A Model of Occupational Choice, Offshoring and Immigration

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Abstract
This paper develops a two-country model of offshoring and immigration with occupational choice and endogenous firm productivity. Individuals in Home choose to become entrepreneurs or workers, whereas those in Foreign can only be employed as workers in either country. Entrepreneurs produce output using a fixed set of tasks that can be performed locally or abroad. The model is used to investigate the impact of offshoring and immigration policies on occupational choice, task allocation, productivity, income inequality, and welfare. The model yields several interesting findings, but most notably it predicts that lowering offshoring costs generates job polarization, and pro-offshoring and pro-immigration policies may not be welfare improving in Home.

JEL Classification: F1, J2, J3, J6, L1
Keywords: Entrepreneurship, Occupational Choice, Inequality, Offshoring, Immigration
1 Introduction

Cheaper labor in developing economies combined with lower trade barriers and improved communication channels over the last three decades have intensified offshoring among companies in the developed countries. For instance, many high-tech firms have shut down their manufacturing process in the U.S. and moved to China and other low-wage Asian countries.\textsuperscript{1} During the same period, immigration to advanced countries has increased substantially as well. For example, immigrants’ share of the total U.S. population rose significantly from about 6 percent in 1980 to more than 13 percent in 2014.\textsuperscript{2} The trends in offshoring and immigration have been hotly debated among policy makers and academics over their economic and social implications.

This paper develops a two-country, general equilibrium model of trade to investigate the impact of offshoring and immigration on occupational choice, firm productivity, income inequality, and welfare. Home is populated by natives with different levels of managerial ability and immigrants from Foreign. Individuals in Home choose to become entrepreneurs or workers depending on their abilities. Home competitively produces two homogeneous goods using labor as the sole factor of production. Good 1 is produced using only native workers and serves as a numeraire in the model. Good 2 is produced by a continuum of heterogeneous firms each run by an entrepreneur using a fixed set of tasks that can be performed locally or abroad as in Grossman and Rossi-Hansberg (2008). Entrepreneurs can improve their firm productivity by investing in managerial capital, and investment is less costly for more able entrepreneurs.

Tasks are imperfect substitutes and can be performed by native, immigrant, or offshore workers as in Ottaviano et al. (2013). Offshoring not only incurs variable (per-unit) costs, but also fixed costs which induce only more able entrepreneurs to offshore. To simplify the analysis, I assume that Foreign is populated by workers (i.e., no entrepreneurs), and produces good 1 and performs tasks for firms in Home. In addition, as indicated above, workers in Foreign can immigrate to Home, but they will be less productive than Home’s native workers because of their communication difficulties with natives and unfamiliarities with rules. Using this unified framework, I investigate implications of further exposure to offshoring and making immigrants more integrated to Home so that they become more

\textsuperscript{1}The Economist’s (2013) special report on outsourcing and offshoring gives several examples of high-tech firms that moved their operations to China and other Asian countries.

\textsuperscript{2}These statistics are calculated using the current population survey (CPS) annual files.
productive. Since individuals in Foreign are identical and employed as workers, I will mainly focus on implications for Foreign.

Reducing variable offshoring costs increases the set of tasks performed by offshore workers by downgrading tasks performed by immigrant workers and upgrading tasks performed by native workers. This policy generates a job polarization by increasing the mass of workers and offshoring entrepreneurs at the expense of the middle-skill entrepreneurs. Under this policy, offshoring firms acquire more managerial capital (hence, have higher firm productivity), whereas non-offshoring firms acquire less managerial capital (hence, have lower firm productivity). Reducing variable offshoring costs increases the income inequality between entrepreneurs and workers, but improves the aggregate welfare.

Another offshoring policy that I consider is a reduction in fixed offshoring costs. This policy also leads to job polarization and increases the income inequality between workers and entrepreneurs as in the previous policy. However, reducing fixed offshoring costs does not have any impact on task allocation across different worker groups. This policy may also lead to a welfare loss when individuals do not have strong preferences for good 2.

The immigration policy that makes immigrants more integrated to Home induces them to perform more complex tasks. Since immigrants are more intensively used by non-offshoring firms, this policy will make non-offshoring firms relatively more profitable; as a result, the number of non-offshoring entrepreneurs increases and that of offshoring ones decreases. Under this policy, non-offshoring (offshoring) firms acquire more (less) managerial capital. Making immigrant workers more integrated to Home reduces the income inequality between workers and entrepreneurs, and generally improves the aggregate welfare. However, when consumers’ preferences for good 2 are too low the policy may lead to a welfare loss.

These results are generally consistent with recent empirical studies. For example, Barba Navaretti et al. (2008) find that offshoring is more prevalent among larger and more productive firms in Italy. Similarly, using Chilean plant-level data, Kasahara and Lapham (2013) find that firms importing intermediate goods tend to be larger and more productive. The finding that lowering variable offshoring costs leads to task upgrading of native workers and task downgrading for immigrants in offshoring firms finds support from Ottaviano et al. (2013). The prediction that making immigrants more integrated to Foreign induces task upgrading of immigrant workers is also consistent with Ottaviano et al. (2013). The finding that reducing variable offshoring costs induces firms to import more intermediate goods is consistent with Goldberg et al. (2010) who find that lower input tariffs increased
new input varieties in India. Using Hungarian firm-level data, Halpern et al. (2015) find that using more imported inputs increases firm productivity significantly (see also Topalova and Khandelwal 2011).

Recent studies in labor economics have documented a substantial change in the pattern of employment in the U.S. and European labor markets over the last three decades. In particular, employment shares have risen in the highest and lowest end of the skill distribution at the expense of the middle-skill employment (Goos et al. 2009, Autor and Dorn 2013). Technology and offshoring are argued to be the main force behind this job polarization (Acemoglu and Autor 2011). My model theoretically complements this literature by showing that offshoring indeed can generate job-polarization. In addition, the possibility that firms can endogenously determine their productivity increases the job-polarization effect of offshoring.

This paper is related to recent literature that explores task trade introduced by Grossman and Rossi-Hansberg (2008). Baldwin and Robert-Nicoud (2014) develop a model with trade in goods and trade in tasks to analyze to what extent gains from trade and theorems in the Heckscher-Ohlin model change. Groizard et al. (2014) incorporate offshoring into Melitz’s (2003) model to study the impact of offshoring on unemployment. Egger et al. (2015) present a monopolistic-competition model of trade with occupational choice, and find that an exposure to offshoring may lead to a welfare loss. Unel (2016) develops a small-open-economy model of offshoring with unemployment to study the impact of credit constraints on offshoring and unemployment.3

There are relatively few papers that have studied offshoring and immigration in a unified framework. Barba Navaretti et al. (2008) develop a simple offshoring model to investigate the impact of offshoring on native and immigrant workers using firm-level data from Italy.4 They find that offshoring decreases the demand for native and immigrant workers. Olney (2012) develops a partial equilibrium model to compare the impact of offshoring and immigration on wages of U.S. native workers, and finds that offshoring has a more positive impact on low-skilled wages than immigration. He assumes that immigrants are identical to natives in performing tasks, and the supply of skilled and unskilled workers are exogenously fixed. Mandelman and Zlate (2014) develop a stochastic growth model where offshoring and

3 This paper also complements the literature that incorporates occupational choice into trade models. See, for example, Monte (2011), Egger and Kreichenmeir (2012), Dinopoulos and Unel (2015 & 2016), and Unel (2015) among many others.

4 Although they empirically investigate the impact of offshoring on immigrant workers, their model does not explicitly considers immigrants.
immigration can jointly generate job polarization. Their model has two large symmetric countries and a small country that is the source of unskilled immigrants. Furthermore, tasks are produced using only skilled workers and there is a two-way offshoring between the two large economies.

A more related paper in this literature is Ottaviano et al. (2013) who, using data on the US manufacturing industries over 2000–2007, investigate how declines in offshoring and immigration costs affect the employment of native workers. They find that lowering offshoring costs leads to task upgrading of natives and task downgrading of immigrants, whereas a reduction in immigration costs leads to task upgrading of immigrants but has no effect on the task complexity of natives. For their empirical analysis, they develop a partial equilibrium model where supply of workers are fixed (i.e., no occupational choice), firms in each sector are identical (i.e., no firm heterogeneity), and offshoring does not incur any fixed costs.

