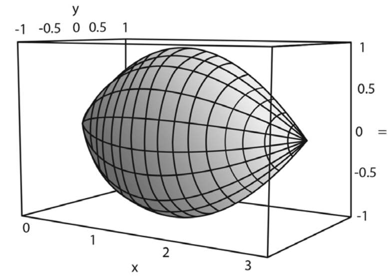
Shell Tutorial Center

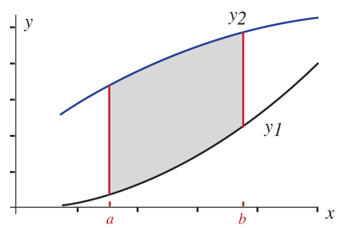


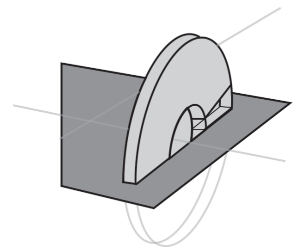
Volume of Revolution

The choice of method will be dependent upon a combination of the independent variable used and the axis of rotation.

1. Washer

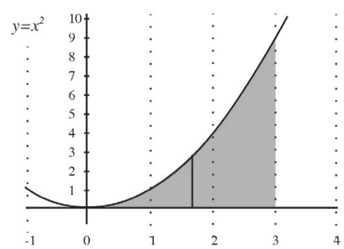
$$\pi \int_a^b (R_2^2 - r_1^2)\,(dx \text{ or } dy)$$

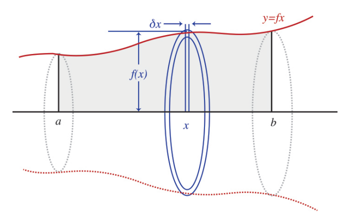




2. Disk : For this method one boundary will be the axis of revolution

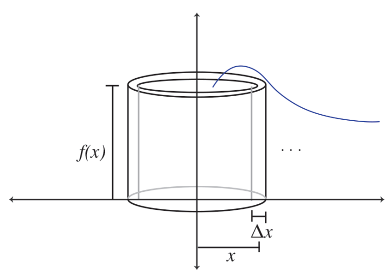
$$\pi \int_a^b R^2\,(dx \text{ or } dy)$$

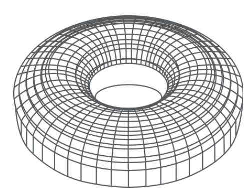




3. Shell

$$2\pi \int_a^b r\,h(dx \text{ or } dy)$$





The choice of method will be dependent upon a combination of the independent variable used and the axis of rotation.

Axis of revolution	Variable	Method
Line y = a (Note x axis is line y = 0)	Function: f(x)	Disk if axis of rotation is a boundary; washer if not
	Function: f(y)	Shell
Line x = b (Note y axis is line x = 0)	Function: f(x)	Shell
	Function: f(y)	Disk if axis of rotation is a boundary; washer if not

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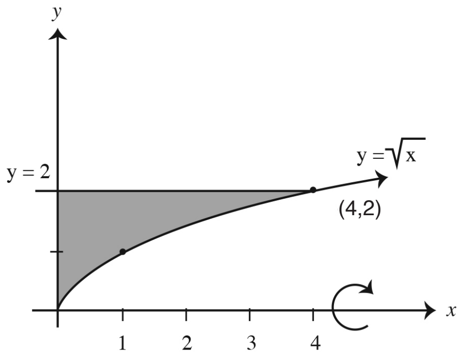
Note: The limits of integration will cooerespond to values for the variable used R in each of the above formula represents the distance from the axis of revolution. If the axis of revolution is x or y axis these values will be f(x), f(y), x or y. For other axes R will be calculated as the difference between f(x), f(y), x or y and the axis. Remember in calculating R to take the smaller value for x or y from the larger one.

Problems may be worked using either method once you have the correct correspondence between the axis of rotation and independent variable. However you may find the integration easier with one method than the other.

Examples: Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$, y=2, x=0 about the given axis:

a. x axis (y = 0)

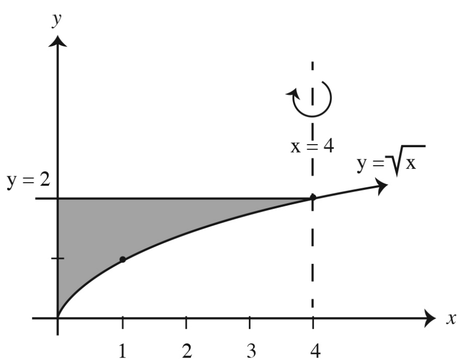


Method 1: With an axis of rotation is y = 0 and the function in the form f(x) we can use disk/washer method. Since the axis of rotation is not a boundary equation we will use washer with R = 2 and r= $\sqrt{x}$ giving $\pi \int_0^4 (2^2 - \sqrt{x}^2)\, dx$

Method 2: To use shell method we must change our equation to the form $x = f(y) = y^2$. Since the axis of rotation is the x axis r = y and h = y^2 - 0 giving

$$2\pi \int_0^2 y(y^2)\, dy$$

b. x = 4



Method 1: With axis of rotation x = 4 and a function of the for f(x) we can use shell method. Since axis of rotation is not an axis we must calculate r: r = 4 – x and h = 2 - $\sqrt{x}$, giving $$2\pi \int_0^4 (4-x)(2-\sqrt{x}\,)dx$$

Method 2: First we must change our equation to the form $f(y)=y^2$. Since x = 4 is not a boundary condition we will be using washers with R = 4 and r = 4 - y^2 giving $$\pi \int_0^2 (4^2 - (4-y^2)^2)dy$$