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Inflation Forecast Targeting: An Alternative Approach to Estimating the Inflation-Output Variability Tradeoff

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Inflation Forecast Targeting: An Alternative Approach to Estimating the Inflation-Output Variability Tradeoff

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Abstract: We suggest a new way of computing the inflation-output variability tradeoff under inflation forecast targeting. Our approach is based on dynamic, stochastic simulations of the average inflation rate over a two-year horizon using the moving average representation of a VAR model. Using real time data over two samples, we estimate the inflation-output variability tradeoff for the U.S. and show that it has shifted favorably over time. We analyze the policy interventions required to achieve target inflation in each sample and compare these interventions over time.
I. Introduction

Over the past decade and a half, most major central banks around the world have adopted monetary policy frameworks that include either explicit or implicit inflation targets. The type of inflation targeting implemented by central banks is best characterized as flexible inflation targeting, where central banks are simultaneously concerned about the variability of output around its natural level as well as the variation of inflation around target. Furthermore, Svensson (1997) has argued that inflation targeting should be implemented as inflation forecast targeting because of lags in the effect of monetary policy on inflation and because of imperfect control of inflation by the central bank. Finally, Bernanke (2004), in comparing and contrasting use of “simple feedback policies” (instrument rules) and “forecast-based policies,” concludes that a forecast-based approach to policy implementation “… has become increasingly dominant in the monetary policymaking of leading central banks…. [T]he Fed relies primarily on the forecast-based approach for making policy.”\(^1\)

Following Taylor’s (1994) suggestion for estimating the inflation-output variability tradeoff for a variety of policy rules, recent research includes estimation of the tradeoff between inflation variability and output variability. This research investigates policy rules in the presence of a loss function for the central bank in which there is often a substantial weight on the variation of inflation around target relative to the weight on output variability. In this literature, a common approach is to vary the weight on the inflation target, derive the associated instrument rule, simulate a structural model within which the rule is embedded, and then compute inflation and output variability for alternative values of the relative weight on the inflation target. For example, Rudebusch-Svensson (1999) estimate empirically the inflation-output variability tradeoff for a variety of rules.\(^2\) For each rule, some of which can be categorized as inflation forecast targeting rules, the tradeoff is derived by varying the relative weight on output.

In contrast to the instrument-rule approach, we present a novel way of estimating the inflation-output variability tradeoff (hereinafter “variability tradeoff”) that utilizes forecast-based counterfactual simulations. Our motivation is the real-world policymaking process described by Svensson (2003) in which policymakers do not write down an explicit loss function but instead evaluate alternative paths for the policy instrument and then pick a path that, in Svensson’s terminology, “looks best”. Given that central banks don’t announce a specific loss function and the weights therein and, following an analogy presented in Svensson (1997), we consider inflation target bands

\(^{1}\) Svensson (2003) provides a detailed critique of instrument rules in an inflation targeting setting.

\(^{2}\) The loss function employed maintains a weight of unity on the inflation objective, allows varying weight on the output goal, and also includes weight for interest rate smoothing.
of varying width as proxies for changes in the relative weight on inflation versus output stabilization in a loss function. In this approach, wider target bands are analogous to a relatively smaller weight on the inflation target.

Following McCallum’s (1988) suggestion that “rules” should be evaluated in a variety of models, we illustrate our technique using a VAR model\(^3\) and compute the policy interest rate needed to keep forecast average inflation within several pre-specified target bands.\(^4\) For each target band, the policy innovations needed to keep average inflation within the band are computed.\(^5\) These innovations, along with typical shocks for the other variables, are then used in a dynamic, stochastic out-of-sample forecast of the VAR model to determine the time paths for the system variables. For each of the inflation target bands, we simulate the model over 1000 trials, allowing us to compute the variances of output and inflation. By specifying several bands, we generate the variability tradeoff, a menu of options from which the policymaker can choose. As with other procedures for empirical policy evaluations, the Lucas critique is a concern, and we consider our results in light of this critique.

We illustrate our procedure in two experiments. In order to mimic the policy process, we construct a real time data set for each experiment and employ Blinder’s (1997) description of the policy process as our template for the setup of each experiment.

The first experiment begins in 1983:10, using a model estimated over the pre-Great Moderation period 1962:1-1983:9 in which there was substantial variation in both inflation and output growth. Though not part of the implicit inflation targeting period denoted by Goodfriend (2005) as emerging under Greenspan, an experiment beginning in 1983:10 is a useful benchmark for the second experiment described below. Specifically, we view the period beginning in the early 1980s, which followed a period of accelerating inflation that culminated with double-digit inflation as the decade began, as one with a relatively high weight on inflation control in the underlying policy maker preference function. Furthermore, as punctuated by the unusual Saturday evening FOMC meeting in October

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\(^3\) Although we illustrate the technique with a VAR model, the technique can be applied to structural models as well.

\(^4\) In practice, central banks appear to be willing to tolerate some variability in inflation resulting from either noise or temporary factors affecting the price indexes, so that at least implicitly the goal can be interpreted as an average inflation target. An example of a central bank that employs average inflation targeting is the Reserve Bank of Australia. Reserve Bank Gov. Fraser argued (1993, p. 2) that “if the rate of inflation in underlying terms could be held to an average of 2 to 3 percent over a period of years, that would be a good outcome.” He reiterated this position a year later, arguing that “In our judgment, underlying inflation of 2 to 3 per cent is a reasonable goal for monetary policy. These figures, incidentally, are not intended to define a (narrow) range; rather, they are indicative of where we would like to see the average rate over a run of years.” Fraser (1994, p. 21)

\(^5\) The inflation target bandwidths are specified as the goal inflation rate, plus or minus pre-specified tolerance bands that include a degenerate bandwidth of zero in which the goal rate is to be met precisely.
1979, substantial concern existed with regard to inflation expectations. In broad terms, then, the objectives at the beginning of the first experiment were much the same as in inflation targeting regimes.

The second experiment computes the variability tradeoff policy makers would have faced in 2001:1 using a model estimated over 1980:1-2000:12, a period characterized by a monetary policy focus on reducing the inflation rate, maintaining it at a low level, and reducing variability in inflation and output compared to 1962:1-1983:9.

The models used in these experiments were estimated over periods with varying intensity of concern for inflation and, especially in the first experiment, substantial variability in both inflation and output. The Lucas critique would thus seem applicable. We address this issue within the context of the discussion by Sims-Zha (2006b) and provide some empirical evidence about the relevance of the critique to our experiment using in part the Leeper-Zha (2003) approach to evaluating modest policy interventions.

In Section II, we present the theoretical underpinnings of our experiments. In section III, we present the intuition behind our counterfactual analysis, with technical detail relegated to an appendix that is available on request. We also discuss antecedents in the literature to our use of counterfactual simulations to evaluate alternative policies. In Section IV, we specify the empirical model, discuss data, and examine the plausibility of the estimated model through a presentation of the impulse responses to a policy shock. We include in Section V the main statistical results, culminating in presentation of the variability tradeoffs for each experiment. In Section VI, we provide evidence on whether the Lucas critique is a concern for our experiments by computing the “modesty statistic” proposed by Leeper-Zha (2003). Our results suggest that the variability tradeoff constructed for the first experiment can be usefully compared to the tradeoff in the second experiment. Finally, in Section VI, we summarize the results and discuss possible explanations for them.