The rest of this paper is organized as follows. The next section introduces the model and discusses its equilibrium properties. Section 3 presents a series of comparative static exercises to study the impact of offshoring and immigration policies on occupational choice, firm productivity, income inequality, and welfare. Section 4 concludes the paper.

2 Setup of the Model

There are two countries, Home and Foreign, that consume two homogeneous goods produced in perfectly competitive markets. Home is populated by *natives* with total mass one and *immigrants* from Foreign whose population is endogenously determined. Natives differ with respect to their managerial ability, and endogenously choose to become an entrepreneur or worker. Immigrants do not have any managerial ability, and thus are employed as workers. In Home, both goods are produced using labor as the only factor of production. Good 1 is produced using only native workers, whereas good 2 is produced by a continuum of entrepreneurs each using her managerial ability and a fixed set of tasks that can be performed by native, immigrant, or offshore workers.\(^5\)

Foreign is initially populated by workers with constant mass \(L^*\), some of whom immigrate to Home. Foreign produces good 1 and performs tasks for firms in Home (and thus, good 2 is not produced there). In my subsequent analysis, all variables related to Foreign

\(^5\)Allowing immigrants to produce good 1 makes the exposition notationally complicated without changing the main results.
are denoted by an asterisk, and for the sake of brevity, I will only show expressions for Home when this causes no confusion.

2.1 Preferences

Individuals’ preferences are described by the following Cobb-Douglas utility function

\[ u = \left( \frac{q_1}{1-\theta} \right)^{1-\theta} \left( \frac{q_2}{\theta} \right)^{\theta}, \]  

(1)

where \( q_i \) is consumption of good \( i = 1, 2 \), and \( \theta \in (0, 1) \) is an exogenous, constant parameter.

Let \( e \) denote individual expenditure. The demand for each good is given by

\[ q_1 = (1 - \theta)e/p_1, \quad q_2 = \theta e/p_2, \]  

(2)

where \( p_i \) is the price of good \( i \). Good 1 is chosen as the numeraire by setting its price equal to one (i.e., \( p_1 = 1 \)), and for notational simplicity, the price of good 2 is denoted by \( p \).

Substituting \( q_i \) from (2) into (1) yields the indirect utility

\[ v(e, p) = p^{-\theta}e, \]  

(3)

and aggregating (3) across all individuals yields \( V(p) = p^{-\theta}E \), where \( E \) denotes aggregate expenditure (income).

2.2 Production

Good 1 in Home is competitively produced using only native workers. Production of one unit of good 1 requires one unit of native worker, and perfect competition implies that their wage equals one, i.e. \( w = 1 \). In Foreign, production of one unit of good 1 requires \( 1/w^* \) units of worker, where \( w^* < 1 \) is an exogenous constant. It is assumed that the final goods are freely traded between two countries, which implies that the wage in Foreign equals \( w^* \).

Good 2 is produced only in Home by a continuum of heterogeneous firms, each owned and managed by an entrepreneur under perfect competition. Entrepreneurs differ with respect to their managerial capital (or firm productivity), and an entrepreneur with managerial capital \( z \) produces output according to

\[ y_2(z) = \left( \frac{z}{1-\eta} \right)^{1-\eta} \left( \frac{L}{\eta} \right)^{\eta}, \]  

(4)
where $\eta \in (0,1)$ is an exogenous parameter that measures the labor share in production and $L$ is a composite labor. The restriction $\eta \in (0,1)$ ensures that firms have finite size, and thus $\eta$ also measures managers’ span of control (Lucas, 1978). The composite labor $L$ is produced by assembling a set of differentiated tasks as follows:

$$L = \left( \int_0^1 l(j)^\frac{\sigma - 1}{\sigma - 1} dj \right)^\frac{\sigma}{\sigma - 1},$$

(5)

where $\sigma$ represents the elasticity of substitution between any two tasks. Task assembly technology (5) is a generalization of Grossman and Rossi-Hansberg’s (2008) model where tasks are perfect complements (i.e., $\sigma \to 0$). I hereafter refer to tasks as gross substitutes (complements) when $\sigma \geq 1$ ($\sigma < 1$).

Each task is produced using only workers under constant returns to scale technology. As in Ottaviano et al. (2013), each task can be performed in one of the following three ways: production by native workers, production by immigrant workers, or production by workers in Foreign. Production of one unit of any task requires one unit of native workers, and thus the unit cost of production of any task by a native worker is one, i.e. $c(j) = 1$.

If task $j$ is performed by immigrant workers, production of one unit of the task requires $\beta_{mtm}(j)/w^*$ units of immigrant workers. The parameter $\beta_m$ covers the cost of immigration as well as immigrants’ communication difficulties with natives and unfamiliarity with institutional rules in Home. Thus, a reduction in $\beta_m$ makes immigration and immigrants’ integration to Home easier. Function $t_m(j)$ measures heterogeneity across tasks, and it is assumed that tasks are indexed in increasing order of complexity, i.e. $d t_m(j)/dj > 0$. Assuming that immigration from Foreign does not entail any other costs, the wage of immigrant workers will be $w^*$, and thus the marginal cost of producing task $j$ by immigrant workers is $c(j) = \beta_{mtm}(j)$.

Entrepreneurs wishing to offshore face both fixed and variable costs of offshoring. They first must pay an irreversible fixed costs $f_o$ measured in terms of good 1. The fixed cost $f_o$ covers foreign market entry costs as well as coordinating the performance of tasks to be produced abroad. Upon paying $f_o$, performing one unit of the task $j$ requires $\beta_o t_o(j)/w^*$

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6Division by $w^*$ is done to simplify the notation and it is assumed that $\beta_{mtm}(j)/w^* > 1$ for all $j \in [0,1]$ so that tasks are performed more efficiently by native workers.

7The above approach slightly differs from Ottaviano et al. (2013) who assume that production of one unit of task $j$ requires $t_m(j)$ units of immigrant and an immigrant worker endowed with one unit of labor is able to provide only $1/\beta_m$ units of labor in Home. In their case, the immigration indifference condition implies that the wage paid to immigrants is $w_m = w^*\beta_m$, and the marginal cost of producing task $j$ is $c(j) = w^*\beta_m t(j)$. Both approaches clearly yield the same conclusions.
units of foreign workers. The parameter $\beta_o$ reflects the overall state of communication technology between the two countries as well as variable trade costs such as transport and tariffs, and $t_o(j)$ captures the heterogeneity in productivity across tasks. It is again assumed that $t_o(j)$ is continuously differentiable with $dt_o(j)/dj > 0$. It then follows that the marginal cost of performing task $j$ in Foreign is $c(j) = \beta_o t_o(j)$.

**Assumption 1.** The following conditions hold.

i. $\beta_m t_m(0) < \beta_o t_o(0) < 1$ and $1 < \beta_o t_o(1) < \beta_m t_m(1)$;

ii. $\beta_m t_m(j)$ and $\beta_o t_o(j)$ intersect only once at some $j_1 \in (0, 1)$ such that $\beta_m t_m(j) \leq \beta_o t_o(j) < 1$ for all $j \leq j_1$.

If a firm does not offshore, it divides its tasks between immigrants and natives. In this case, as shown in Figure 1, all tasks with $j < j_2$ will be performed by immigrants, whereas the rest by natives. Consequently, the marginal cost of producing a task is given by

$$c_d(j) = \begin{cases} 
\beta_m t_m(j) & j \in [0, j_2), \\
1 & j \in [j_2, 1].
\end{cases}$$
Using the above cost function, the cutoff $j_2$ is given by

$$t_m(j_2) = \frac{1}{\beta_m}.$$  

(7)

Given that tasks are ranked in increasing order of complexity, the task cutoff $j_2$ decreases as the parameter $\beta_m$ increases. In addition, the cutoff $j_2$ is independent of the managerial capital $z$, i.e. the set of tasks performed by immigrants is the same for all entrepreneurs producing domestically.

