Shell Tutorial Center

Implicit Differentiation

Equations may be written explicitly where the variable y is explicitly written as a function of x ($y=3x^2 + 4x - 10$). Others are only implied. For example the function $y=1/x$ is written explicitly but if we rewrite it as $xy=1$we have an implicit form. If an equation is given in an implicit form we can sometime rewrite it in its explicit form by solving for y and then finding the derivative as usual. However some equations cannot be easily converted into an explicit form. In this case we would use implicit differentiation.

Remember that differentiation is done with respect to x so that when you differentiate terms involving x alone, you differentiate as usual but when differentiate terms involving y you must use the chain rule because y is an implied differentiable function of x.

Examples: **Differentiating with respect to x**

a. $\frac{d}{dx}[x^3] = 3x^2$ — Variable sgree: Use simple power rule

b. $\frac{d}{dx}[3y^3] = 6y\frac{dy}{dx}$ — Variables disagree: Use chain rule

c. $\frac{d}{dx}[x^2y^2] = x^2\frac{d}{dx}[y^2] + y^2\frac{d}{dx}[x^2]$ — Product rule

$=x^2\left(2y\frac{dy}{dx}\right) + y^2(2x)$ — Chain rule

$=2x^2y\frac{dy}{dx} + 2xy$ — Simplify

Guidelines for implicit differentiation

1. Differentiate both sides with respect to x
2. Collect all terms involving $\frac{dy}{dx}$ on one side of the equation and all other terms to other side
3. Factor out $\frac{dy}{dx}$ and divide both sides by the other factor

Example: Find $\frac{dy}{dx}$ *if* $y3 + y2 - 5y - x^2 = -4$

1. Differentite both sides with respect to x: $3y^2\frac{dy}{dx}+2y\frac{dy}{dx} - 2x = 0$
2. Collect $\frac{dy}{dx}$ terms: $3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$
3. Factor and divide: $\frac{dy}{dx}(3y^2 + 2y - 5) = 2x$

$$\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}$$