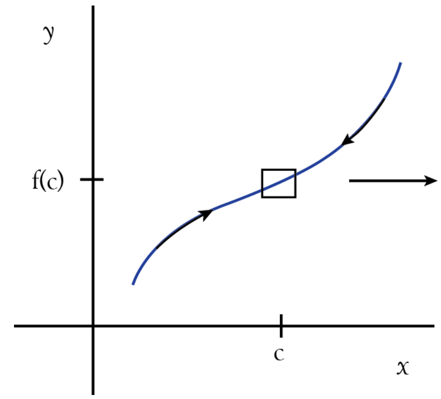
Shell Tutorial Center

Limits



Limits describe the general trends in the y values generated by substituting in values of x close to a given number. To say $\lim_{x \to a} f(x) = L$ means that as we substitute in values of x just smaller than **a** and just bigger than **a**, the y values will be approaching ***L***. Since we are looking at **trends** the function does not have to actually have a value at ***a***.

For continuous functions (Note: A continuous functions is one that you can draw without removing your pencil from the paper) the limit will equal the functional value at a: $\lim_{x \to a} f(x) = f(a)$. Since this covers a great many of the problems you will encounter a good starting point is to determine the functional value at a.

If substituting **a** into the function gives a result of the form $\frac{\text{(nonzero number)}}{0}$ the function is discontinuous at a and the limit DNE or goes to $\pm\infty$

Indeterminate forms: If upon substituting **a** into the function you get $\frac{0}{0}, \frac{\infty}{\infty}, 0 * \infty, \infty - \infty, 0^0, \infty^0, 1^\infty$

- Form $\frac{0}{0}$:
  - If the function is the quotient of polynomials then factor the polynomials, reduce and substitute into the reduced form of the fraction. The graphs of the original function and the reduced for agree everywhere except at a where one function has a value and the other doesn't.
  - If the function is a quotient containing radicals, try multiplying numerator and denominator by the conjugate of the expression with the radical, simplify and substitute.
- Form $\frac{\infty}{\infty}$: This form will generally arise when considering $\lim_{x \to \infty} f(x)$
  - Generally begin by dividing all terms in both the numerator and denominator by the largest power of x **in the denominator**. This will generate several terms of the form $\frac{k}{x^p}$. As long as p is positive then $\lim_{x \to \infty} \frac{k}{x^p} = 0$
    - Quick reference: $\lim_{x \to \infty} \frac{a_m x^m + a_{m-1} x^{m-1} + \ldots}{b_n x^n + b_{m-1} x^{n-1} + \ldots}$ will equal 0 if m < n, $\frac{a_m}{b_n}$ if m = n, $\pm\infty$ if m > n
  - A term of the form $\sqrt{ax^2 + bx + c}$ is considered to have degree 1 but when dividing this term by x we would need to express it as $\sqrt{x^2}$. This creates a problem when considering the $\lim_{x \to -\infty} f(x)$ as the x will have a negative value while $\sqrt{x^2}$ has a positive value. So to keep values consistent use $-\sqrt{x^2}$ to divide by in this instance.

Note the above forms may also be determined using l'Hospital's Rule: $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$ **provided** $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$

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- Form 0*∞: Rewrite the product as a quotient ($fg = \frac{f}{\frac{1}{g}}$ or $\frac{g}{\frac{1}{f}}$) which will put it into the form $\frac{0}{0}$ **or** $\frac{\infty}{\infty}$

  Proceed using as above or use l'Hospital Rule

- Form $\infty - \infty$
  - If the difference involves 2 fractions find a common denominator and rewrite as a single fraction. Substitute into this new form and proceed as dictated by the results
  - If the difference involves a radical, multiply and divide by the conjugate of the expression, substitute into this new form and proceed as dictated by the results.
- Forms 0^0, ∞^0, 1^∞
  - These forms are solved using ln. Set $y = f(x)^{g(x)}$ then taking the ln of both sides we would get ln $y = g(x)\ \ln(f(x))$. Find the limit of $g(x)\ \ln(f(x))$. (This should have one of the forms discussed above.) This result is the value for ln y not y itself. To convert to the value of $y = e^{Answer}$

If the function involves a product or quotient where the limit of one of the factors DNE try the squeeze theorem.

Limits you need to know

- $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1$
- $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$
- $\lim_{x\to -\infty} e^x = \lim_{x\to\infty} e^{-x} = 0$ and $\lim_{x\to\infty} e^x = \lim_{x\to -\infty} e^{-x} = \infty$
- $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$ and $\lim_{x\to -\infty} \tan^{-1} x = -\frac{\pi}{2}$
- $\lim_{x\to\infty} \ln x = \infty$ and $\lim_{x\to 0^+} \ln x = -\infty$