Assumption 1 implies that for an offshoring firm the tasks are divided into three groups as shown in Figure 1. All tasks with $j < j_1$ are performed by immigrants, tasks with $j \in [j_1, j_3)$ by foreign workers, and tasks with $j \geq j_3$ by natives. Thus, the marginal cost of producing a task is given by

$$c_o(j) = \begin{cases} 
\beta_m t_m(j) & j \in [0, j_1), \\
\beta_o t_o(j) & j \in [j_1, j_3), \\
1 & j \in [j_3, 1]. 
\end{cases}$$  

(8)

Using this cost distribution, the cutoffs $j_1$ and $j_3$ are given by

$$\frac{t_o(j_1)}{t_m(j_1)} = \frac{\beta_m}{\beta_o}, \quad t_o(j_3) = \frac{1}{\beta_o}.$$  

(9)

Given that $t_o'(j) > 0$, the cutoff $j_3$ decreases as the variable offshoring cost $\beta_o$ increases. Since $t'_m(j_1) > t'_o(j_1)$ as shown in Figure 1, the task cutoff $j_1$ decreases in $\beta_m$ and increases in $\beta_o$. Note that both cutoffs are independent of the fixed offshoring cost $f_o$ and the managerial capital $z$. The following lemma summarizes these results (see Appendix A.1 for formal proofs).

**Lemma 1.** Consider Home just described.

i. $d j_1 / d \beta_m < 0$, $d j_2 / d \beta_m < 0$, and $d j_3 / d \beta_m = 0$;

ii. $d j_1 / d \beta_o > 0$, $d j_2 / \beta_o = 0$, and $d j_3 / d \beta_o < 0$;

iii. $d j_i / dz = 0$ and $d j_i / d f_o = 0$ for $i = 1, 2, 3$.

The division of tasks across worker types differs from Ottaviano et al. (2013) in one important aspect. Their model does not include firm heterogeneity or fixed offshoring costs (since they empirically investigate the effects of immigration and offshoring on the average task assignments across worker types). Consequently, all firms offshore and tasks
are performed by three different worker groups as discussed above. In the present set-
up, firm heterogeneity and fixed offshoring costs will induce only more productive firms to
offshore; as a result, there will be non-offshoring firms that only use native and immigrant
workers to perform their tasks.

I now turn to discuss formation of managerial capital \( z \). If an individual chooses to
become an entrepreneur, she can invest to improve the managerial capital (firm productiv-
ity). Following insights from the human capital theory (Becker 1994, Acemoglu 1996), I
assume that an entrepreneur with managerial ability \( a \) must spend \( \lambda z^2/2a \) units of good \( 1 \)
to acquire \( z \) units of managerial capital, where \( \lambda > 0 \) is an exogenous parameter. The costs
of managerial capital decline with the level of ability and increase with the level of man-
gerual capital. Ability levels are drawn from a common, exogenous cumulative distribution
\( G(a) \) with density \( g(a) \) and support \([1, \infty)\). To simplify the subsequent exposition, following
Helpman et al. (2010), I assume that ability levels follow the following Pareto distribution:

\[
G(a) = 1 - a^{-k},
\]
where \( k \) is the shape parameter. To ensure aggregate variables have finite values I assume
that \( k > 1 \).

An entrepreneur with managerial ability \( a \) maximizes her earnings \( e_2(a) \) by choosing
the amount of task \( l(j) \) and whether to offshore:

\[
e_2(a) \equiv \max_{l_o, l(j), z} \left\{ py_2(z) - \int_0^1 c(j)l(j) dj - \mathbb{I}_o f_o - \frac{\lambda z^2}{2a} \right\},
\]
where \( y_2(z) \) is given by (4) and \( \mathbb{I}_o \) is an indicator function that equals one if the firm
offshores, and zero otherwise.

Let \( L_d \) and \( L_o \) denote composite labor inputs used by non-offshoring and offshoring
firms, respectively; and \( W_d \) and \( W_o \) denote aggregate wages associated with these labor
inputs so that \( W_v L_v = \int_0^1 c_v(j)l_v(j) dj \) (hence, \( W_v \) denotes the unit composite-labor cost).
As shown in Appendix A.2, these aggregate wages are given by

\[
W_d = \left[ \int_0^{j_2} [\beta_m t_m(j)]^{1-\sigma} dj + 1 - j_2 \right]^{\frac{1}{1-\sigma}},
\]
\[
W_o = \left[ \int_0^{j_1} [\beta_m t_m(j)]^{1-\sigma} dj + \int_{j_1}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj + 1 - j_3 \right]^{\frac{1}{1-\sigma}}.
\]

Note that \( W_o \) does not depend on firm productivity, and thus \( W_d \) is the same for all non-
offshoring firms and \( W_o \) is the same for offshoring ones. In addition, Appendix A.2 shows
that offshoring firms face a lower aggregate wage than domestic firms, i.e. $W_o < W_d$.

Since markets are perfectly competitive, production technology (4) implies that $y_{2v}(z) = W_v L_v / (\eta p)$, and substituting this back into (4) yields

$$L_v = \frac{\eta z}{1 - \eta} \left( \frac{p}{W_v} \right)^{\frac{1}{1-\eta}}, \quad y_{2v}(z) = \frac{z}{1 - \eta} \left( \frac{p}{W_v} \right)^{\frac{\eta}{1-\eta}},$$

where the second expression is obtained by substituting $L_v$ back into (4). Substituting $y_{2v}(z)$ from (13) and $W_v L_v = \eta p y_{2v}(z)$ into profit function (11), and then maximizing the resulting expression with respect to $z$ yields

$$z_v(a) = \frac{a}{\lambda} \left( \frac{p}{W_v^\eta} \right)^{\frac{1}{1-\eta}}, \quad v = d, o.$$  \hfill (14)

Finally, substituting $z$ back into profit function (11) yields

$$e_{2v}(a) = \frac{a}{2\lambda} \left( \frac{p}{W_v^\eta} \right)^{\frac{2}{1-\eta}} - I_o f_o.$$  \hfill (15)

### 2.3 Occupational Choice

I begin my analysis by considering allocation of ability. Immigrants choose to become workers, since they have the lowest ability level. A native individual chooses to become an entrepreneur if her entrepreneurial income is greater than the wage that she earns as a worker, i.e. $e_2(a) \geq w = 1$. The ability cutoff (denoted by $a_d$) at which an individual is indifferent between being an entrepreneur or a worker is given by $e_{2d}(a_d) = 1$. Using (15) with $I_o = 0$ and $v = d$ yields

$$a_d = 2\lambda \left( \frac{W_d^\eta}{p} \right)^{\frac{2}{1-\eta}}.$$  \hfill (16)

Now consider the decision to offshore. An entrepreneur chooses to offshore if $e_{2o}(a)\{I_o = 1\} \geq e_{2d}(a)\{I_o = 0\}$, where $e_{2v}(a)$ is given by (15). The ability cutoff level (denoted by $a_o$) necessary for offshoring is determined when the inequality holds with equality, which yields

$$a_o = \frac{2\lambda f_o}{(W_d/W_o)^{2\eta/(1-\eta)} - 1} \left( \frac{W_d^\eta}{p} \right)^{\frac{2}{1-\eta}},$$  \hfill (17)

where $W_d$ and $W_o$ are given by (12a) and (12b), respectively. Combining equations (16) and (17) yields

$$a_o = A a_d, \quad A \equiv \frac{f_o}{(W_d/W_o)^{2\eta/(1-\eta)} - 1}.$$  \hfill (18)
It is assumed that $f_o$ is sufficiently high so that $A > 1$; as a result, only more able entrepreneurs offshore.

With these ability cutoffs, native workers in Home are divided into three groups: those with $a < a_d$ become workers; those with $a \in [a_d, a_o)$ become entrepreneurs and produce domestically; and finally, those with $a \geq a_o$ become entrepreneurs and offshore the same set of tasks to Foreign. All immigrants are workers. Using the above ability cutoffs, one can express the managerial capital and entrepreneurial income in Home as follows:

$$z_v(a) = a \left( \frac{2}{\lambda a_d} \right)^{\frac{1}{2}} \left( \frac{W_d}{W_v} \right)^{\frac{1}{1-\eta}}, \quad (19a)$$

$$r_{2v}(a) = \frac{2a}{(1-\eta)a_d} \left( \frac{W_d}{W_v} \right)^{\frac{2\eta}{1-\eta}}, \quad (19b)$$

$$e_{2v}(a) = a \frac{W_d^{\frac{2\eta}{1-\eta}}}{a_d W_v^{\frac{2\eta}{1-\eta}}} - f_o, \quad (19c)$$

where $r_{2v}(a) = py_{2v}(a)$ represents firm revenue and $W_v$ is the aggregate price index given by (12).