II. Theoretical Background

Our empirical analysis is based on two premises. First, in keeping with the Fed’s dual mandate, we assume the Fed takes output stabilization into account, at least in the short-run, in monetary policy decisions even if the primary goal of the Fed is price stability. In the context of our experiments, the narrower (wider) the inflation bandwidth, the less (more) concern is implied for output stabilization. In addition, the shorter (longer) the horizon for moving the average inflation rate to target, the less (more) concern for output stability. Second, we assume Blinder’s (1997) idealized description of policy making is appropriate as a template of the policy process.
Attaining inflation objectives has long been a goal of monetary policy, and it can be argued that the Fed was an implicit inflation targeter from the early 1980s until at least the onset of the current financial crisis. At the beginning of the 1980s, Fed policy was clearly focused on disinflationary strategies. More recently, as inflation targeting policy frameworks have become popular around the world, Goodfriend (2005) has argued (p. 321) that “… the Greenspan Fed adopted, gradually and implicitly, an approach to monetary policy characterized as inflation targeting.” Goodfriend cites Congressional testimony by Alan Greenspan that the inflation objective would be achieved when “the expected rate of change of the general level of prices ceases to be a factor in individual and business decision making.” Thus, that the U.S. used a targeting framework, at least implicitly, during the time periods for our experiments is a reasonable assumption.

With regard to the policy process, Blinder (1997) describes a two-step procedure for policymakers:

“First, you must plan an entire hypothetical path for your policy instrument, from now until the end of the planning horizon, even though you know you will activate only the first step of the plan. It is simply illogical to make your current decision in splendid isolation from what you expect to do in subsequent periods. Second, when next period actually comes, you must appraise the new information that has arrived and make an entirely new multiperiod plan. If the surprises were trivial, that is, if the stochastic errors were approximately zero, step one of your new plan will mimic the hypothetical step two of your old plan. But if significant new information has arrived, the new plan will differ notably from the old one. Third, you must repeat this reappraisal process each and every period.”

We interpret this as follows. When the FOMC meets, it should evaluate, *inter alia*, the Greenbook forecasts and the various policy options contained in the Bluebook. This is the ‘first step of the plan’ by which ‘an entire hypothetical path for the policy instrument, from now until the end of the planning horizon’ is considered. Note that the forecasts and assessments of the policy alternatives in the planning horizon extend well beyond the next FOMC meeting. It is this ‘first step’ of the Blinder two step procedure we are empirically modeling. The second step of the

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6 In detail, Goodfriend argues that the Greenspan Fed: (1) approached its inflation goal gradually, trying to avoid disruptions to output, consistent with flexible inflation targeting; (2) would not have *deliberately* (Goodfriend’s emphasis) undertaken policies that would raise inflation above 2% after achieving PCE inflation in that range in the mid-1990s; (3) would have been unlikely to *deliberately* (his emphasis) aim at inflation below 1% given the costs associated with deflation and problems associated with the zero nominal interest rate bound; and (4) that when actual and expected inflation were well-contained, the Fed “aggressively” pursued countercyclical policy.

7 Interestingly, by the time of our second experiment in 2001, the Bluebook presentation of policy alternatives presents long-range policy paths for the federal funds rate and the implications for broad macro aggregates through 2005, an “entire hypothetical path for the policy instrument … even though you know you will activate only the first step of the plan.” Thus, at least part of the strategy suggested by Blinder in 1997, which we model below, seemed to be part of the operating procedures at the time of our second experiment. In 1983, during our first experiment, the longer-range horizon was shorter, however, about a year. There was also a heavier focus on
Blinder procedure would come at the next FOMC meeting, when ‘new information’ has arrived and ‘an entirely new multiperiod plan’ is implemented.

Svensson (1997, 1999) has developed a simple, analytical model of inflation targeting that provides a useful technical background for the first step of Blinder’s policy description under the dual mandate. Specifically, he presents a model of inflation targeting implemented as inflation forecast targeting. We consider the key points in the Svensson model to fix ideas prior to our empirical implementation. While in Svensson’s model the lag between the policy variable and inflation is two periods, in our empirical work we allow the policy variable to influence inflation over the 24 month period we adopt as Blinder’s planning horizon.

Svensson’s model specifies a setting where the policy interest rate affects inflation with a two period lag. Accordingly, the interest rate is set at a level today consistent with forecasts of inflation two periods later. The two period lag arises by assuming that the current interest rate setting affects the output gap with a one period lag, which in turn affects inflation with a subsequent one period lag. Recall from the introduction that Bernanke (2004) characterized forecast-based approaches as “dominant” in policymaking.

In Svensson’s setting, the aggregate supply/Phillips curve relationship is given by

\[ \pi_{t+1} = \pi_t + \alpha_y y_t + \epsilon_{t+1} \]  

(1)

while the aggregate demand/IS is given by

\[ y_{t+1} = \beta_y y_t + \beta_z z_t - \beta_i (i_t - \pi_{t+1} - \bar{r}) + \eta_{t+1} \]  

(2)

where \( \pi_{t+1} \) is the inflation rate in period t+1, \( y_t \) is the output gap, \( \epsilon_{t+1} \) is a random shock to aggregate supply, \( i_t \) is the nominal interest rate, \( \pi_{t+1} \) is the expected inflation rate conditioned on information at time t, \( \bar{r} \) is the equilibrium real interest rate, \( \eta_{t+1} \) is a random shock to aggregate demand, and \( z_t \) is a vector of exogenous variables given by

\[ z_{t+1} = \gamma_z z_t + \theta_{t+1} \]

with \( \gamma_z \) being a conformable vector and \( \theta_t \) a random vector with the same dimensionality as \( z_t \). The period loss function, reflecting the dual mandate, is

\[ -\frac{1}{2} [\lambda y_t^2 + (\pi_t - \pi^*)^2] \]  

\( \pi^* \)  

(3)

monetary aggregates though with qualitative discussion of the interest rate paths consistent with the alternative forecasts.
where $\lambda$ is the relative weight on output stability and $\pi^*$ is the target inflation rate. Svensson (1997) shows that the first-order condition for a minimum is

$$\pi_{t+2p} - \pi^* = \frac{\alpha_c c(\lambda)}{1 - c(\lambda)} y_{t+\tau}$$  \hspace{1cm} (4)$$

where $c(\lambda)$ is a function of model parameters with the properties that $0 \leq c(\lambda) < 1$, $\frac{\partial c}{\partial \lambda} > 0$, $c(0) = 0$, and $c(\infty) \equiv \lim_{\lambda \to \infty} c(\lambda) = 1$. The form of equation (4), with the two-period-ahead inflation forecast and the one-period-ahead output gap forecast, is the result of the lag structure in the model; setting the interest rate today influences only future values of these variables.

Consider the meaning of equation (4). As a result of minimizing the specified loss function subject to the existing tradeoff of inflation for output in equation (1), equation (4) is a consequence of the equality between the marginal rate of substitution in the policy maker preferences with the marginal rate of technical substitution. In practice, Svensson (2003, p. 451) notes “the loss function is not specified in this detail.” Rather, given available information and any judgment to be applied, various forecasts for alternative paths for the policy interest rate are generated as in the Fed’s Bluebook, and the policymaking committee then picks the “combination of forecasts that ‘looks best,’ in the sense of achieving the best compromise between stabilizing the inflation gap and stabilizing the output gap, that is, implicitly minimizing [the loss function].” (p. 451). Our empirical work below will exploit this interpretation of the policy process rather than explicitly specifying a loss function.

