Formula Sheet for LSU Physics 2101, Final Exam, Spring '13

Units:

$1\,\text{m} = 39.4\,\text{in} = 3.28\,\text{ft}$ $\quad 1\,\text{mi} = 5280\,\text{ft}$ $\quad 1\,\text{min} = 60\,\text{s}$, $\quad 1\,\text{day} = 24\,\text{h}$ $\quad 1\,\text{rev} = 360° = 2\pi\,\text{rad}$

$1\,\text{atm} = 1.013\times10^5\,\text{Pa}$ $\quad T = \left(\frac{1\,K}{1°\text{C}}\right) T_C + 273.15\,\text{K}$ $\quad T_F = \left(\frac{9\,°\text{F}}{5°\text{C}}\right) T_C + 32°\text{F}$

Constants:

$g = 9.8\,\text{m/s}^2$ $\quad R_{Earth} = 6.37 \times 10^6\,\text{m}$ $\quad M_{Earth} = 5.98\times10^{24}\,\text{kg}$

$G = 6.67\times10^{-11}\,\text{m}^3/(\text{kg}\cdot\text{s}^2)$ $\quad R_{Moon} = 1.74 \times 10^6\,\text{m}$ $\quad M_{Moon} = 7.36\times10^{22}\,\text{kg}$

Earth-Sun distance $= 1.50\times10^{11}\,\text{m}$ $\quad M_{Sun} = 1.99\times10^{30}\,\text{kg}$ $\quad$ Earth-Moon distance $= 3.82\times10^8\,\text{m}$

$k = 1.38 \times 10^{-23}\,\text{J/K}$ $\quad R = 8.31\,\text{J/(mol}\cdot\text{K)}$ $\quad$ Avogadro's # $= 6.02\times10^{23}$ particles/mol

Properties of H_2O:

Density: $\rho_{\text{water}} = 1000\ \text{kg/m}^3$

Specific heat: $c_{\text{water}} = 4187\ \text{J/(kg K)}$ $\quad c_{\text{ice}} = 2220\ \text{J/(kg K)}$

Heats of transformation: $L_{\text{vaporization}} = 2.256 \times 10^6\ \text{J/kg}$ $\quad L_{\text{fusion}} = 3.33 \times 10^5\ \text{J/kg}$

Quadratic formula: for $ax^2 + bx + c = 0$, $\quad x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot Product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = |\vec{a}|\,|\vec{b}| \cos(\phi)$ $\quad$ (ϕ is smaller angle between $\vec{a}$ and $\vec{b}$)

Cross Product: $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$, $\quad |\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin(\phi)$

Equations of Constant Acceleration:

linear equation along x	missing	missing	rotational equation
$v_x = v_{ox} + a_x t$	$x - x_o$	$\theta - \theta_o$	$\omega = \omega_o + \alpha_x t$
$x - x_o = v_{ox} t + \frac{1}{2} a_x t^2$	v_x	ω	$\theta - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2$
$v_x^2 = v_{ox}^2 + 2a_x(x - x_o)$	t	t	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$
$x - x_o = \frac{1}{2}(v_{ox} + v_x)t$	a_x	α	$\theta - \theta_o = \frac{1}{2}(\omega_o + \omega)t$
$x - x_o = v_x t - \frac{1}{2} a_x t^2$	v_{ox}	ω_o	$\theta - \theta_o = \omega t - \frac{1}{2}\alpha t^2$

Vector Equations of Motion for Constant Acceleration: $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a} t^2$, $\quad \vec{v} = \vec{v}_o + \vec{a} t$

Projectile Motion: (with + direction pointing up from Earth)

$x - x_o = (v_o \cos\theta_o)t$ $\quad y - y_o = (v_o \sin\theta_o)t - \frac{1}{2} g t^2$

$v_x = v_o \cos\theta_o$ $\quad v_y = (v_o \sin\theta_o) - gt$

$y = (\tan\theta_o)x - \dfrac{gx^2}{2(v_o\cos\theta_o)^2}$ $\quad v_y^2 = (v_o \sin\theta_o)^2 - 2g(y - y_o)$

Newton's Second Law: $\sum \vec{F} = m\vec{a}$

Uniform circular motion: $F_c = m a_c = \dfrac{mv^2}{r}$, $\quad T = \dfrac{2\,\pi\,r}{v}$

