

Avoid These Common Pitfalls on the Road to Algebra Success

Errors Related to Order of Operations and Basic Properties

1. The distributive property is one of our basic properties of real numbers. It tells us that multiplication or division “distributes” over addition or subtraction. There are several common mistakes made in the application of this property
 - a. Not distributing to all terms: $6(x+5) \neq 6x + 5$ instead of the correct form $6(x+5) = 6x + 6(5)$. This mistake occurs often when the expression is in the form $-(3x+2)$. When the distributive property is applied correctly to this example you should get $-3x - 2$ not the $-3x+2$ which is often given.
 - b. Applying it to a multiplication: $3(2x)$ does not become $(3*2)(3*x)$. Different properties come into play when you are dealing with a single type of operation. Here we would want to employ the associative property to regroup the multiplication $(3*2)x = 6x$
 - c. Failure to include parentheses when substituting: If $f(x) = x^2 - x$ and you want to find $f(x+2)$ the correct substitution should look like $f(x+2) = (x+2)^2 - (x+2)$ and the understood -1 factor for the second term would then be distributed over the expression. However many times the substitution is written as $f(x+2) = (x+2)^2 - x+2$ so that it does not appear that the distributive rule needs to be applied.
2. We can all probably recite the acronym PEMDAS for the order of operations. However many mistakes are made in failing to recognize when we need to employ this ordering. For example if we have $3*2^2$ most people recognize the need to do the exponent before the multiplication. However with the expression $3(x+4)^2$ many people focus on the parentheses and thinking that they have to deal with those first, want distribute first rather than dealing with the exponent before distributing. The correct order of operations here would produce $3(x^2 + 8x + 16) = 3x^2 + 24x + 48$

Errors Related to Exponents

One of the basic rules of exponents is $(a*b)^n = a^n*b^n$. This rule is very specific about the operation to which it applies. We can extend it to include division since division problems can be rewritten as a multiplication. Many mistakes result from attempting to apply this rule to additions or subtractions

3. $(x-3)^2 \neq x^2+9$. To correctly evaluate this expression since the rule does not apply we must go back to the definition of an exponent which tells us that $(x - 3)$ will be used as a factor twice. So we get $(x - 3)(x - 3) = x^2 - 6x + 9$.
4. We have a basic definition of ${}^n\sqrt{a^m} = a^{\frac{m}{n}}$. Thus $\sqrt{(x^2-4)} = (x^2-4)^{\frac{1}{2}}$. As in (3) the expression being raised to a power involves a subtraction so the basic rules do not apply.

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5. Suppose we wish to rewrite the $\frac{1}{3x}$ so that we can apply the power rule for finding the derivative. We know the basic definition $\frac{1}{a^n} = a^{-n}$. The error occurring most often in this process is to apply the rule to one factor but not the other getting the incorrect expression $3x^{-1}$. There are several ways the expression could be written correctly: $3^{-1}x^{-1}$ or $(3x)^{-1}$ or $\frac{1}{3} x^{-1}$. The last one would be the preferred expression.

Errors Related to Fractions

Most errors related to fractions stem from a lack of understanding of basic operations – why do we need common denominators, what operations require them and which don't. Denominators basically tell us into how many pieces a whole is divided so they determine the size of the pieces. To add or subtract the pieces need to be the same size to determine how much of the whole we obtain, so we need a common denominator. Once we have all the pieces the same size the addition or subtraction occurs only in the numerator which tells how many pieces we have. In multiplying by another fraction, you are subdividing the pieces from the first division into smaller parts. Since the number of pieces for this second subdivision is independent of the first, a common denominator is not necessary. Multiplication across the denominators will give us the new number of pieces into which the whole is now divided and multiplication across the numerator will indicate the number of those pieces that we have.

6. $\frac{1}{(x+y)} \neq \frac{1}{x} + \frac{1}{y}$. Since this is an addition we need a common denominator which would be xy not $x+y$.
7. $\frac{2x+6}{2} \neq x + 6$. A basic property of numbers is that if we multiply that number by 1 we do not change its value, $a \cdot 1 = a$. That 1 can take on infinitely many forms if we think of it in terms of a fraction, $\frac{2}{2}$, $\frac{-10}{-10}$, $\frac{x+y}{x+y}$, etc. Since multiplication of fractions involves multiplication in both the numerator and denominator, we now have a common factor. In reducing fractions we are taking out the common factors. $\frac{2x+6}{2} = \frac{2(x+3)}{2} = \frac{2}{2} * \frac{x+3}{1} = 1(x+3) = x + 3$
8. $\frac{(x+3)}{x} * x \neq \frac{x^2+3x}{x^2}$. Here we are actually multiplying the first fraction by $\frac{x}{1}$ not $\frac{x}{x}$ so that the correct form would be $\frac{x+3}{x} * \frac{x}{1} = \frac{x^2+3x}{x}$

Errors related to solving equations

There are three basic ideas involved in solving equations we can add or subtract the same quantity from both sides of the equation; we can multiply or divide both side of the equation by a **nonzero** number; if we can get the equation in the for $a * b = 0$ then one of the quantities a or b must be 0.

9. $\frac{3x}{x-1} + 2 = \frac{3}{x-1}$. There are three common mistakes made in the solution of equation like this one. The first is step in the solution is to multiply both sides of the equation by the common denominator

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of the fractions. One mistake is a failure to multiply all terms on one side of the equation by that quantity. The second involves “bookkeeping”. In multiplying $(x-1) \cdot \frac{3x}{x-1}$ what is happening is $\frac{x-1}{1} \cdot \frac{3x}{x-1} = \frac{(x-1) \cdot 3x}{x-1} = \frac{(x-1)}{(x-1)} \cdot \frac{3x}{1} = 3x$. We see that in the end result the denominator has been “canceled”. The mistake made by many students is not noting that in the end it is also “canceled” from the numerator as well, getting $(x-1) \cdot \frac{3x}{x-1} = 3x(x-1) + 3x^2 - 3x$. The third error stems from the fact that in multiplying by $x - 1$ we do not know its value at the time we multiply. Our rules specify that our solution is valid only if the number we multiply by is **not** 0. The third error is not checking that this criterion has been met. For this equation we would get $x = 1$. For this value of x we would be multiplying by zero so there is no valid solution.

10. $x=7$ so $x=\sqrt{7}$. In looking for solutions we are trying to find all numbers which can be substituted into the equation and produce a true statement. Here we have only considered the positive solution. The correct answer should include both the positive and negative solution $x=\pm\sqrt{7}$

11. $3x^3=2x^2$. Here the mistake is dividing by the common factor x^2 to get $3x=2$ or $x=\frac{2}{3}$. While multiplying by an expression involving the variable as in 9 can introduce invalid (extraneous) solutions, dividing by an expression involving the variable can cause you to lose valid solutions. Two-thirds is a valid solution but so is 0 which has been “lost”. The correct steps to finding all solutions should be: $3x^3 - 2x^2=0$ so $x^2(3x-2) = 0$. Since one of these factors must be 0 we get $x = 0$ or $x = \frac{2}{3}$.