### 2.4 Income Distribution and Welfare

Income of a native worker is $w = 1$ and the entrepreneurial income is given by (19c). It then follows that the income distribution across natives in Home is given by

$$e(a) = \begin{cases} 
1 & \text{if } a < a_d, \\
\frac{a}{a_d} & \text{if } a \in [a_d, a_o), \\
\frac{a W_d^{\frac{2\eta}{1-\eta}}}{a_d W_v^{\frac{2\eta}{1-\eta}}} - f_o & \text{if } a \geq a_o. 
\end{cases} \quad (20)$$

Using $v(a, p) = p^{-\theta} e(a)$ together with (20) yields the welfare distribution in Home. The income of an immigrant or offshore worker is $w^*$, and thus her welfare is given by $v^* = w^* p^{-\theta}$.

With this distribution assumption, one can calculate the average entrepreneurial income to make between-group comparisons. Note that the mass of entrepreneurs in Home is $a_d^{-k}$. Using (20) and (10), it is easy to show that the average entrepreneurial income is given by

$$\bar{e}_2 = \frac{k + f_o A^{-k}}{k-1}, \quad \bar{e}_{2o} = \frac{f_o + kA}{k-1}, \quad (21)$$

*For example, combining equations (12) and (16) yields $z(a)$; and combining (15) and (16) yields $e_2(a)$. 
where \( A \) is given by (18) and \( \bar{e}_{2o} \) is the average income of offshoring entrepreneurs. In addition, using (20) and (10), the aggregate income in Home is given by

\[
E = 1 + w^* M + \frac{1 + f_o A^{-k}}{(k-1) a_d^k},
\]

where \( M \) is the mass of immigrants that will be determined shortly.

Home’s aggregate welfare is given by

\[
V = p - \theta E.
\]

However, from a policy perspective it is more interesting to investigate how immigration and offshoring policies affect Home’s national welfare, which is defined as the aggregate welfare across all individuals born in Home, which is given by

\[
V^n = p - \theta + \left(1 + f_o A^{-k}\right)p - \theta (k-1) a_d^k,
\]

where \( p \) and \( a_d \) are related with each other through equation (16).

### 2.5 The Open-Economy Equilibrium

I begin my analysis by determining the total amount of labor used in each sector. As shown in Appendix A.3, the total number of natives and immigrants working in sector 2 are respectively given by

\[
L_2 = \frac{bW^o_{d}^{-1}}{a_d^k} \left[ (1 - A^{1-k})(1 - j_2) + (1 - j_3)A^{1-k} \left(\frac{W_o}{W_d}\right)^{\frac{\sigma - \frac{1+n}{1-\eta}}{1-\sigma}} \right],
\]

\[
M = \frac{bW^o_{d}^{-1}}{w^*a_d^k} \left[ (1 - A^{1-k}) \int_0^{j_2} [\beta_{m} t_{m}(j)]^{1-\sigma} dj + A^{1-k} \left(\frac{W_o}{W_d}\right)^{\frac{\sigma - \frac{1+n}{1-\eta}}{1-\sigma}} \int_0^{j_3} [\beta_{m} t_{m}(j)]^{1-\sigma} dj \right],
\]

where \( b = 2k\eta/[(k-1)(1-\eta)] \). The number of natives workers in sector 1 is \( L_1 = G(a_d) - L_2 \).

The total amount of labor used in performing tasks in Foreign is

\[
L_o^* = \frac{bA^{-k+1}W^o_{d}^{\frac{2\eta}{1-\eta}} \int_0^{j_3} [\beta_{o} t_{o}(j)]^{1-\sigma} dj}{w^*a_d^k W_o^{\frac{\sigma - \frac{1+n}{1-\eta}}{1-\sigma}}},
\]

where \( b = 2k\eta/[(k-1)(1-\eta)] \), \( W_d \) and \( W_o \) are given by (12a) and (12b), and \( A \) is given by (18) (Appendix A.3). The amount of labor used in production of good 1 then is \( L_1^* = L^* - L_o^* - M \).

Since production of each unit of good 1 uses one unit of native worker, the total amount of good 1 produced in Home equals \( Y_1 = L_1 \), which also equals the total revenue generated

\[
E^* = w^* (L^* - M) \quad \text{and} \quad V^* = p^{-\theta} E^*.\]

Since individuals are identical in Foreign, welfare analysis at individual level is more informative.
by this sector. Furthermore, using \( pY_2 = \int py_2(a)da \), the total revenue generated by each sector is given by

\[
Y_1 = G(a_d) - L_2, \quad pY_2 = \frac{2k(1 + f_oA^{-k})}{(k - 1)(1 - \eta)a_d^k}, \tag{26}
\]

where \( A \) is given by (18) and \( L_2 \) is given by (24a). Since production of one unit of good 1 requires \( 1/w^* \) units of workers in Foreign, the total amount of good 1 produced in Foreign is \( Y_1^* = w^*(L^* - L_o^* - M) \), where \( L_o^* \) is given by (25).

Demand function (2) implies that \( pQ_2 = \theta E \), where \( Q_2 \) is the total quantity demanded for good 2 in Home and \( E \) is the aggregate income given by (22). It then follows that

\[
pQ_2 = \theta(E + E^*), \tag{27}
\]

where \( Q_2^* \) and \( E^* \) are the corresponding aggregate variables for Foreign.

In equilibrium, the total (net) supply of each good must be equal to the total demand for the good; as a result, \( Q_2 + Q_2^* = Y_2 \), since Foreign does not produce good 2.\(^{10} \)

Substituting \( Q_2 + Q_2^* = Y_2 \) into (27) yields

\[
pY_2 = \theta(E + E^*).
\]

Substituting (22) and (26) into the above equilibrium condition yields

\[
a_d = \left\{ \frac{[2k - \theta(1 - \eta)](1 + f_oA^{-k})}{\theta(k - 1)(1 - \eta)(1 + w^*L^*)} \right\}^{\frac{1}{k}} \tag{28}
\]

**Lemma 2.** The ability cutoff \( a_d \) is uniquely determined and given by equation (28).

Once the ability cutoff \( a_d \) is determined, one can easily determine other endogenous variables. For example, substituting \( a_d \) into (16) yields

\[
p = \left\{ \frac{\theta(k - 1)(1 - \eta)(1 + w^*L^*)(2\lambda)^k W_d^{2k\eta/(1-\eta)}}{[2k - \theta(1 - \eta)](1 + f_oA^{-k})} \right\}^{\frac{1-\eta}{2k}}, \tag{29}
\]

where \( W_d \) and \( A \) are given by (12a) and (18), respectively. Similarly, substituting \( a_d \) into (17) yields the ability cutoff \( a_o \).

\(^{10}\)For good 1, the corresponding equation is \( Q_1 + Q_1^* = Y_1 - C_1 + Y_1^* \), where \( C_1 \) is the amount of good 1 used in formation of managerial capital and foreign-market entry (see Appendix A.3 for calculation of \( C_1 \)).
3 Offshoring and Immigration Policies

This section presents a series of comparative static exercises to investigate implications of offshoring and immigration policies for occupational choice, firm productivity, income inequality, and welfare in Home.

3.1 Offshoring Policies

I begin my analysis by considering a reduction in the variable offshoring cost $\beta_o$. According to Lemma 1, a reduction in $\beta_o$ increases the set of tasks offshored, but has no impact on the set of tasks performed by immigrant workers at non-offshoring firms. Consequently, the aggregate wage index $W_o$ decreases, but $W_d$ remains unchanged.

Reducing $\beta_o$ intuitively makes offshoring cheaper and more profitable, which in turn induces more entrepreneurs to offshore (i.e., $a_o \downarrow$). Since offshoring entails fixed entry costs, the increased demand for good 1 by firms wishing to offshore increases the relative price of good 1 (i.e., $p \downarrow$), and thus forces less able entrepreneurs to become workers (i.e., $a_d \uparrow$).

In sum, reducing the variable offshoring cost $\beta_o$ decreases the mass of entrepreneurs, but increases the mass of offshoring entrepreneurs (see Appendix A.4 for formal proofs).