Force of Friction: Static: $f_s \le f_{s,max} = \mu_s F_N$, Kinetic: $f_k = \mu_k F_N$

Spring (elastic) Force: Hooke's Law $F_{\text{spring}} = -kx$ $\quad$ (k = spring (force) constant)

Kinetic Energy: Translational $\quad K = \frac{1}{2} m v^2$

Work:

$W = \int_{x_i}^{x_f} F(x)dx$ (variable 1-D force), $\; W = \int_{r_i}^{r_f} \vec{F}(\vec{r}) \cdot d\vec{r}$ (variable 3-D force), $\; W = \vec{F} \cdot \vec{d}$ (constant force)

$W = \vec{F} \cdot \vec{d}$ (constant force)

Work - Kinetic Energy Theorem: $W = \Delta K = K_f - K_i$

Work done by weight (gravity close to the Earth surface): $W = m\,\vec{g}\cdot\vec{d} = -m\,g\,\Delta y$

Work Done by a Spring Force: $W_s = -k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right)$

Power:

Average: $P_{avg} = \frac{W}{\Delta t}$, $P = \vec{F}\cdot\vec{v}_{avg}$ (const. force) Instantaneous: $P = \frac{dW}{dt}$, $P = \vec{F}\cdot\vec{v}$ (const. force)

Potential Energy Change: $\Delta U = -W$

Gravitational Potential Energy: $U_g(y) = mgy$

Elastic (Spring) Potential Energy: $U_s = \frac{1}{2}kx^2$ (relative to the relaxed spring)

Potential-Force Relation: $F(x) = -\frac{dU(x)}{dx}$ **Mechanical Energy:** $E_{mec} = K + U$

Change of Mechanical Energy due to Non-Conservative (nc) Forces:

$$W_{\text{nc}} = \Delta E_{\text{mec}} = (K_f + U_f) - (K_i + U_i)$$

Conservation of Energy: $W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}} + \Delta E_{\text{int}}$, W_{ext} is the *net, external* work done by external forces on the system, $\Delta E_{\text{th}} = -W_{f_k}$ is the thermal energy change, ΔE_{int} is the internal energy change

Center of mass: $M = \sum_{i=1}^{N} m_i$, $x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$, $y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$, $z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$

$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i \qquad \vec{v}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i \qquad \vec{a}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{a}_i = \frac{1}{M}\sum_{i=1}^{N}\vec{F}_i$$

Definition of Linear Momentum: one particle: $\vec{p} = m\vec{v}$, system of particles: $\vec{P} = \sum_{i=1}^{N}\vec{p}_i = M\vec{v}_{\text{com}}$

Newton's 2nd Law for a System of Particles: $\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} = \frac{d\vec{P}}{dt}$

Conservation of Linear Momentum of an Isolated System: $\sum\vec{p}_i = \sum\vec{p}_f$

Impulse - Linear Momentum Theorem: $\Delta\vec{p}_1 = \vec{J}_{12} = \int_{t_1}^{t_2}\vec{F}_{12}(t)dt = \vec{F}_{\text{avg,12}}\Delta t$

