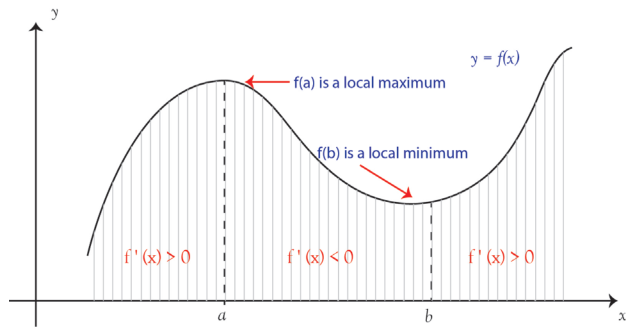
Shell Tutorial Center

Uses of the Derivative



The first derivative of a function can be used to tell us where the function is increasing and decreasing and thus where the function will reach a maximum or minimum value.

We use the second derivative to tell us about the concavity of the graph and locate the inflection points.

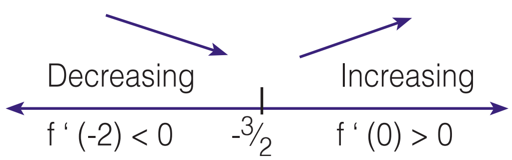
Finding the intervals where the function is increasing or decreasing

1. Find the critical numbers
   a. Find the derivative.
   b. Set the derivative = 0 and solve. The solution(s) will be the critical numbers.

Example: f(x) = x2 +3x – 2

   a. f '(x) = 2x + 3
   b. 2x + 3 = 0 ⟶ $x = -\frac{3}{2}$. The only critical number is $-\frac{3}{2}$.

2. Create a number line extending from the smallest number in the domain to the largest number in the domain. In the above example the domain of the function is all real numbers so we would use the entire line. Place the critical number on the line if it falls within the desired domain.



3. Choose test number within each interval on the number line you created. Evaluate the first derivative at each test number. (Note: You are only interested in the sign on the result not the actual value)

f ' (-2) < 0 so the function will be decreasing on the interval containing -2, $(-\infty, -\frac{3}{2})$

f ' (0) > 0 so the function will be increasing on the interval containing 0, $(-\frac{3}{2}, \infty)$

Finding maximum and minimum

The process used here will slightly different depending upon whether we want to find an absolute or local max or min. In problems looking for absolute max or min you will generally be give a specific interval that is of interest while for local max or min we generally are interested in the entire domain.

Procedure for absolute max or min

1. Find the critical numbers as above.
2. Evaluate the **original function** at the endpoints of the given interval and any critical numbers within the interval. The largest answer will3 be the absolute max and the smallest value will be the absolute min. Example: f(x) = x2 +3x – 2 on [- 2, 3]. From above the critical number is -3/2 which falls within the interval.

   $f(-2) = 4 - 6 - 2 = -4$, $f(-\frac{3}{2}) = 9/4 - 9/2 - 2 = -\frac{17}{4}$, $f(3) = 9 + 9 - 2 = 16$

   Absolute max = 16, Absolute min = $-\frac{17}{4}$

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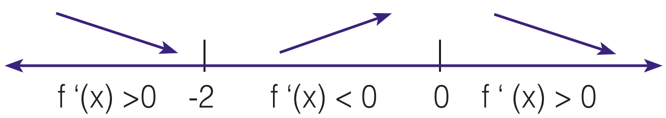
Procedure for local max or min

1. Determine the intervals over which the function is increasing and decreasing (See part 1)
2. If the function is increasing on the left of the critical point and decreasing on the right then a **local max** will occur at that critical number. If the function is decreasing on the left and increasing on the right then a **local min** will occur at that critical number. You will still need to evaluate the original function at these numbers to find the actual max or min.

Example: f(x) = x3 + 3x2 -4

f '(x) = 3x2 +6x → Critical numbers: -2, 0



This function will have a **local max** of f(-2) = -8 + 12 – 4 = 0 at x = -2 and a **local min** of f(0)= -4 at x = 0. If you are only interested in where the local max or min will occur then you will not need to evaluate the function.

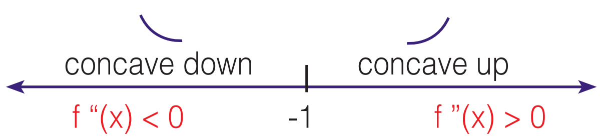
Concavity and Inflection points

To determine concavity we will need to find the **second derivative** of the function. The process is basically the same as when we were determining where the function was increasing or decreasing.

1. Find the second derivative and determine the values that make this derivative 0. These will be **possible inflection points**.
2. Make a number line extending over the domain of the function and place the possible inflection points on this line.
3. Using test numbers from each interval evaluate the **second derivative**. Again you are only interested on the sign of the result not the actual number.
4. If the value of f "(x) > 0 then the function will be concave up and if f "(x) < 0 then the function will be concave down.
5. **Inflection points** will occur where there is a change in concavity regardless of direction.

Example: f(x) = x3 + 3x2 -4

f"(x) = 6x + 6 → Possible inflection point: -1



Since the concavity changed at -1, we have an inflection point here.

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