Lemma 3. In the open-economy equilibrium, $da_d/d\beta_o < 0$ and $da_o/d\beta_o > 0$.

As the relative price of good 1 rises, acquiring managerial capital becomes more expensive; consequently, non-offshoring firms acquire less managerial capital and generate lower entrepreneurial income as indicated by (14) and (15), respectively. For offshoring firms, however, as the cost of offshoring $\beta_o$ falls, the unit-labor cost falls as well ($W_o \downarrow$). As argued by Grossman and Rossi-Hansberg (2008), this cost reduction can be thought much the same as an increase in the productivity of workers. In the present context, reducing $\beta_o$ also directly increases the productivity of offshoring firms, because additional profits obtained from lower production costs induce these firms to increase their managerial capital $z$. With improved productivity, incumbent offshoring firms generate higher entrepreneurial income as well.\(^\text{11}\)

\(^{11}\)Differentiating $T = a_o W_o^{2\eta/(1-\eta)}$ with respect to $\beta_o$ and using (A.8) from the appendix yields

$$\frac{dT}{d\beta_o} = \frac{2\eta(1-A^{1-k})T}{(1-\eta)(1+f_o A^{-k})W_o} \frac{dW_o}{d\beta_o} > 0,$$

where $dW_o/d\beta_o > 0$ is given by equation (A.6) in the appendix.
Figure 2 shows the impact of reducing $\beta_o$ on income distribution in Home. There are two important points related to the distributional change. First, the policy leads to a job polarization in the sense that two ends of the ability distribution (i.e., workers and offshoring entrepreneurs) increase their shares in labor supply and aggregate income, whereas the middle group in the distribution experiences a reduction in the corresponding shares. Second, since a reduction in $\beta_o$ reduces $A$ as shown in Appendix A.4, equations in (21) imply that the average income of all entrepreneurs increases, but the average income of offshoring entrepreneurs decreases. Thus, the income inequality between all entrepreneurs and workers increases, but that between offshoring entrepreneurs and workers decreases. The latter one decreases because the supply of offshoring entrepreneurs increases and the entrants make lower income than the incumbents.

Under this policy, workers and entrepreneurs with $a > \bar{a}$ become better off, while entrepreneurs with $a < \bar{a}$ become worse off. Thus, there is a welfare polarization as well. However, national welfare $V^n$ still increases. A reduction in the relative price $p$ clearly makes everyone from Foreign better off.

12Note that $v = ep^{-\theta} \propto p^{2/(1-\eta)-\theta}$ for non-offshoring firms. Since $2/(1-\eta) - \theta > 0$, a reduction in $p$ reduces their welfare.

13It follows from (22) that Home’s national income is given by $E^n = 1 + (a_{d1}^{-k} f_{o} a_{w}^{-k})/(k - 1)$. Differentiating $E^n$ with respect to $\beta_o$ and using $da_{d1}/d\beta_o$ and $da_{o2}/d\beta_o$ from Appendix A.4 yields $dE^n/d\beta_o = 0$, i.e. a reduction $\beta_o$ does not change the national income. Since $V^n = p^{-\theta} E^n$ and $dp/d\beta_o > 0$, it then follows that national welfare $V^n$ increases.
Proposition 1. Consider the two trading economies as described. A reduction in the variable cost of offshoring $\beta_o$ has the following effects in Home. It

a. generates a job polarization by increasing the mass of workers and offshoring entrepreneurs;

b. increases (decreases) productivity of offshoring (non-offshoring) firms;

c. increases (decreases) the inequality between workers and all (offshoring) entrepreneurs;

d. improves the aggregate welfare.

The results are largely consistent with the recent empirical studies. The finding that lowering variable offshoring costs leads to task upgrading of native workers and task downgrading for immigrants in offshoring firms are in line with Ottaviano et al. (2013). The prediction that reducing the variable offshoring cost $\beta_o$ induces offshoring firms to import more intermediate goods is consistent with Goldberg et al. (2010) who, using firm-level data from India, show that lower input tariffs increases new input varieties. Topalova and Khandelwal (2011) analyze India’s externally imposed trade reform in 1991, and show that lower input tariffs appear to have increased firm-level productivity. Relatedly, using Hungarian micro data, Halpern et al. (2015) find that using more imported inputs increase firm productivity significantly. In particular, they attribute a quarter of Hungarian productivity growth over the 1993–2002 period to imported inputs.

The prediction that lowering the variable offshoring cost $\beta_o$ generates a job polarization in Home is interesting, because it resembles what has happened in the U.S. and European labor markets over the past three decades. Specifically, the recent empirical studies have documented a substantial increase in the share of high-skill and low-skill employment in the US and many other developed countries at the expense of middle-skill employment (Goos et al. 2009, Acemoglu and Autor 2011, Autor and Dorn 2013). In addition, the wage distribution across these skill groups has followed a similar pattern as well. Technology and globalization are argued to be the main force behind this job polarization (e.g., Goos et al. 2009, Acemoglu and Autor 2011). The present model theoretically complements these studies by showing that offshoring indeed can generate job-polarization.15

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14 Similarly, Gopinath and Neiman (2014) show that the within-firm drop in imported varieties explains about 45 percent of the decline in imports during the Argentine crisis in 2001-2002.

15 It is important to emphasize that the job-polarization result is mainly driven by the general-equilibrium
Next I will consider implications of a reduction in the fixed-offshoring cost $f_o$. According to Lemma 1, this policy has no impact on any of the task cutoffs; consequently, it has no impact on the aggregate price indexes $W_d$ and $W_o$. However, a reduction in $f_o$ makes offshoring more profitable, and thus induces more able entrepreneurs to offshore (i.e., $a_o \downarrow$). The increased demand for good 1 by new offshoring firms raises the relative price of good 1 (i.e., $p \downarrow$), and thus exerts a negative effect on firm profits. In this case, less able entrepreneurs exit the market (i.e., $a_d \uparrow$). In sum, as in the previous offshoring policy, a reduction in the fixed-offshoring cost $f_o$ leads to a job polarization by increasing the shares of the two ends of the ability distribution (i.e., workers and offshoring entrepreneurs) in the labor supply.

**Lemma 4.** In the open-economy equilibrium, $da_d/df_o < 0$ and $da_o/df_o > 0$.

Since a reduction in $f_o$ increases the relative price of good 1, the cost of acquiring managerial capital goes up; as a result, all entrepreneurs acquire less managerial capital. The incomes of entrepreneurs who continue on producing domestically decline as shown in Figure 3. The impact of this policy on incomes of offshoring entrepreneurs is ambiguous. To see this, differentiating income function (20) with respect to $f_o$ and substituting $da_d/df_o$ from (A.10) from Appendix A.5 yields

$$\frac{\partial e_2(a)}{\partial f_o} = \frac{(a^{1-k} + f_o A^{-k})a}{(1 + f_o A^{-k})a_o} - 1,$$

where $A$ is given by (18). Note that $\partial e_2/\partial f_o > 0$ for sufficiently high values of $a$, i.e. entrepreneurs earning very top-income experience an income loss due to lower firm productivity. Furthermore, it is easy to see that $\partial e_2(a)/\partial f_o < 0$ at $a = a_o$, i.e. the income of entrepreneur with managerial talent $a_o$ increases as shown in Figure 3. Since reducing $f_o$ reduces $A$, equations in (21) imply that the inequality between workers and entrepreneurs increases, but that between workers and offshoring entrepreneurs decreases.

Since the relative price $p$ falls, reducing $f_o$ makes workers and entrepreneurs with $a \in [\bar{a}_1, \bar{a}_2]$ better off. Entrepreneurs with $a \in [\bar{a}_{d2}, \bar{a}_{o2}]$ and those with the high end of the structure of the model. If Home were a small, open economy so that the relative price $p$ is exogenously fixed, lowering offshoring cost $\beta_o$ would not generate the above job polarization. To see this, note that when $p$ is fixed, equation (16) implies that $a_d$ is independent of the offshoring costs; as a result, lowering $\beta_o$ has no impact on the ability cutoff $a_d$. It then follows from (20) that the income of a non-offshoring entrepreneur $e_{2d}(a) = a/a_d$ is also independent of offshoring cost $\beta_o$.
ability distribution will experience a welfare loss.\footnote{For non-offshoring entrepreneurs, differentiating $v$ from (3) with respect to $f_o$ yields $$\frac{\partial v}{\partial f_o} = \frac{(2 - \theta(1 - \eta))v \, da_d}{2a_d} \frac{da_d}{df_o} > 0,$$ because $da_d/df_o < 0.$} For all other individuals, the welfare effect is ambiguous and depends on the parameter $\theta$ that measures the share of good 2 in expenditure. Aggregating across all natives yields the same conclusion that reducing $f_o$ has an ambiguous impact on national welfare $V^m$ (Appendix A.6). As in the previous case, a reduction in $p$ makes everyone from Foreign better off.

