

Shell Tutorial Center

Laplace Transforms

	$f(t)$	$F(s)$	Page
1.	1	$\frac{1}{s}$	116
2.	t	$\frac{1}{s^2}$	
3.	t^n ($n = 0, 2, 3, \dots$)	$\frac{n!}{s^{n+1}}$	116
4.	t^α ($\alpha > 0$)	$\frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}$	118
5.	e^{at}	$\frac{1}{s - a}$	118
6.	te^{at}	$\frac{1}{(s - a)^2}$	
7.	$t^n e^{at}$ ($n = 1, 2, 3, \dots$)	$\frac{n!}{(s - a)^{n+1}}$	119
8.	$\sin bt$	$\frac{b}{s^2 + b^2}$	118
9.	$\cos bt$	$\frac{s}{s^2 + b^2}$	118
10.	$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$	120
11.	$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$	120
12.	$\frac{\sin t}{t}$	$\tan^{-1} \frac{1}{s}$	358
13.	$\frac{\sin at}{t}$	$\tan^{-1} \left(\frac{a}{s} \right)$	365
14.	$\frac{e^{bt} - e^{at}}{t}$	$\ln \left(\frac{s - a}{s - b} \right)$	365

Shell Tutorial Center

$$15. \quad 2 \frac{\cos bt - \cos at}{t} \qquad \ln \left(\frac{s^2 + a^2}{s^2 + b^2} \right) \qquad 365$$

$$16. \quad 2 \frac{\cos bt - \cos at}{t^2} \qquad s \ln \left(\frac{s^2 + b^2}{s^2 + a^2} \right) - 2b \tan^{-1} \left(\frac{b}{s} \right) \qquad 365$$

Laguerre polynomials

$$17. \quad \ell_n(t) = \sum_{k=0}^n (-1)^k \binom{n}{k} \frac{t^k}{k!} \qquad \frac{(s-1)^n}{s^{n+1}} \qquad 361$$

$$18. \quad \ell_n(at) \qquad \frac{(s-a)^n}{s^{n+1}} \qquad 366$$

The Heavyside function

$$19. \quad h(t-c) \qquad \frac{e^{-sc}}{s} \qquad 404$$

The on-off switch

$$20. \quad \chi_{[a,b)} \qquad \frac{e^{-as}}{s} - \frac{e^{-bs}}{s} \qquad 405$$

The dirac delta function

$$21. \quad \delta_c \qquad e^{-cs} \qquad 428$$

The square-wave function

$$22. \quad \text{sw}_c \qquad \frac{1}{1+e^{-cs}} \frac{1}{s} \qquad 456$$

The sawtooth function

$$23. \quad \langle t \rangle_p \qquad \frac{1}{s^2} \left(1 - \frac{spe^{-sp}}{1-e^{-sp}} \right) \qquad 457$$

Periodic dirac delta functions

$$24. \quad \delta_0(\langle t \rangle_p) \qquad \frac{1}{1-e^{-ps}} \qquad 459$$

Alternating periodic dirac delta functions

$$25. \quad (\delta_0 - \delta_p)(\langle t \rangle_{2p}) \qquad \frac{1}{1+e^{-ps}} \qquad 459$$

The matrix exponential

$$26. \quad e^{At} \qquad (sI - A)^{-1} \qquad 459$$

Shell Tutorial Center

Convolutions

	$f(t)$	$g(t)$	$(f * g)(t)$	Page
1.	$f(t)$	$g(t)$	$f * g(t) = \int_0^t f(u)g(t-u) du$	187
2.	1	$g(t)$	$\int_0^t g(\tau) d\tau$	190
3.	t^m	t^n	$\frac{m!n!}{(m+n+1)!} t^{m+n+1}$	193
4.	t	$\sin at$	$\frac{at - \sin at}{a^2}$	
5.	t^2	$\sin at$	$\frac{2}{a^3} (\cos at - (1 - \frac{a^2 t^2}{2}))$	
6.	t	$\cos at$	$\frac{1 - \cos at}{a^2}$	
7.	t^2	$\cos at$	$\frac{2}{a^3} (at - \sin at)$	
8.	t	e^{at}	$\frac{e^{at} - (1 + at)}{a^2}$	
9.	t^2	e^{at}	$\frac{2}{a^3} (e^{at} - (a + at + \frac{a^2 t^2}{2}))$	
10.	e^{at}	e^{bt}	$\frac{1}{b-a} (e^{bt} - e^{at}) \quad a \neq b$	192
11.	e^{at}	e^{at}	te^{at}	192
12.	e^{at}	$\sin bt$	$\frac{1}{a^2 + b^2} (be^{at} - b \cos bt - a \sin bt)$	195
13.	e^{at}	$\cos bt$	$\frac{1}{a^2 + b^2} (ae^{at} - a \cos bt + b \sin bt)$	195
14.	$\sin at$	$\sin bt$	$\frac{1}{b^2 - a^2} (b \sin at - a \sin bt) \quad a \neq b$	195
15.	$\sin at$	$\sin at$	$\frac{1}{2a} (\sin at - at \cos at)$	195

Shell Tutorial Center

16.	$\sin at$	$\cos bt$	$\frac{1}{b^2 - a^2}(a \cos at - a \cos bt) \quad a \neq b$	195
17.	$\sin at$	$\cos at$	$\frac{1}{2}t \sin at$	195
18.	$\cos at$	$\cos bt$	$\frac{1}{a^2 - b^2}(a \sin at - b \sin bt) \quad a \neq b$	195
19.	$\cos at$	$\cos at$	$\frac{1}{2a}(at \cos at + \sin at)$	195
20.	f	$\delta_c(t)$	$f(t - c)h(t - c)$	444
21.	f	$\delta_0(t)$	$f(t)$	445