Given period $t$ information, equation (2) implies that the one-period-ahead output gap forecast is

$$y_{t+1} = \beta_1 y_t + \beta_2 z_t - \beta_3 (i_t - \pi_{t+\tau} - \bar{\tau})$$  \hspace{1cm} (5)$$

and from equation (1) the two-period-ahead inflation forecast is

$$\pi_{t+2p} = \pi_{t+\tau} + \alpha_c Y_{t+\tau}$$  \hspace{1cm} (6)$$

Subtracting $\pi^*$ from both sides of (6) and using (4) to substitute for $\alpha_c Y_{t+\tau}$ gives

$$\pi_{t+2p} - \pi^* = c(\lambda)(\pi_{t+\tau} - \pi^*)$$

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8 Svensson (2009) argues that equation (4) implies $\pi_{t+2p} - \pi^* = c(\lambda)(\pi_{t+2\tau} - \pi^*) = c(\lambda)^2 (\pi_t - \pi^*)$ where $\pi_{t+2p}$ is the $t + \tau$ inflation forecast that “looks good” in period $t$. In Svensson’s analysis, the policy is engineered so inflation asymptotically approaches its target; driving inflation to the target within a specified policy horizon may not be efficient. In our empirical analysis, we adopt relatively long (48 and 36 month) transition periods to the target in the spirit of the asymptotic approach.
Substituting the right-hand-side of (6) for $\pi_{t+1|y}$ and then the right-hand side of (5) for $y_{1,+}\bar{y}$ in the resulting expression and then solving for $i$, yields

$$i_t = \bar{T} + \left(1 + \frac{1 - c(\lambda)}{\alpha_i \beta_i}(\pi_{t+1|y} - \pi^*) + \pi^* + \beta_{1} \frac{\beta_{1}}{\beta_{i}} y_i + \beta_{1} \frac{\beta_{1}}{\beta_{i}} z_i \right) \quad (7)$$

Finally, taking the expectation of equation (1) conditional on time $t$ information and substituting for $\pi_{t+1|y}$ gives

$$i_t = \bar{T} + \left(1 + \frac{1 - c(\lambda)}{\alpha_i \beta_i}(\pi_t - \pi^*) + \pi^* + \beta_{1} \frac{\beta_{1}}{\beta_{i}} y_i + \beta_{1} \frac{\beta_{1}}{\beta_{i}} z_i \right) \quad (7)$$

where $\bar{T} = \bar{r} + \pi^*$. Equation (7) is the setting for the interest rate that satisfies the first order condition, equation (4), and implies that the optimal interest rate setting depends not only on the output gap and the deviation of inflation from target, but also on other model variables included in $z$. Note also that equation (7) is not a modified Taylor rule since (i) $z$ represents the remaining variables in the system and (ii) this interest rate setting is implied by an optimization process (as reflected in the coefficients on the inflation and output gaps). The Taylor rule is generally simpler (i.e., omits the variables in $z$ or includes only a small subset) and has coefficients on the inflation and output gap terms that are policy decision variables.

As noted, Svensson (2003) argued that central bankers generally don’t explicitly reveal a loss function, a step we avoid here as well. Nonetheless, two comments about extending models such as that above to include non-degenerate inflation bands are in order.

First, in a typical loss function, such as equation (3), $\lambda$ represents the relative weight on output. Svensson (1997) states that a “wide [inflation] band could then potentially indicate that the central bank has a relatively high $\lambda$ and hence a significant output stabilization goal. A narrow [inflation] band could indicate a commitment to a low or even zero $\lambda$.” (p. 1135). Alternatively, we could normalize directly on inflation variability, where $\Lambda$ is the relative weight, with a loss function such as $-\frac{1}{2}\left[y_t^2 + \Lambda(\pi_t - \pi^*)^2\right]$. Our bandwidth parameter, $\tau$, is negatively related to $\Lambda$; the narrower our bandwidth, the less tolerant is the policy maker to inflation variability, and implicitly the higher the loss associated with inflation variability.\(^9\) As in equation (4), where the setting of $\lambda$ determines the slope

\(^9\) For the limiting case of $\tau$ approaching 0, $\Lambda$ would become arbitrarily large. In this case, the policy maker would be a strict inflation targeter, abandoning the dual mandate.
of the tradeoff between inflation and output deviations from desired levels, as we set \( \tau \) to various values to trace the variability trade-off, we are implicitly varying the value of \( \Lambda \).

Second, Flood-Isard (1989) present a model quite similar to that of Svensson detailed above with a positive weight on inflation variability, such as \( \Lambda \) in the above paragraph. The policy maker pursues one type of policy when the target variable (the exchange rate in their case) is within a band but another policy when this variable is shocked outside the band. They derive optimal policies with a loss function for two practical cases: discretion and partially state contingent. They demonstrate the existence of a mixed strategy (a probability weighted average of the two) which is time consistent under certain conditions. That is, an optimal policy exists that is different outside that band than inside and still consistent with optimization. Thus, theoretical models underlying our empirical application consistent with optimization exist.

III. Methodology

a. Estimation of the the Variability Tradeoff

We next present the basic intuition behind the methodology used to estimate the variability tradeoff for our inflation target. Consistent with Blinder, we plan a nominal interest rate path over the planning horizon; consistent with Svensson, we compute the policy interest rate needed to attain a forecast of the average inflation rate over a 24 month period on or within a prespecified band. Technical details on the computation of the interest rate setting (analogous to equation 7) needed to attain the inflation target are contained in the appendix.

We specify the inflation goal in terms of a 24-month average inflation forecast.\(^{10,11}\) That is, policy makers are forward-looking, planning interest rate policy to maintain what we will call the forecast average inflation rate (FAIR) to be consistent with the inflation target on or within a specified inflation band. Aiming for inflation on or within a band is consistent with current practice by some inflation-targeting central banks, and reveals a willingness to accept deviations from the mid-point of the target band. That is, if the forecast for inflation over the next 24

\(^{10}\) Svensson (2003) notes that both the Bank of England and the Sveriges Riksbank have used a two-year inflation forecast horizon for conducting monetary policy.

\(^{11}\) Note that there is no necessary reason the ‘planning horizon’ need be equal to the time frame over which the average inflation rate is targeted by the policy authority. We set both equal to 24 months to roughly mimic reality. Specifically, the Greenbook forecasts are now for a period of at least several years. And, as in the previous footnote, some inflation-targeting central banks employ two year forecasts for inflation as the inflation objective. There is no reason, for example, that the forecasts evaluated at a policy meeting could not extend to, say, three years while announcing and maintaining a goal for a two-year average inflation rate over this planning horizon.
months is consistent with the inflation target, i.e., on or within the band, no policy intervention is undertaken. If the FAIR is outside the band, then an intervention is used to return this measure of inflation to the band.\textsuperscript{12}

Since a forecast of the inflation process is needed, we need to specify the source of these forecasts in our analysis. We begin with a structural model

\[ y_t = A_0 y_t + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \varepsilon_t \]  

(8)

In equation (8), \( y_t \) is an (N×1) vector of variables, including the inflation rate and the federal funds interest rate.\textsuperscript{13}

The elements of the \( A_i \) matrices represent the structural coefficients and the elements of \( \varepsilon \) are structural shocks. We assume that \( E(\varepsilon \varepsilon') = \Omega \) is diagonal. The reduced form of (8) is \( \Pi(L) y_t = \varepsilon_t \), where \( \Pi(L) = I - \Pi_1 L - \ldots - \Pi_p L^p \).