**Proposition 2.** Consider the two trading economies as described. A reduction in the fixed cost of offshoring $f_o$ has the following effect on Home. It

- generates a job polarization by increasing the mass of workers and offshoring entrepreneurs;
- decreases productivity of all firms;
- increases (decreases) the inequality between workers and all (offshoring) entrepreneurs;
- has an ambiguous effect on the aggregate welfare.

Baldwin and Robert-Nicoud (2014) analyze the Heckscher-Ohlin (HO) model with offshoring, and show that starting from an open-economy equilibrium with no task trade,
allowing for offshoring may reduce welfare because it may adversely affect the terms-of-trade. Similarly, Egger et al. (2015) develop a monopolistic competition model of trade with occupational choice, and show that moving from non-offshoring to offshoring may lead to a welfare loss when the share of offshoring firms is below a certain level. A move from no task trade to offshoring may reduce welfare in the present model as well. The present model complements these studies by showing that a further exposure to trade in the form of a reduction in the fixed offshoring costs may also be welfare reducing.

3.2 Immigration Policy

This section investigates the impact of a reduction in $\beta_m$ on each economy. As discussed earlier, a reduction in $\beta_m$ can be interpreted as reducing immigration cost, improving immigrants’ communication channels and skills with managers, making them more familiar with rules in Home. According to Lemma 1, a reduction in $\beta_m$ increases the cutoffs $j_1$ and $j_2$, but has no impact on $j_3$. Thus, the set of tasks performed by immigrants increases, but that performed by foreign workers decreases.

A reduction in $\beta_m$ intuitively decreases the unit-labor costs measured by $W_d$ and $W_o$. Since markets are perfectly competitive, the relative price $p$ falls as well. Appendix A.7 shows that a reduction $\beta_m$ decreases $a_d$ and increases $a_o$ if and only if

$$\psi = 1 - \frac{W_o^{\sigma-1} \int_0^{j_1} [\beta_m t_m(j)]^{1-\sigma} dj}{W_d^{\sigma-1} \int_0^{j_2} [\beta_m t_m(j)]^{1-\sigma} dj} > 0.$$  \hspace{1cm} (30)

Since $W_d > W_o$, the above condition always holds when tasks are gross substitutes, i.e. $\sigma \geq 1$. When tasks are gross complements ($\sigma < 1$), however, the above condition holds unless $j_2 - j_1$ is too small (see Appendix A.7). Note that the task cutoff $j_1$ gets closer to $j_2$ when function $t_m(j)$ gets very steep around the task cutoff $j_2$, an unlikely case that can happen in practice.

Lemma 5. Suppose that condition (30) holds. In the open-economy equilibrium, $da_d/d\beta_m > 0$ and $da_o/d\beta_m < 0$.

---

Equilibrium with no task trade is found by setting $f_o \to \infty$. In this case, $A^{-k} = 0$ and equation (23) becomes

$$V_n = p_n - \frac{2k}{k-1} (2\lambda)^k W_d^{2k/(1-\eta)}.$$  

where $p_n$ denotes the relative price with no task trade and is given by (30) with $A^{-k} = 0$. Note that $p_n > p$. Since welfare function $V_n$ is a convex, U-shaped function of the relative price $p$, a move to offshoring can be welfare reducing.
When tasks are gross substitutes, a reduction in $W_d$ is stronger than that in $W_o$ and $p$; as a result, more individuals become entrepreneurs and some offshoring firms choose to produce locally. When tasks are gross complements, condition $\psi > 0$ ensures that the fall in $W_d$ is still stronger than that in $p$ and $W_o$, and thus the same conclusion still holds. In sum, under the mild restriction that $\psi > 0$, lowering $\beta_m$ decreases $a_d$ and increases $a_o$; as a result, it increases the mass of all entrepreneurs, while decreasing the mass of offshoring ones.

Using result in Lemma 5, equations (19a) and (19c) imply that reducing $\beta_m$ increases managerial capital $z$ and entrepreneurial income $e(a)$ of non-offshoring entrepreneurs. As the relative price of good 1 raises ($p \downarrow$), acquiring managerial capital becomes more expensive; consequently, these entrepreneurs have less incentive to acquire managerial capital as shown in equation (14). However, reducing $\beta_m$ reduces the unit-labor cost $W_d$ as well, and this effect dominates the former one; as a result, non-offshoring entrepreneurs acquire more managerial capital. In the case of offshoring entrepreneurs, the reduction in the unit-labor cost $W_o$ is not as substantial as the fall in $p$; consequently, they acquire less managerial capital and generate lower entrepreneurial income as shown in Figure 4 (see Appendix A.8).

Furthermore, reducing $\beta_m$ increases $A$; as a result, equations in (21) imply that this policy reduces the inequality between entrepreneurs and workers, while increasing that between offshoring entrepreneurs and workers. The inequality between all entrepreneurs and
workers decreases because of a substantial increase in the mass of entrepreneurs \((a_d \downarrow)\), and the inequality between offshoring entrepreneurs and workers increases because the number of offshoring entrepreneurs decreases \((a_o \uparrow)\).

A reduction in \(\beta_m\) makes all workers better off because of a decline in the relative price \(p\). It makes entrepreneurs with \(a < \bar{a}\) better off as well, because both the relative price \(p\) decreases and their income increases. The welfare impact of this policy on entrepreneurs with \(a > \bar{a}\) is ambiguous, because both the relative price \(p\) and their entrepreneurial income decrease. The policy is likely to reduce the welfare of very able entrepreneurs when \(\psi\) is high and (in particular) \(\theta\) is small. Interestingly, aggregating individual preferences yields the same result that reducing \(\beta_m\) may reduce aggregate welfare when \(\psi\) is high and \(\theta\) is small. Since this policy lowers the relative price \(p\), it then follows that all workers in Foreign are better off. The following proposition summarizes these results.

**Proposition 3.** Consider the two trading economies as described, and suppose that condition (30) holds. A reduction in \(\beta_m\) has the following effects on Home. It

a. increases (decreases) the mass of non-offshoring (offshoring) entrepreneurs;

b. increases (decreases) productivity of non-offshoring (offshoring) firms;

c. decreases (increases) the inequality between workers and all (offshoring) entrepreneurs;

d. has an ambiguous effect on the aggregate welfare.

The finding that a reduction in \(\beta_m\) increases the set of tasks that immigrants perform is consistent with Ottaviano et al. (2013) who, using employment data on immigrants, natives, and offshore workers from the US over 2000–2007, find that easier immigration leads to task upgrading of immigrants. They also find that lower immigration costs have no impact on the average task complexity of native workers. In the present model, this policy has no impact on native tasks in offshoring firms, while pushing natives toward more complex tasks in non-offshoring firms. The analysis then suggests that when the share of offshoring firms in an industry raises, the impact of easier immigration will have a limited effect on the average complexity of native tasks. Ottaviano et al. consider only manufacturing industries where offshoring has increased dramatically.
4 Conclusion

This paper develops a model of task trade with occupational choice and immigration to investigate the impact of offshoring and immigration on entrepreneurial activity, income inequality, and welfare. Tasks can be performed by native, immigrants, or offshore workers, and offshoring entails variable and fixed costs. Furthermore, markets are perfectly competitive and firms can invest to improve their productivity.

I find that reducing offshoring costs leads to a job polarization by increasing the supply of workers and offshoring entrepreneurs, while reducing the mass of entrepreneurs who produce domestically. It also increases the income inequality between entrepreneurs and workers. Lowering variable offshoring costs increases (decreases) productivity of offshoring (non-offshoring) firms, and improves the aggregate welfare. Lowering fixed offshoring costs reduces productivity of all firms and has an ambiguous effect on the aggregate welfare. Implementing an immigration policy that makes immigrants more integrated to the economy reduces the supply of workers and offshoring entrepreneurs, while increasing the supply of entrepreneurs producing locally. The policy also increases (decreases) productivity of non-offshoring (offshoring) firms, lowers the income inequality between workers and entrepreneurs, and has an ambiguous impact on the aggregate welfare.