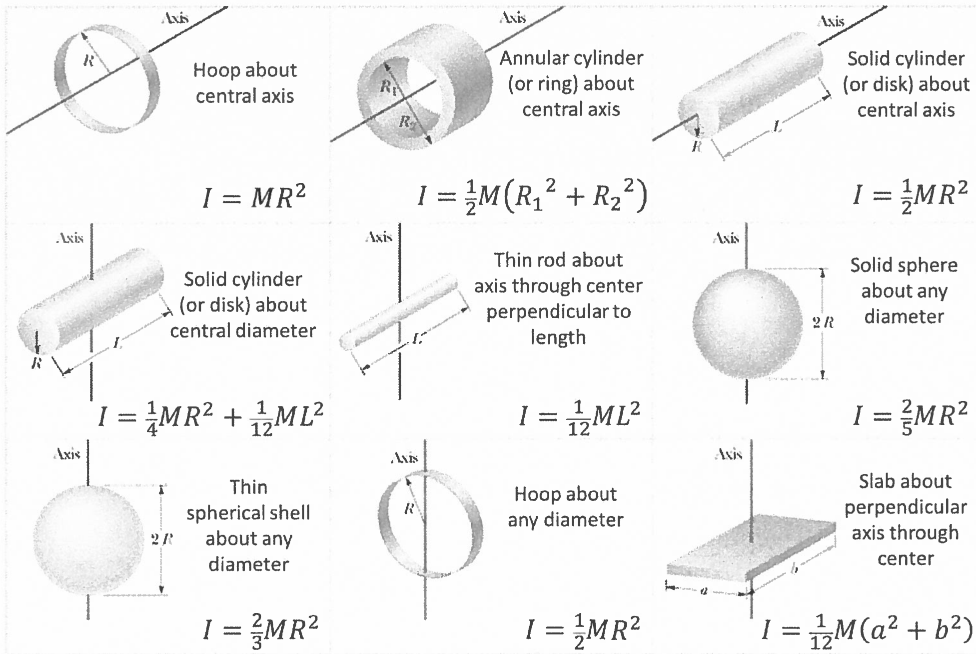
Elastic Collision (1 Dim): $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$ $v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$

Linear and Angular Variables Related:

$s = r\theta$ $v = \omega r$ $a_t = \alpha r$ $a_r = \frac{v^2}{r} = \omega^2 r$ (magnitude of the radial or centripetal acceleration)

Rotation: Rotational Inertia (I_{com}) for Simple Shapes: see next page

Rotational Inertia:	Descrete particles: $I = \sum_{i=1}^{N} I_i$	Continuous object: $I = \int r^2 dm$
Parallel Axis Theorem:	$I = I_{\text{com}} + Mh^2$	
Torque:	$\vec{\tau} = \vec{r}\times\vec{F}$	$\tau = rF_t = r_\perp F = rF\sin\phi$
Angular Momentum:	rigid body, fixed axis: $\vec{L} = I\vec{\omega}$	point-like particle: $\vec{L} = \vec{r}\times\vec{p}$
Newton's 2nd Law:	$\vec{\tau}_{\text{net}} = I\vec{\alpha}$	$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$
Conservation Law	(isolated system, $\sum\tau = 0$):	$\sum\vec{L}_i = \sum\vec{L}_f$
Rotational Work:	$W = \int_{\theta_i}^{\theta_f}\tau d\theta = \tau_{avg}\Delta\theta$	Kinetic Energy: $K = \frac{1}{2}I\omega^2$
Rotational Power:	Instantaneous: $P = \frac{dW}{dt} = \tau\omega$	Average: $P_{avg} = \frac{W}{t} = \tau_{avg}\omega_{avg}$
Rolling:	$v_{\text{com}} = \omega R$	$a_{\text{com}} = \alpha R$
	Kinetic Energy of Rolling:	$K = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I_{\text{com}}\omega^2$



Static equilibrium: $\vec{F}_{net} = 0 \qquad \vec{\tau}_{net} = 0$

Gravity:

Newton's law: $|\vec{F}| = G\frac{m_1 m_2}{r^2}$ Gravitational acceleration (planet of mass M): $a_g = \frac{GM}{r^2}$

Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ Potential Energy: $U = -G\frac{m_1 m_2}{r}$

Potential Energy of a System (3 or more masses): $U = -\left(G\frac{m_1 m_2}{r_{12}} + G\frac{m_1 m_3}{r_{13}} + G\frac{m_2 m_3}{r_{23}} + ...\right)$

Static Fluids:

Density: $\rho = \frac{\Delta m}{\Delta V}$ Pressure: $p = \frac{\Delta F}{\Delta A}$ Absolute Pressure: $p = p_o + \rho g h$

Pressure Variation with Height or Depth: $p_2 = p_1 + \rho\, g\, (y_1 - y_2)$ Gauge pressure: $p - p_o$

Archimedes' Principle: $F_b = \rho_f V_{\text{displaced}} g = m_f g$ weight$_{\text{apparent}} = mg - F_b$

Simple Harmonic Motion (SHM): $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Linear:
$x(t) = x_m \cos(\omega t + \phi)$
$v(t) = -x_m \omega \sin(\omega t + \phi)$
$a(t) = -x_m \omega^2 \cos(\omega t + \phi) = -\omega^2 x(t)$