Reduced-form coefficient matrices are given by \( \Pi_i = (I - A_i)^{-1} A_i \) and reduced-form shocks by \( \varepsilon_t = (I - A_i)^{-1} \varepsilon_t \). The moving average matrix is defined as \( C(L) = [\Pi(L)]^{-1} \), with \( C_0 = I \). Define \( D_t = C_s (I - A_i)^{-1} \). The moving average representation (MAR) of equation (8), expressed in terms of the structural shocks, is

\[ y_t = \sum_{s=0}^{\infty} D_s \varepsilon_{t-s} \]  

(9)

Fundamental to our analysis is the historical decomposition, which in its basic form is found by advancing equation (9) by \( n \) periods and then decomposing the resulting expression into two terms:

\[ y_{t+n} = \sum_{s=0}^{n-1} D_s \varepsilon_{t+s} + \sum_{s=n}^{\infty} D_s \varepsilon_{t+n-s} \]  

(10)

The second term on the right hand side of equation (10) is the dynamic forecast or base projection (BP) of \( y_{t+n} \) conditional on information at time \( t \). The first term on the right hand side shows the influence on \( y_{t+n} \) of the shocks to the variables in the system between periods \( t+1 \) and \( t+n \). Even though the expected values of these shocks are zero, policy makers know that the realizations of these shocks over any particular period are likely to be nonzero, which provides the motivation for the stochastic part of our simulation. These shocks, drawn from the estimated residuals, represent the source of variability around the base projection. Given a set of shocks to the system, we obtain monthly inflation rates from the relevant equation in system (10), which are then averaged to obtain the FAIR.

\textsuperscript{12} While we do not do so in our simulations, it would be straightforward in practice to allow for judgment in the forecast by including an adjustment for factors that are outside the model but deemed by policymakers to be important for the immediate policy exercise.

\textsuperscript{13} For simplicity, we do not explicitly consider expectational variables in our analysis. We do note, however, that under some conditions models with expectations of variables can be solved for a VAR of the type estimated here; see Fernandez-Villaverde-Rubio-Ramirez-Sargent-Watson (2007) and Ireland (2004) for examples.
If the FAIR deviates from target, a policy action is called for. In an obvious extension of equation (3) the goal for policy is to remain on or within the range \((\pi^* \pm \tau)\) where \(\tau\) is half the bandwidth (including the case of a zero bandwidth), with policy aiming at \((\pi^* - \tau)\) when inflation is below the band and aiming at \((\pi^* + \tau)\) when inflation is above the band. For a non-degenerate bandwidth, our policy experiments return FAIR to the edge of the band rather than the midpoint for four reasons. First, although we don’t model the loss function explicitly, our presumption about the dual mandate means that a more aggressive policy action needed to return the FAIR to the midpoint of the band rather than the edge induces additional variability in output, raising the overall loss.\(^{14}\) Second, if there is multiplicative uncertainty about the economy, in the sense of Brainard (1967), then the policy authority may not necessarily aim at the midpoint of the range. That is, if there is not certainty equivalence, then aiming at the midpoint is no longer necessarily optimal.\(^{15}\) Third, if policy makers want to minimize their impact on financial markets, returning to the edge of the inflation band requires a smaller interest rate innovation, and thus helps minimize interest rate movements. That is, we undertake the smallest policy action needed to attain the inflation objective over the two year horizon. Of course, the tradeoff is that our smaller financial market interventions may be more frequent than relatively aggressive actions aimed at returning to the midpoint of the band. Fourth, as Blinder noted, during his Federal Reserve tenure there was a lack of consensus on the desirable inflation rate among the governors as well as a tendency to wait for opportunistic shocks to lower the inflation rate, in practice restraining inflation that is “too high” but not necessarily moving it aggressively toward a firm number such as 2\%.

We emphasize that in our analysis the policy objective is the FAIR over the 24-month period rather than either the current or any particular future monthly inflation rate. Current inflation is the result of past decisions by both policy makers and private agents in the economy and is presumably not directly affected by current policy. Reported inflation, or a forecast of a particular monthly inflation rate, may deviate from the inflation target without necessarily calling for a policy action as long as the FAIR suggests that the longer-run objective will be satisfied. However, if the FAIR deviates from target or lies outside the band, a policy action is called for. In our application,

\(^{14}\) This is essentially the point of opportunistic policy described in Orphanides-Small-Wieland-Wilcox (1997). For additional discussion, see also Result 12 in Clarida-Gali-Gertler (1999).

\(^{15}\) Despite the fact that the “Brainard conservatism principle” (as Blinder dubs it) does not apply in all cases, he notes that as a central banker, he viewed the principle as “extremely wise.” The wisdom of the basic Brainard result is included as well in Clarida-Gali-Gertler (1999), result 11. Specifically, they argue that “parameter uncertainty may reduce the response of the policy instrument to disturbances in the economy.” That is, the reduction in the response may lead to aiming for the edge of the band rather than the midpoint. Further, Barlevy (2009) has shown that when robust control techniques are applied in circumstances similar to those emphasized by Brainard, robust control implies an even more “conservative” policy response than does Brainard.
we will use the federal funds rate as the policy instrument to control the inflation rate, and a policy action in a particular month is defined as an intervention in the funds rate equation in that month.\textsuperscript{16} Due to interaction with other system variables via system dynamics, a policy action in a particular month will affect inflation over the remainder of the horizon. That is, even if the funds rate has a relatively small contemporaneous effect on inflation, marginal changes in this rate can still have substantial effects on long-run inflation.

Diagram 1 gives a schematic presentation of our model of the first stage of the Blinder process in an inflation-forecast targeting regime that desires to keep inflation on or within a target band. Period 1 on the horizontal axis is the first period of the planning horizon, which in practice would be the date of an FOMC meeting. The vertical axis is the inflation rate.

Two alternative inflation bands are sketched on Diagram 1. The band centered at 2\%, with ±\(\tau\)% bands around this midpoint, represents our presumed long-run goal for policy. Of course, it is possible that at the time an inflation target is adopted, actual inflation will be above this long run target range; after all, inflation targets are adopted to try to control an inflation problem. In Diagram 1, the negatively sloped lines (for simplicity) represent a linear transitional inflation band in which inflation is to be gradually lowered, so that after some (possibly publicly announced) period of time, inflation will be brought into the long-run range. In Diagram 1, a policy of gradualism over 48 months is depicted. This period could be shorter or longer depending on policy maker preferences.

Consider period 1 in Diagram 1. The policy maker has data through period 0 and is interested in policy simulations conditional on data through period 0, a procedure roughly similar to that undertaken at a given FOMC meeting. Assume that the inflation rate is about 10\%. The policy maker makes a forecast of inflation over the next 24 months and computes the average of the monthly inflation forecasts. The period 1-24 is “underlined” beneath the horizontal axis. If the FAIR is within the transitional inflation band, such as point “x”, then no policy intervention is undertaken. Alternatively, if the FAIR is at a point like “w” a policy intervention is needed that will bring the current forecast to the upper edge of the band.\textsuperscript{17} During period 0, in planning for period 2 a new simulation is made conditional on what was done in period 1, intervention or not. If an intervention was needed in period 1, the

\textsuperscript{16} As is shown in the technical appendix, the desired change in the funds rate is implemented by replacing the residual term in the funds rate expression with an appropriately-sized shock that brings the funds rate to the desired level.