The current model can be extended by incorporating skill heterogeneity among immigrants and allowing them to become entrepreneurs. This extension is important because recent studies have shown skilled immigrants significantly contribute to entrepreneurship and economic growth (Peri 2012, Kerr and Kerr 2015). Another interesting extension will be to incorporate labor market frictions leading to equilibrium unemployment, and investigate how offshoring and immigration policies affect unemployment of each group.
A Appendix

A.1 Proof of Lemma 1

Differentiating equations (7) and (9) with respect to $\beta_m$ yields

$$\frac{dj_1}{d\beta_m} = \frac{t_m(j_1)}{\beta_m t'_m(j_1) - \beta_m t'_o(j_1)} < 0, \quad \frac{dj_2}{d\beta_m} = -\frac{t_m(j_2)}{\beta_m t'_m(j_2)} < 0, \quad \frac{dj_3}{d\beta_m} = 0,$$

(A.1)

where the first inequality follows from $\beta_m t'_m(j_1) > \beta_m t'_o(j_1)$, because at $j_1$ the green curve in Figure 1 is steeper than the red one.

Similarly, differentiating equations in (7) with respect to $\beta_o$ yields

$$\frac{dj_1}{d\beta_o} = \frac{t_o(j_1)}{\beta_m t'_m(j_1) - \beta_o t'_o(j_1)} > 0, \quad \frac{dj_2}{d\beta_o} = 0, \quad \frac{dj_3}{d\beta_o} = -\frac{t_o(j_3)}{\beta_o t'_o(j_3)} < 0,$$

(A.2)

where the first inequality follows again from $\beta_m t'_m(j_1) > \beta_o t'_o(j_1)$.

Finally, according to equations (7) and (9), $j_1, j_2,$ and $j_3$ are clearly independent of $z, M,$ and $f_o$.

A.2 Aggregate Wage Indexes

The aggregate wage index is given by

$$W_v = \min \left\{ \int_0^1 c_v(j) l_v(j) dj \mid L_v = 1 \right\}.$$

Solving this cost minimization problem yields

$$W_v = \left[ \int_0^1 c_v(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}, \quad v = d, o.$$

Substituting $c_v(j)$ from (6) and (7) into the above equation yields the dual price indexes $W_d$ and $W_o$ given by equations (12a) and (12b).

To show that $W_o < W_d$, suppose that $\sigma < 1$. Using equations (12a) and (12b) implies that $W_o < W_d$ if and only if

$$\int_{j_1}^{j_2} [\beta_o t_o(j)]^{1-\sigma} dj + \int_{j_2}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj < \int_{j_1}^{j_2} [\beta_m t_m(j)]^{1-\sigma} dj + \int_{j_2}^{j_3} dj.$$

According to Figure 1, $\beta_o t_o(j) < \beta_m t_m(j)$ for $j \in [j_1, j_2]$, and $\beta_o t_o(j) < 1$ for $j \in [j_2, j_3]$. Since $\sigma < 1$, it then follows that the above inequality always holds.

Now suppose that $\sigma > 1$. In this case, $W_o < W_d$ if and only if

$$\int_{j_1}^{j_2} [\beta_o t_o(j)]^{1-\sigma} dj + \int_{j_2}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj > \int_{j_1}^{j_2} [\beta_m t_m(j)]^{1-\sigma} dj + \int_{j_2}^{j_3} dj.$$

Using Figure 1 together with $\sigma > 1$ immediately implies that the above inequality holds.
A.3 Aggregation of Variables

Maximizing profit function (11) with respect to \(x_v(j)\) and using \(W_v \equiv \eta \psi_2(v) = \eta r_2(v)\) yields

\[
l_v(j; a) = \frac{\eta r_2(v)c_v(j)^{-\sigma}}{W_v^{1-\sigma}} \frac{2\eta a}{(1-\eta) a_d} \left( \frac{W_d}{W_v} \right)^{\frac{2\eta}{1-\eta}} c_v(j)^{-\sigma} W_v^{1-\sigma},
\]

where the second equality follows from equation (19b). Aggregating \(l_v(j; a)\) over \([j_2, 1]\) and \([j_3, 1]\) yields the amount of native workers used by an entrepreneur with ability \(a\)

\[
\ell(a) = \frac{2\eta a W_d^{\frac{2\eta}{1-\eta}}}{(1-\eta) a_d} \times \begin{cases} (1-j_2) W_d^{\sigma - \frac{1+\eta}{1-\eta}} & a \in [a_d, a_o) \\ (1-j_3) W_o^{\sigma - \frac{1+\eta}{1-\eta}} & a \geq a_o. \end{cases} \tag{A.3}
\]

Integrating (A.3) across all entrepreneurs yields (24a).

Since production of one unit of task \(j\) requires \(\beta_m t_m(j)/w^*\) units of immigrants, aggregating \(\beta_m t_m(j)l_v(j; a)/w^*\) over \([0, j_1]\) and \([0, j_2]\) yields the amount of immigrant workers used by an entrepreneur with ability \(a\)

\[
m(a) = \frac{2\eta a W_d^{\frac{2\eta}{1-\eta}}}{(1-\eta) w^* a_d} \times \begin{cases} W_d^{\sigma - \frac{1+\eta}{1-\eta}} \int_{j_2}^{j_1} [\beta_m t_m(j)]^{1-\sigma} dj & a \in [a_d, a_o) \\ W_o^{\sigma - \frac{1+\eta}{1-\eta}} \int_{j_0}^{j_1} [\beta_m t_m(j)]^{1-\sigma} dj & a \geq a_o. \end{cases} \tag{A.4}
\]

Integrating (A.4) across all entrepreneurs yields (24b).

If a firm offshores task \(j\), its unit production requires \(\beta_o t_o(j)/w^*\) units of workers in Foreign. Aggregating \(\beta_o t_o(j)l_o(j; a)/w^*\) over \([j_1, j_3]\) yields the amount of offshore workers used by an entrepreneur with ability \(a\)

\[
p_o(a) = \frac{2\eta a W_d^{\frac{2\eta}{1-\eta}}}{(1-\eta) w^* a_d} \int_{j_0}^{j_1} [\beta_o t_o(j)]^{1-\sigma} dj \tag{A.5}
\]

and integrating this function over \([a_o, \infty)\) yields

\[
L_o^* = \frac{2k\eta A^{-k+1} W_d^{\frac{2\eta}{1-\eta}} \int_{j_0}^{j_1} [\beta_o t_o(j)]^{1-\sigma} dj}{(k-1)(1-\eta) w^* a_d^k W_o^{\sigma - \frac{1+\eta}{1-\eta}}}.
\]

Good 1 is used in acquiring managerial capital and entry to Foreign. The cost of acquiring \(z\) units of managerial capital is given by \(\lambda z^2/2a\). Substituting \(z\) from (24a) into the latter, and then aggregating across all entrepreneurs and using \(A\) from (18) yields

\[
C_{1z} = \frac{k[1+f_o A^{-k}]a_d^{-k}}{k-1}.
\]

The total amount of good 1 used in entry to Foreign is \(C_{1e} = f_o a_o^{-k} = f_o A^{-k} a_d^{-k}\). It then
follows that
\[ C_1 = C_{1z} + C_{1e} = \frac{k + (2k - 1)f_oA^{-k}}{(k - 1)a_d^k}. \]

A.4 Proof of Lemma 3

Note that \( W_d \) does not depend on \( \beta_o \). Using Leibniz rule, differentiating \( W_o \) from (12b) with respect to \( \beta_o \) yields
\[ \frac{dW_o}{d\beta_o} = \frac{W_o^\sigma}{\beta_o} \int_{j_1}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj > 0. \] (A.6)

Differentiating \( A \) from (18) with respect to \( \beta_o \) yields
\[ \frac{dA}{d\beta_o} = \frac{2\eta A(W_d/W_o)^{2\eta}}{(1 - \eta)W_o[(W_d/W_o)^{2\eta} - 1]} \frac{dW_o}{d\beta_o} > 0, \] (A.7)

\( dW_o/d\beta_o \) is given by (A.6). Differentiating (28) with respect to \( \beta_o \) yields
\[ \frac{da_d}{d\beta_o} = -\frac{f_oA^{-k-1}a_d}{1 + f_oA^{-k}} \frac{dA}{d\beta_o} < 0. \] (A.8)

Differentiating \( a_o = Aa_d \) with respect to \( \beta_o \) yields
\[ \frac{da_o}{d\beta_o} = \frac{a_d}{1 + f_oA^{-k}} \frac{dA}{d\beta_o} > 0. \]

Finally, using (16) and (A.8) yields
\[ \frac{dp}{d\beta_o} = \frac{(1 - \eta)f_oA^{-k-1}p}{2(1 + f_oA^{-k})} \frac{dA}{d\beta_o} > 0, \] (A.9)

where \( dA/d\beta_o \) is given by (A.7).

