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Summary for Tests for Series

Test	Series	Condition(s) of Convergence	Conditions(s) of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{x\to\infty} a_n \neq 0$	This test can not be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\|r\| < 1$	$\|r\| \geq 1$	$\text{Sum}: S = \frac{\text{a}}{1-\text{r}}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{x\to\infty} b_n = L$		$\text{Sum}: S = b_1 - L$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$\text{p} > 1$	$\text{p} \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\|R_N\| \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ *diverges*	$0 < R_N < \int_N^{\infty} f(x)\,dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} < 1$	$\lim_{n\to\infty} \sqrt[n]{\left\|\frac{a_n+1}{a_n}\right\|} > 1$	This test is inconclusive if: $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = 1.$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| < 1$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| > 1$	This test is inconclusive if: $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = 1.$
Direct Comparison (a_n, $b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison (a_n, $b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

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Definitions of Absolute and Conditional Convergence

1. $\sum a_n$ is **absolutely convergent** if $\sum |a_n|$ converges.
2. $\sum a_n$ is **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ diverges.

*n*th term test

Is $\lim a_n \neq 0$ → Yes → Series diverges

No ↓ (Series may converge or diverge)

Geometric Series test

Is $\sum a_n = a + ar + ar^2 + \ldots$ → Yes → Series converges if $|r| < 1$; Series diverges if $|r| > 1$

No ↓

Nonnegative terms and/or absolute Convergence

Does $\sum |a_n|$ converge? (apply a Comparison test, Integral test, Ratio test or Root test to $\sum |a_n|$ → Yes → Original series converges

No ↓

→ Alternating Series

Is $\sum a_n = b_1 - b_2 + b_3 - \ldots$ (an alternating series) → Yes → Is $b_1 > b_2 > b_3 > \ldots > b_n$ (sequence decreasing)

- Yes ↓ Series converges if $b_n \to 0$; Series diverges if $b_n \not\to 0$
- No → Investigate the partial sums

No ↓

Investigate the partial sums