\textsuperscript{17} If the forecast inflation rate is below the band, then in a transitional period such as that characterized by the negatively sloped inflation band in Diagram 1, the policy maker may choose to conduct policy consistent with opportunistic disinflation. This would entail lowering the transitional range of the inflation target so that the period over which the gradualist policy need be conducted is shortened.
magnitude of this intervention must be incorporated into the analysis to properly forecast inflation over the next 24 month period in which average inflation is to be on or inside the band. That is, the period 0 forecast for inflation over periods 2-25 is computed, as indicated by the second “underline” beneath Diagram 1. If the forecast is within the band, similar to “x” in period 1, then no intervention is needed; if it is above the band, similar to “w” in period 1, an appropriate intervention is conducted to bring the 24 month average inflation forecast within the band. If, at a given policy meeting in period 0, Blinder’s ‘planning horizon’ is 2 years (‘even though you know you will only activate the first step of the plan’) then inflation forecasts are conducted through the end of this horizon, so a period 0 forecast is needed for month 24, which extends over months 25-48, the last underlined period in Diagram 1.

Our choice of a 48 months as a gradual transition to the long run inflation target is based on both theoretical considerations and observation of central bank practices. Though not suggesting a specific length of the transition period, Svensson (1997) argues theoretically that a positive weight on the output gap in the loss function implies that optimal disinflationary policy will be one of gradualism, as in our downwardly sloped transition bands in diagram 1. Given the “dual mandate,” U.S. policymakers should then approach inflation targets gradually. In practice,
according to Bernanke-Mishkin (1997), central bankers behave as suggested by Svensson. They note (p. 99): “Initial announcements of inflation targeting generally allow for a gradual transition from the current level of inflation to a desired steady state, usually the level deemed consistent with price stability.” Furthermore, Bernanke-Mishkin later note that after the 1979 oil shock, the German Bundesbank “announced the ‘unavoidable’ inflation rate to be 4 percent, then moved its target gradually down to 2 percent over a six-year period.” (p. 101). In the U.S., Goodfriend (2005) indicated that an “inflation scare” in 1987 due to the infusion of liquidity after the October 1987 stock market crash took the Greenspan Fed “… about five years to overcome” (p. 8). Our choice of 48 months as the transition period is a bit shorter than, but not at great odds with, these descriptions of the behavior of inflation-targeting central banks. However, we also consider a shorter transition of 36 months to the inflation target; this shorter transition is consistent with a higher relative weight on achieving the inflation target as opposed to output stabilization.

We now present the setup of our experiments, each of which produces a point on the variability tradeoff the central bank would have faced at a particular time. Each experiment corresponds to our model of the ‘first step’ in the Blinder policy process.

For each of our two time periods, we estimate a VAR model using real-time data that ends with the period before the start of the simulation. We compute the base projection at the end of the estimation period, mimicking the real-time forecasting process just prior to an FOMC meeting. This forecast of $y_{t+1}$ through $y_{t+24}$ is represented by the second right-hand-side term in equation (10) and is estimated from the lagged historical residuals from the VAR. Since the base projection is based on historical residuals, it does not change across the trials of a given experiment.

For each time period, we conduct four experiments for both the 48 month and 36 month transition periods. Each experiment has 1000 trials with a pre-specified bandwidth. For each trial, we draw (with replacement) a vector of residuals of length 48 from the estimated residuals for each equation in the system.18

Each experiment starts with the actual inflation rate in the period prior to the experiment and gradually lowers the inflation target to 2% over a 48 (36) month period (as in Diagram 1), subject to the bandwidth. The alternative bandwidths are 0, 1, 2, and $\infty$ where a bandwidth of 0 means that the focus is on the target itself and a bandwidth of $\infty$ represents a stochastic simulation using the draw from the historical residuals without any policy intervention. So, in Diagram 1, given the negatively sloped transition path, the dashed lines could assume one of the indicated bandwidth values. The initial 24 terms of the drawn vector of shocks are used to compute the first term on

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18 As noted above, we need 48 residuals since during the last month of the two year planning horizon, policy makers want to know the FAIR for the subsequent 24 months.
the right-hand-side of (10). When combined with the base projection, equation (10) gives the path the economy, as represented by the system of equations, would follow under this trial. Combining the base projection and the initial 24 elements of the vector of draws from the residuals gives the policy maker a forecast of inflation for each of the next 24 months for this particular trial. The policy maker averages these 24 individual monthly inflation rates to compute the FAIR. If this rate is inside the band or equal to target in the case of a 0% band, no policy intervention is needed. On the other hand, if this inflation rate is outside the band, a preemptive policy action of sufficient magnitude to return the 24 month average forecast inflation rate to the closest edge of the band is calculated.\footnote{That is, if a policy intervention is needed to attain the band, the drawn residual from the interest rate equation is replaced with one that is computed to assure that the FAIR attains the policy objective.} This policy action, computed as detailed in the appendix, produces an interest rate analogous to equation (7) above. Furthermore, this policy intervention is carried along for the remainder of the trial, affecting all the system variables in later time periods. Also, as is shown in the appendix, the computation by which the needed intervention is done is conditional on the shocks from the random draw for later time periods; the current policy action needed to attain the inflation target is “identified” using the remaining residuals from the draw. Finally, note that since the residuals in equation (10) are structural, the residual drawn for the policy equation can be replaced with the needed policy action without implications for the random shocks to the other equations since it was assumed there is no contemporaneous correlation among the structural shocks.\footnote{We note that attempting to use the funds rate to control the inflation rate at very short horizons may lead to instrument instability. The intuition behind this statement is that the interest rate is not an important component of measured prices and that the contemporaneous effect of a change in the interest rate on aggregate demand is very small. Consequently, an interest rate change would have a relatively small near-term impact on the inflation process, requiring large interest rate movements to affect short-term inflation. With a longer-term inflation objective, say one of several years as we employ here, a current interest rate change has lagged effects on the inflation rate, consistent with system dynamics. This point is recognized by central bankers, who generally implement policy via interest rate innovations which are allowed to work their way through the dynamics of the economy.}

\footnote{As we will discuss further below, this procedure is the same as that adopted by Leeper-Zha (2003).}

\footnote{An alternative approach for obtaining a desired average inflation rate would be to employ a “constant interest rate” approach, which would take the base projection and adjust it by imposing a constant interest rate over the 24-month horizon that brings about the desired average inflation rate. This approach thus implicitly imposes an entire path for the shocks to the interest rate equation. In our analysis, we identify the current policy shock needed to attain the objective (given the rest of the draw) while the constant interest rate approach implicitly identifies a vector of shocks, current and for the remainder of the horizon, needed to maintain a constant interest rate and simultaneously attain the inflation objective. The constant interest rate approach thus imposes more policy action than needed to attain the policy objective. It imposes interest rate smoothness while our approach allows the path of rates to be determined by the response of the policy maker to the forces that may drive the FAIR outside the band.}

\footnote{It is possible to model correlations among the structural shocks, as in Bernanke-Mihov (1998). If such modeling included contemporaneous correlation between the policy innovation and other variables, then other structural}
For each month in the planning period, a policy action is either needed or not. Either way, the dynamic path of the economy is computed. After passing through the planning horizon, intervening as needed, we obtain at the end a path for the system of variables in which policy is used to attain the inflation target. By construction, this counterfactual path over the planning horizon is consistent with the inflation objective of the policy authorities. With 1000 trials for each bandwidth, we can compute the variance of each element of the vector of variables.

Over the 1000 trials in a given experiment we determine the variances of inflation (around the trial mean) and the output gap (also around the trial mean). At a point in time and for a given inflation band, we use these variances of inflation and output to represent a point on the variability tradeoff. Using the four alternative bandwidths then allows us to plot the available inflation-output variability frontier. The policy maker can then select the value for \( r \) that ‘looks best’ in the characterization of Svensson (2003, 2009). By using different time periods, we can also discover how the point estimate of the frontier has changed over time.