Angular:
$\Theta(t) = \Theta_m \cos(\omega t + \phi)$
$\Omega(t) = -\Theta_m \omega \sin(\omega t + \phi)$
$\alpha(t) = -\Theta_m \omega^2 \cos(\omega t + \phi) = -\omega^2 \Theta(t)$

Linear Oscillator: Spring-Block: $\omega = \sqrt{\frac{k}{m}}$ Horizontal Spring-Block: $E_{\text{mec}} = \frac{1}{2}kx_m^2$

Pendulums: Torsion: $\omega = \sqrt{\frac{\kappa}{I}}$ Simple: $\omega = \sqrt{\frac{g}{L}}$ Physical: $\omega = \sqrt{\frac{mgh}{I}}$

Torsion torque: $\tau = -\kappa\Theta$

Waves:

$y(x,t) = y_m \sin(kx \mp \omega t + \phi)$ Angular Frequency: $\omega = \frac{2\pi}{T}$ Wave Number: $k = \frac{2\pi}{\lambda}$

Speed: $v = \frac{\omega}{k} = \lambda f$ **Stretched String Speed:** $v = \sqrt{\frac{\tau}{\mu}}$ Linear Density: $\mu = \frac{m}{L}$

Interference (same direction): $y'(x,t) = [2y_m \cos\frac{\phi}{2}] \sin(kx \mp \omega t + \frac{\phi}{2})$

Standing waves: $y'(x,t) = [2y_m \sin(kx)]\ \cos(\omega t)$ Resonance: $f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}, \quad n = 1, 2, ...$

Thermodynamics:

Linear Expansion: $\Delta L = L\alpha\Delta T$ Volume Expansion: $\Delta V = V\beta\Delta T = 3\alpha V\Delta T$

Heat of Warming/Cooling: $Q = mc\Delta T$ Heat of Transformation: $Q = mL$

Work Done **by** the System: $W = \int_i^f p\,dV$

First Law: $\Delta E_{int} = Q - W$ $\Delta E_{int} = E_{int,f} - E_{int,i}$ $dE_{int} = dQ - dW$

Ideal Gas Law: $pV = nRT = NkT$ $\frac{pV}{T} = \text{const}$ for n=const

rms-speed: $v = \sqrt{\frac{3RT}{M}}$

Change in Entropy: $\Delta S = \int_i^f \frac{dQ}{T}$ ...(reversible path) $\Delta S = S_f - S_i$

Second Law: $\Delta S \geq 0$ ...(closed system)

Solids/Liquids: $\Delta S = \frac{mL}{T}$ (transformation) $\Delta S = mc\ln\frac{T_f}{T_i}$ (warming/cooling)

Molecule	Monoatomic	Diatomic	Polyatomic
C_V	$(3/2)R$	$(5/2)R$	$3R$
C_p	$(5/2)R$	$(7/2)R$	$4R$

$\gamma = C_p/C_V$, $E_{\text{int}} = nC_VT$

Process Type	**Const. Quant.**	**Useful Relations**
Any path		$\Delta E_{\text{int}} = nC_V\Delta T = (C_V/R)(p_fV_f - p_iV_i)$ $\Delta S = nR\ln(V_f/V_i) + nC_V\ln(T_f/T_i)$
Isochoric	V	$Q = \Delta E_{\text{int}} = nC_V\Delta T,\ W = 0$
Isobaric	p	$Q = nC_p\Delta T, W = p\Delta V, \Delta S = nC_p\ln(V_f/V_i)$
Isothermal	T	$W = nRT\ln(V_f/V_i)$, $\Delta E_{\text{int}} = 0$
Cyclic		$Q = W$, $\Delta E_{\text{int}} = 0$, $\Delta S = 0$
Adiabatic	pV^γ, $TV^{\gamma-1}$	$Q = 0,\ W = -\Delta E_{\text{int}}, \Delta S = 0$

Engines:

1st Law for Eng. and Refrig.: $0 = |Q_H| - |Q_L| - |W|$

Efficiency: $\epsilon = \frac{|W|}{|Q_H|}$ Carnot (ideal) efficiency: $\epsilon_C = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H}$