chain rule so it will involve identification of an outside function and inside function. Since application of the chain rule to determine the derivative begins with the derivative of the outside function and ends with the derivative of the inside function in backtracking we want to begin with clearing the derivative of the inside function and ending with clearing the derivative of the outside function.

- Let **u = 'inside' function** and $\frac{du}{dx}$ = **it's derivative**. Solve this for **dx**
- Replace the 'inside function' with u and the dx with the expression found in the previous step. Simplify the expression. You should now have an expression only involving u. If not you may need to do a second substitution. Solve the equation u = 'inside function' for x and make this substitution.
- You should be able to integrate the resulting function. If not then try u =a different function or use a different technique!
- To finish replace the u with the expression it represents.

Example: $\int sin^2(3x)\ \cos(3x)\ dx$

Since $\sin^2(3x) = (\sin 3x)^2$, the inside function sin 3x = u. Its derivative du = 3 cos 3x dx. Solving this expression for dx we get: $dx = \frac{du}{3\cos 3x}$ Substituting we get: $\int u^2 \cos 3x \frac{du}{3\cos 3x} = \int \frac{1}{3} u^2\, du = \frac{1}{3}\left(\frac{1}{3}\, u^3\right) + C$
Finishing by replacing u: $\frac{1}{9}\,(\sin 3x)^3 + C$.

Note: Had we chosen u = 3x then du = 3 dx and the substituted integral would be $\int (\sin u)^2 \cos u\, \frac{du}{3}$ which is not one of the basic formulas.