To summarize, in the first step of Blinder’s process, the policy maker takes into account the results of the entire 24-month planning horizon since (in his words) “[i]t is illogical to make your current decision in splendid isolation from what you expect to do in subsequent periods.” This longer range planning process, analogous to the evaluation of the Green and Bluebooks at the FOMC meeting, may be aimed at issues such as whether there is instrument instability for a given policy which might not show up at shorter horizons, whether there are undesirable characteristics of the implied interest rate interventions (are they too frequent? do they impart too much variability into the financial market?), and so on. At the same time, the policy maker also knows that it is likely to ‘activate only the first [portion] of the plan.’

In the second step of Blinder’s procedure, ‘new information that has arrived’ is assessed, and an ‘entirely new multiperiod plan’ is made. In real time, this would include additional economic data arriving subsequent to an FOMC meeting, and the new plan would incorporate information contained in the shocks to these data. If we were to proceed to this second step in our analysis, we would collect a new real-time data set and then at the time of the next FOMC meeting re-do the experiments described above.

b. Comparison with Previous Studies

As referenced earlier, the use of counterfactual analysis to evaluate policy alternatives has several precedents in the literature. Fackler-Rogers (1995) were the first to suggest the general approach used here, though shocks would be affected when a policy shock needed to attain the FAIR is imposed. We do not model such contemporaneous correlations here.
their analysis was in a simpler setting and was conducted in-sample in contrast to the out-of-sample analysis in the current paper. Specifically, in the Fackler-Rogers analysis a policy shock in a period was selected to attain a target exchange rate for that period rather than aiming at a 24-month average for the target variable. Christiano (1998) used a similar in-sample approach, computing the shocks to the interest rate equation needed in a given period to keep the money supply on a constant growth rate path during the Great Depression. He reported that the resulting path for the system variables other than the money supply “oscillated so wildly” that his presented results used an ad hoc method of combining a weighted average of the counterfactual shocks with the historical shocks rather than pure counterfactual residuals as is done here.

Leeper-Zha (2003) evaluated the effects of changes in the target funds rate beginning in 1990 and in 1994-95. Using a different empirical model that employed a richer specification of the reserves market than Leeper-Zha, Fackler-McMillin (2002) compared with no-change policies the effects of specified changes in the target fed funds rate on the time paths of output, the price level, and other model variables in 1995, and again in 1998. In contrast to these two studies which computed the shocks to the funds rate required to achieve an arbitrary target funds rate, the current paper computes the shocks to the funds rate required to generate the time path of the funds rate that achieves an inflation target. Finally, we note that Leeper-Zha contributed importantly to the policy evaluation literature by constructing a “modesty statistic” that allows one to test whether the Lucas critique is a concern for policy analysis, and we employ this statistic to evaluate our counterfactual experiments.

Sims-Zha (2006b) considered the effects of changes in the monetary policy reaction function in a VAR model on the economy’s response to shocks to non-policy variables. They first computed the effect of the policy and non-policy shocks on the model variables assuming a monetary policy reaction function that featured lagged response of the policy variable (the funds rate or alternatively the 3-month T-bill rate) to all model variables and contemporaneous response to only a subset of the model variables. They then separately computed the response of the economy to non-policy shocks assuming the central bank keeps the policy rate constant. Our analysis differs in several regards: (1) as noted earlier, we compute the policy shocks required to generate the (non-constant) path of the funds rate that achieves the inflation target and (2) since shocks hit all sectors of the economy simultaneously, we compute the funds rate shocks in the presence of shocks to the entire system rather than consider shocks to each non-policy variable as a separate case.
IV. Empirical Model

As noted earlier, the variables in the VAR model we estimate include those in the typical New Keynesian model: the output gap, the inflation rate, and the federal funds rate. Additionally, we include the rate of change in a commodity price index for two reasons. First, we add commodity prices following earlier literature that addresses the well-known “price puzzle” often found in VAR models. Second, since commodity price volatility is often used to represent supply shocks, as a first (and likely crude) approximation, we use this variable to help control for changes in output and inflation volatility emanating from sources outside the policy process. In order to establish the usefulness of the model for monetary policy evaluation, the macroeconomic effects of monetary policy are estimated by computing impulse response functions (IRFs) for shocks to the federal funds rate.

The model is estimated using monthly real time data over two time periods: 1962:1-1983:9 and 1980:1-2000:12. Our first set of counterfactual inflation targeting experiments begins in 1983:10, a year after the end of reserve targeting that characterized the October 1979-October 1982 period, thus allowing for adjustment to the new operating procedure to be basically completed before initiating the experiments. The second counterfactual begins in 2001:1. This starting point was chosen for two reasons. One is that there was considerable uncertainty about the macroeconomic effects of the decline in stock prices that began in 2000. The second is that it allows a year’s transition from the temporary effects of the Y2K preparations of the Federal Reserve and the subsequent volatility in the growth rate of the monetary base. In estimating the VAR, twelve lags of all variables are employed.

The transformations of the variables in the model follow the transformations of the variables in the typical New Keynesian model. Measuring the output gap at a monthly frequency is problematic, and we considered three alternative measures. The first measure was constructed by subtracting the quadratic trend of log real GDP from the log of real GDP at a quarterly frequency and then interpolating to monthly values. The second measure was the log of real GDP at a quarterly frequency and then interpolating to monthly values. The second measure was the log of real GDP minus the log of Hodrick-Prescott filtered real GDP, again interpolated from quarterly to monthly.
frequency. This filter is commonly used despite the potential problems in using this filter noted by Cogley-Nason (1995), among others. The third alternative was the monthly total unemployment rate. Results for the model with the first measure are reported in the text, and results for the models using the other two real macroeconomic activity measures, which are quite similar to the first model, are in the appendix. Since central banks tend to focus on longer-term inflation, the inflation rate is measured by the year-over-year rate of the change in the personal consumption expenditure deflator, a key series in the Fed’s evaluation of inflation. This avoids filtering out longer-run inflation information as would occur, for example, if we had used the annualized monthly rate of change in the price level. The federal funds rate is the monthly average of the daily rate. The rate of change in commodity prices is calculated as the annual difference of the log of this series. A description of the real time data and sources of the data is provided in the data appendix which is available on request.

Monetary policy shocks are identified using a Choleski decomposition. The ordering is: rate of change in commodity prices, output gap, inflation rate and then federal funds rate. Placing the funds rate last is based on a suggestion by Bernanke-Blinder (1992), and allows a contemporaneous response by the Fed to movements in the other three variables while simultaneously imposing a lagged effect of monetary policy on these variables.

The IRFs for a shock to the federal funds rate for the model with the quadratic trend output gap for both estimation periods are presented in Figure 1. In each panel, the solid line is the point estimate and the dotted lines are one standard deviation confidence intervals computed using Monte Carlo simulations employing 10,000 draws. The general pattern of results is similar for each sample period, but the timing and magnitude of effects differs across samples. The magnitude of the one standard deviation federal funds rate shock is comparable across the two

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26 Although policy makers focus on the core personal consumption expenditure deflator, the core series was not used in the model because it was not possible to construct a real-time version of this series for use in the experiments.

