

# Shell Tutorial Center

## Polynomial Division

A rational number, such as  $\frac{4}{6}$ , is not considered to be in simplified form since the numerator and denominator have a common factor of 2. Likewise a quotient of polynomials is not in simplest form if there are common factors between the numerator and denominator.

Look at the fraction  $\frac{x^2 - 4x + 3}{x - 1}$ . We know that we can factor the numerator:  $x^2 - 4x + 3 = (x - 3)(x - 1)$  so that the fraction can be rewritten in the form  $\frac{(x - 3)(x - 1)}{x - 1}$ .

We can then eliminate the common factor and get the simplified form  $\frac{(x - 3)}{1}$ .

This is used extensively in calculus for evaluating limits of the indeterminate form  $\frac{0}{0}$ .

Notice that when we substitute either 3 or 1 into the expression for  $x$  we get a value of zero, thus these numbers are referred to as zeros of the polynomial. If a number, "a", makes the value of a polynomial zero then  $x - a$  will be a factor of the polynomial. Looking at the polynomial  $x^3 + x^2 + x - 3$ , we note that substituting 1 for  $x$  will yield a value of 0. This tells us that  $x - 1$  is a factor of the polynomial but what is the other factor? From arithmetic we know that 2 is a factor of 12 because  $2 \overline{)12}$  with a remainder of 0. The quotient 6 would be the other factor. Using this idea we could divide the polynomial  $x^3 + x^2 + x - 3$  by the known factor  $x - 1$  to give us the second factor.

Fractions such as  $\frac{8}{3}$  are considered "improper" since the numerator is greater than or equal to the denominator. Likewise, algebraic fractions in which the degree of the numerator is greater than or equal to the degree of the denominator are considered improper. These improper expressions are changed into "mixed numbers" through division. So  $\frac{8}{3} = 2\frac{2}{3}$  because  $\frac{8}{3} = 2$  with a remainder of 2. This process is required in calculus for the evaluation of some integrals.

### How to do the Division

The process of polynomial division is similar to that of arithmetic division so let's review that process first. The division of  $\frac{14}{3}$  would be computed as follows:

- Divide
  - Multiply
  - Subtract
- $$\begin{array}{r} 4 \\ 3 \overline{)14} \\ \underline{12} \\ 2 \end{array}$$
- Divisor  $\longrightarrow$  3      Dividend  
 Remainder

So  $\frac{14}{3} = 4 + \frac{2}{3}$ . For polynomial division we must also match exponents.

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Example

$\frac{x^2 - 4x + 3}{x - 1}$  is computed as follows:

1. To match the leading term of the divisor with the leading term of the dividend we must multiply the whole divisor by  $x$ . (We get this by dividing the first term of the dividend by the first term of the divisor)
2. Subtract  $x^2 - x$  from  $x^2 - 4x + 3 \longrightarrow 0x^2 - 3x + 3$
3. Repeat. To match the leading term of  $-3x$  we must multiply the divisor by  $-3$  to get  $-3(x - 1) = -3x + 3$
4. Subtracting the result in 3 from the result in 2 we get a remainder of 0
5. We are finished since the degree of the divisor is larger than the degree of the remainder

$$\begin{array}{r}
 \begin{array}{c} \textcircled{1} \quad \textcircled{3} \\ \downarrow \quad \downarrow \\ x + (-3) \end{array} \\
 x - 1 \overline{) x^2 - 4x + 3} \\
 \underline{x^2 - x} \phantom{+ 3} \\
 0x^2 - 3x + 3 \quad \textcircled{2} \\
 \underline{-3x + 3} \\
 0 \quad \textcircled{4}
 \end{array}$$

Example

$$\frac{x^3 + x^2 + x - 3}{x - 1}$$

1. Divide
2. Multiply
3. Subtract
4. Repeat

$$\begin{array}{r}
 \begin{array}{c} \text{1. } \frac{x^3}{x} = x^2 \\ \text{Repeat 1: } \frac{2x^2}{x} \end{array} \\
 \begin{array}{r} x^2 + 2x + 3 \\ x-1 \overline{) x^3 + x^2 + x - 3} \\ \underline{(x^3 - x^2)} \\ 2x^2 + x - 3 \\ \underline{(2x^2 - 2x)} \\ 3x - 3 \\ \underline{(3x - 3)} \\ 0 \end{array} \\
 \begin{array}{c} \text{Repeat 1: } \frac{3x}{x} \\ \text{2. } x^2(x - 1) \\ \text{Repeat 2: } 2x(x - 1) \\ \text{Repeat 2: } 3(x - 1) \\ \text{Repeat 3: } (3x - 3) - (3x - 3) \end{array}
 \end{array}$$

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