A.5 Proofs of Lemma 4

Since \( j_i \) is independent of the fixed offshoring cost \( f_o \), it then follows that \( W_d \) and \( W_o \) are independent of \( f_o \) as well; as a result, \( dA/df_o = A/f_o \). Differentiating \( a_d \) from (27) with respect to \( f_o \) and using \( dA/df_o = A/f_o \) yields
\[ \frac{da_d}{df_o} = -\frac{A^{-k}a_d}{1 + f_oA^{-k}} < 0. \] (A.10)
Finally, differentiating $a_o = Aa_d$ with respect to $f_o$ and using $dA/df_o = A/f_o$ yields

$$\frac{da_o}{df_o} = \frac{a_o}{f_o} + \frac{Aa_d}{df_o} = \frac{a_o}{f_o(1 + f_oA^{-k})} > 0.$$  

**A.6 Proof of Proposition 2**

Differentiating $E^n$ with respect to $f_o$ and using (A.10) yields

$$\frac{dE^n}{df_o} = \frac{1}{(k-1)a_o^k} > 0.$$  

Differentiating $V^n = p^{-\theta}E^n$ with respect to $f_o$ and using $dp/d\beta_o = -(1-\eta)p/2a_d da_d/df_o$ from (16) yields

$$\frac{dV^n}{df_o} = \frac{p^{-\theta}}{(k-1)a_o^k} \left[ 1 - \frac{\theta(1-\eta)(k-1)Ea_k}{2(1+f_oA^{-k})} \right],$$  

and note that as $\theta$ gets smaller the expression in the square brackets becomes positive.

**A.7 Proof of Lemma 5**

To determine the impact of a reduction in $\beta_m$ on ability cutoffs $a_d, a_o,$ and $a_d^*$, I first need to determine its impact on aggregate wages $W_d, W_o,$ and the world relative price $p$. Using Leibniz rule, differentiating $W_d$ from (8) with respect to $\beta_m$ yields

$$\frac{dW_d}{d\beta_m} = \frac{W_d^\sigma}{\beta_m} \int_0^{j_2}[\beta_m t_m(j)]^{1-\sigma} dj > 0.$$  

Similarly, differentiating $W_o$ from (9) with respect to $\beta_m$ yields

$$\frac{dW_o}{d\beta_m} = \frac{W_o^\sigma}{\beta_m} \int_0^{j_1}[\beta_m t_m(j)]^{1-\sigma} dj > 0.$$  

For future references, combining these expressions yields

$$\frac{W_o}{W_d} \frac{d(W_d/W_o)}{d\beta_m} = \psi \frac{dW_d}{W_d} \frac{d\beta_m}{W_d},$$  

where $\psi$ is given by (30). Thus, $d(W_d/W_o)/d\beta > 0$ if and only if $\psi > 0$. Note that $\psi > 0$ when $\sigma \geq 1$. In addition, it is always less than 1, i.e. $\psi < 1$.

To determine $dp/d\beta_m$, first note that

$$\frac{dA}{d\beta_m} = -\frac{2\eta(A + A^2 f_o^{-1})\psi dW_d}{(1-\eta)W_d} d\beta_m.$$  

26
and note that $dA/d\beta_m \leq 0$ if and only if $\psi \geq 0$. Differentiating $p$ from (29) with respect to $\beta_m$ and using (A.14) yields

$$\frac{dp}{d\beta_m} = \left[ \frac{1 - \psi A^{1-k} + (1 - \psi)f_o A^{-k}}{1 + f_o A^{-k}} \right] \eta p \frac{dW_d}{d\beta_m} > 0, \quad (A.15)$$

where inequality follows from $\psi < 1$ and $A^{1-k} < 1$. Thus, a reduction in $\beta_m$ always reduces the relative price $p$.

Now differentiating $a_d$ from (27) with respect to $\beta_m$ yields

$$\frac{da_d}{d\beta_m} = \frac{2\eta \psi (1 - k + f_o A^{-k}) a_d}{(1 - \eta)(1 + f_o A^{-k})} \frac{dW_d}{d\beta_m}, \quad (A.16)$$

and thus $da_d/d\beta_m > 0$ iff $\psi > 0$.

Substituting $W_d$ and $W_o$ from (12a) and (12b) into $\psi$ in (30) and rearranging the terms yields

$$\psi > 0 \iff \frac{\int_{j_2}^{j_3} [\beta_m t_m(j)]^{1-\sigma} dj}{\int_{j_2}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj} + f_{ij_1} \frac{[\beta_o t_o(j)]^{1-\sigma} dj - (j_3 - j_2)}{1 - j_2} \geq 0,$$

and clearly $\psi > 0$ if $\int_{j_2}^{j_3} [\beta_o t_o(j)]^{1-\sigma} dj \geq j_3 - j_2$. Note that if $j_2 - j_1$ is very small, the first term in the second expression will be close to 0 and $j_3 - j_2$ will be bigger; consequently, the whole expression may be negative, i.e. $\psi < 0$.

Differentiating $a_o = A a_d$ with respect to $\beta_m$ and using (A.14) and (A.16) yields

$$\frac{da_o}{d\beta_m} = -\frac{2\eta \psi (1 + A f_o^{-1}) a_o}{(1 - \eta)(1 + f_o A^{-k})} \frac{dW_d}{d\beta_m},$$

and thus $da_o/d\beta_m < 0$ iff $\psi > 0$.

### A.8 Proof of Proposition 3

**Part b.** Let $\hat{e} = a_d^{-1}(W_d/W_o)2^\eta/(1-\eta)$. Differentiating $\hat{e}$ with respect to $\beta_m$ and using the results from the previous section yields

$$\frac{d\hat{e}}{d\beta_m} = \frac{2\eta \psi (1 - A^{1-k}) \hat{e}}{(1 - \eta)(1 + f_o A^{-k})} \frac{dW_d}{d\beta_m},$$

and thus $d\hat{e}/d\beta_m > 0$ if and only if $\psi > 0$.

**Part d.** Differentiating $E^n = 1 + (a_d^k + f_o a_o^k)/(k-1)$ with respect to $\beta_m$ yields

$$\frac{dE^n}{d\beta_m} = \frac{2k\eta \psi (A + f_o)}{(k-1)(1 - \eta)(1 + f_o A^{-k})} a_o^k \frac{dW_d}{d\beta_m}, \quad (A.17)$$

and thus $dE^n/d\beta_m > 0$ iff $\psi > 0$. 

27
Differentiating $V^n = p^{-\theta}E^n$ with respect to $\beta_m$ yields
\[
\frac{dV^n}{d\beta_m} = p^{-\theta} \left[ \frac{dE^n}{d\beta_m} - \frac{\theta E^n}{p} \frac{dp}{d\beta_m} \right].
\]
Substituting $dp/d\beta_m$ from (A.15) and $dE^n/d\beta_m$ from (A.17) into the above equation yields
\[
\text{Sign} \left\{ \frac{dV^n}{d\beta_m} \right\} = \text{Sign} \left\{ \frac{2k\psi(A^{1-k} + f_oA^{-k})}{(k-1)(1-\eta)d^k_a} - \theta E^n \left[ 1 + f_oA^{-k} - \psi(A^{1-k} + f_oA^{-k}) \right] \right\}.
\]
Note that when $\psi$ is high and $\theta$ is very small, $dV/d\beta_m > 0$.

References