27 One concern about this ordering is that it does not allow monetary policy to have a contemporaneous effect on the commodity price index which is comprised of auction-market type variables that may well respond within the period to monetary policy shocks. Other concerns include (a) the assumption that the central bank responds contemporaneously to current period movements in output and the price level whereas data (even preliminary) on current period values of these variables is available only with a lag and (b) the constraint that output isn’t allowed to respond contemporaneously to a shock to monetary policy. Imposing a lag in the effect of monetary policy on inflation is not controversial. Because of these concerns, we estimated a Bernanke (1986)-type structural VAR which differed from the Choleski described in the text by allowing a contemporaneous effect of monetary policy on commodity prices, by allowing a concurrent effect of monetary policy on output, and by imposing no contemporaneous response of the federal funds rate to output and inflation shocks. The federal funds rate was, however, allowed to respond contemporaneously to commodity price shocks. The point estimates of the IRFs for this structural VAR for a shock to the federal funds rate were plotted along with the confidence intervals for the Choleski decomposition. The point estimates for a monetary policy shock for all variables for both samples were within the Choleski confidence intervals except for a few very minor departures in the very short-run for output. Based on these results, we used the Choleski decomposition in all experiments.
samples: 0.56 for 1962:1-1983:9 and 0.48 for 1980:1-2000:12. A positive shock to the federal funds rate persists briefly, but the confidence interval for the funds rate spans zero within 5-6 months, which we interpret as a return to the initial value. There is a transitory negative effect on the rate of change in commodity prices, and the effect is stronger and more persistent for the 1962:1-1983:9 sample. The output gap becomes negative after several months, but returns to its initial value over time. The magnitude of the effect is greater for the 1962:1-1983:9 sample than the second sample, but the time required for output to return to its trend and stay there is comparable for both samples. There is a transitory negative effect on the rate of change in the personal consumption expenditure price index but while the magnitude of the effect differs between the two periods, the time required before the effect becomes significant and the time that lapses until the rate of inflation returns to its initial value is about the same across samples.

Since the VAR models are used to assess the quantitative implications of inflation targeting, it is important that the VARs produce paths of the model variables for shocks to monetary policy that are consistent with macro models in which monetary policy shocks can temporarily affect real variables. This appears to be the case for the VAR models used in this paper.

V. Results

In this section, we present a variety of results from the inflation targeting experiments and discuss their economic interpretations. The discussion focuses on what policy makers in real time would have seen had they employed our methodology. Specifically, we investigate the nature of the available tradeoffs between inflation and output variability and how these tradeoffs have changed in the two periods we consider. As detailed earlier, in each experiment we assume that a policy of gradualism to reduce inflation is employed, first for a 48 month transition period and then for a 36 month period. Since the standard deviations of output, inflation and the interest rate for each target band in both experiments was only slightly greater for the 36-month period than the 48-month period and the inferences were the same, we present only the 48-month results. The 36-month results are in the appendix. The benchmark policy is for the midpoint of each inflation band to approach 2% over a 48 month period with bandwidths varying between 0% and one that is arbitrarily large.

The first experiment begins in 1983:10. The top part of Figure 2 shows the actual inflation rate through 1983:9 and the base projection of the inflation rate along with the ±1% and ±2% bands moving to the long-run inflation target. The inflation rate as measured by the personal consumption expenditures deflator at the outset of
this experiment was approximately 3.8%. Note that while the actual inflation rate was relatively low, the base projection suggested that inflation would quickly move outside the ±1% bands and approach the upper 2% band. Thus, for policy officials using real-time data in late 1983, the need for restrictive monetary policy looked highly likely. Such a policy would likely raise the specter of another recession following on the two at the outset of the decade, making empirical estimation of the variability tradeoff an important consideration.

The second experiment begins in 2001:1. Even though inflation was reasonably well contained at approximately 2.5% when our second experiment begins and the base projection in the bottom part of Figure 2 puts inflation within the 2% inflation bands, uncertainty about the macroeconomic effects of the decline in stock prices that began in 2000 suggests it is worth considering the implications of inflation targeting in 2001.

Summary statistics and basic results for the four bandwidths for both periods are presented in Tables 1 and 2. As detailed earlier, for each period and each bandwidth, these results are from 1000 trials starting with draws from the estimated residuals. Note that while the FAIR relative to the inflation band is used as the criterion of whether to intervene in a particular month, in order to be comparable to inflation data as commonly reported, the inflation statistics from our experiments reported in Tables 1 and 2 are for the underlying inflation rates for each particular month rather than the FAIR. The results in these tables are based on our 24-month characterization of Blinder’s ‘planning horizon.’ That is, the information in these tables is analogous to the FOMC Greenbook forecasts on the longer-run implications of the current or proposed policy path.

Table 1 includes basic results. We note three main points. First, for both experiments, the percentage of trials with any intervention and the average number of interventions per trial both fall as the width of the bands increases. This result is as expected; the wider the inflation band the more likely the FAIR falls within the band and the less likely an intervention is needed to maintain average inflation inside the band. The number of interventions per trial starts at the maximum of 24 months (the planning horizon) when the bandwidth is zero and is zero when the band is arbitrarily wide (in which case it is not necessary to intervene). The percentage of trials with any intervention for the 1% and 2% bands is substantially smaller in the 2001 experiment than in the 1983 experiment.

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28 We have excluded from the statistics in Tables 1 and 2 those trials in which a negative (nominal) interest rate would occur. Generally, the results that include trials in which negative interest rates occur are nearly identical to those reported below. Note that negative nominal rates do show up in real-world data on occasion. For example, Cecchetti (1988) discusses negative nominal interest rates on some Treasury securities in the 1930s and, more recently, Fleming-Garbade (2004) discuss repurchase agreements with negative interest rates. Casual analysis of our trials in which negative interest rates occur suggest that they were about the same order of magnitude as those appeared in Cecchetti and Fleming-Garbade.
and for each of these bands the average number of interventions is substantially lower in 2001. As bandwidth increases, the decline in both the percentage of trials with any intervention and the average number of interventions is faster in the 2001 experiment than the 1983 experiment.

Second, for both samples, the average number of interventions from above the band are substantially greater than from below the band. Given the inflationary pressures suggested by the base projections in the two experiments, it is not surprising that for the ±1% and ±2% bands, the number of interventions needed to restrain inflation (positive policy shocks to the interest rate equation) outnumber the interventions needed to stimulate inflation in order to maintain inflation within the bands.

Third, for both samples, the average maximum number of consecutive interventions per trial falls as the width of the band increases, and the average maximum number of consecutive interventions for the 1% and 2% bands are much fewer in number for the 2001 experiment than the 1983 experiment. Consecutive interventions stem from our imposition of a mechanical “commitment” to the inflation target objective. In our experiments, there is no option for the policymaker to deviate from this objective when computing the intervention. Once the inflation rate breaches the edge of the inflation band, depending on the inflation inertia several policy shocks may be needed to return average, long-run inflation to the specified level. Also note that even when there is a pattern of several consecutive interventions, given an initial intervention, it is less clear that there will necessarily be inertia in the interest rate itself, since (i) the interventions are partly a function of the random draws for all the variables, which can entail consecutive interventions but not necessarily of the same sign, and (ii) there is an endogenous component to the funds rate equation over and above the intervention term.

Table 2 provides a more detailed look at the policy interventions. Recall that each trial is initiated with a random draw from the estimated residuals, replacing the interest rate shock with a computed policy residual whenever the FAIR is outside the bounds of the inflation target band. The estimated residuals are, of course, zero mean since they are the result of OLS regressions. The computed policy residuals need not be zero mean, and in our setting where the general pattern is for inflation to breach the upper bound of the target range (see Figures 2 and 3) the average policy intervention is a positive interest rate shock to restrain the economy and lower the average inflation.

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29 See equations (A2) and (A3) in the appendix.
30 Because each trial in each experiment that eventually builds up to the variability frontier always selects the policy innovation needed to attain the inflation goal, there is a risk of instrument instability. While our approach does not rule out instrument instability, it is also possible that inertia in the interest rate will result. It turns out that there is substantial smoothing of the interest rate in our counterfactual simulations, even though we have not imposed any features that would explicitly limit the magnitude of interest rate movements.
inflation rate. In Table 2, we take the entire set of residuals with which we end up from the trials for the 1% and 2% bands and separate them into those that were drawn from the historical residuals and the policy residuals we computed in order to keep average inflation at the acceptable level.\textsuperscript{31} For the 5th and 95th percentile shocks (used to avoid any outliers) we find that the 95th percentile policy shock is about one and a half to two times the size of the comparable estimated residual. Furthermore, note that the average policy shock is well within the range of the shocks of the estimated residuals. Policy innovations outside the historical norm range occur, but not on average.

Table 3 shows the fundamental results: the standard deviations of the key variables for each bandwidth of each experiment. These standard deviations, also plotted in Figures 3 and 4, show the basic results of the paper: the estimated tradeoffs over time between inflation and output variability and inflation and interest rate variability. We note, for both periods, that as we move from the stochastic simulation of the historical policy in the estimated VAR (the infinity band, where no interventions are called for) to the 2% target band, inflation variability falls and output variability rises since more tightly controlling inflation implies the policy maker must accept the higher output volatility as a consequence. The absolute value of the change in output variability is less than the change in inflation variability— their ratio is .7 in both experiments. As we move from the 2% band to a 1% band, the absolute value of the ratio of the change in output variability to the change in inflation variability is 1.3 for the 1983 experiment and 1.4 for the 2001 experiment. Finally, as we move from a 1% band to achieving the target without variability (the 0% band), the absolute value of the ratio of the change in output variability to the change in inflation variability is 2 for the 1983 experiment and 3 for the 2001 experiment. The point on the tradeoff that would be chosen by the central bank clearly depends on the central bank’s preferences regarding output and inflation variability – the point that, in the earlier words of Svensson, “looks best.”\textsuperscript{32} Our technique provides a data-driven methodology for estimating the objective tradeoff faced by the central bank.\textsuperscript{33}

Figure 3 plots the tradeoff data from Table 3 and shows a roughly parallel downward and inward shift of the inflation-output variability tradeoff over time. For example, in the 2001 experiment, inflation variability equal to

\textsuperscript{31} Note that for the 0% band, all the innovations are (almost certainly) policy innovations and for the arbitrarily wide band, there are no policy innovations, so the 1% and 2% bands discussed in Table 2 are the only relevant comparisons.

\textsuperscript{32} Implicitly, movement along the tradeoff reflects alternative policy maker preferences. In Svensson’s setup as summarized in equation (4), the tradeoff between inflation and output is a function of $\lambda$, his relative weight on output variability. An analogous point holds here where we have instead normalized the implicit loss function on inflation variability as expressed in $A$ in section II above.

\textsuperscript{33} Results analogous to those in Table 3 for the case of a 36-month transition horizon are included in appendix table A2. The implications are virtually the same as those reported in the text for the 24-month case.
.006 (1% band) is associated with output variability of .011 whereas for the 1983 experiment output variability implicitly associated with the same inflation variability is approximately .019 (0% band), about 70% higher than for the 2001 experiment.

In Figure 4, similar results are found for the inflation-interest rate variability tradeoffs. Smaller inflation targeting bands are associated with greater interest rate variability since more frequent and larger interest rate adjustments are required for more precise inflation control. As in Figure 3, the inflation–interest rate variability tradeoff also shifts inward in a roughly parallel fashion; for given inflation variability, we observe much lower interest rate variability in the second period.

Given the differences in methodology, data, and estimation periods, only rough comparisons between our analysis and other work can be made. A particularly difficult methodological issue is the translation of weights in a loss function into explicit values for the width of inflation target bands; absence of an exact correspondence between the width of target bands and specific weights on output stabilization make direct comparison with research based on loss function weights difficult. However, we will discuss our variability tradeoff with another in the literature that, as closely as we can find, approximates our presentation.

Rudebusch-Svensson (1999) (RS) start with a loss function that has inflation deviations from target, the output gap, and the change in the interest rate as arguments. They minimize this loss function subject to a model of the economy that includes a Phillips curve/aggregate supply specification and an IS/aggregate demand equation. Policy focuses on the relevant feedback function for the nominal interest rate in the IS curve. Among the eleven simulated policy rules (some with several variants) that are compared with the optimal setting derived from the formal minimization are two that focus on inflation forecast targeting, their “FIFT” and “FIFTS” policy rules. Their FIFT rules assume no interest smoothing while the FIFTS rules allow various degrees of such smoothing. As our model does not preclude such smoothing, and our interest rate paths show that interest rate variations are relatively small (i.e., the data seem to select a path for the policy interest rate that exhibits patterns consistent with partial adjustment) we will briefly compare our results with the RS FIFTS model.

Since we have targeted a two-year inflation forecast, and given that RS use quarterly data, our comparison focuses on their eight-quarter forecast rule (FIFTS(8)). As noted in Table 3 and as plotted in Figure 3, when the standard deviation of inflation in our experiments is .006 (for the 1% bandwidth in the 2001 experiment and for the 0% bandwidth for the 1983 experiment) output variability was .011 (2001) and .019 (1983). From the RS results in
their Tables 5.3 – 5.7 (using various sets of weights on loss function arguments), for about the same inflation variability,\(^{34}\) the RS output variability\(^{35}\) varies from .0187 (.0066 inflation variability) to .0267 (.0050 inflation variability). We also note that as a rough point estimate, when we examine Fig. 5.2 which plots the variability tradeoff for FIFTS(8) as well as other rules, we see that inflation variability of .006 (2.4 in the figure) is associated with output variability of about .002. Thus, our results seem reasonably close to theirs for this particular comparison, given the differences in methodologies and differences in sample periods for estimation. However, as noted earlier, conclusions from this comparison should be tempered by the fact there is no straightforward way to directly translate loss function weights in their study to the bandwidths in ours.

Finally, an important question is: What is the source of the inward shift in the inflation-output variability tradeoff between the two periods that we find? Although we cannot definitively identify the reason(s) for the shift, we note that much of the sample used in the estimation of the VAR for the later period spans the period of the “Great Moderation” in macroeconomic volatility. Explanations for the Great Moderation include good luck in terms of reduction in the frequency and magnitude of economic shocks and better monetary policy, and evidence has been presented in support of both explanations. The consensus from studies such as Stock-Watson (2002), Ahmed-Levin-Wilson (2004), and Sims-Zha (2006a) is that “good luck” in the form of reduced variability of non-monetary policy shocks is primarily responsible for the increased stability of the real economy since the mid-1980s. However, Stock-Watson (2002) attribute from 10-25% of the reduced variability in the real economy to improved monetary policy, and Ahmed-Levin-Wilson (2004) find that, although “good policy” doesn’t seem to explain much of the reduced variability in real output, it is important in understanding the reduction in inflation variability.

In regard to the “good policy” explanation, we note that much of the recent literature on monetary policy suggests that inflation targeting allows central banks to gradually gain credibility. Clarida-Gali-Gertler (1999) suggest that credible policy “… enables the central bank to stabilize the economy with relatively modest movements in the short rate” (pp. 1689-90), and Carlstrom-Fuerst (2005) present simulations showing that central bank credibility allows the Fed to achieve given objectives with smaller policy interventions than in the case where credibility is lacking. Cecchetti-Flores-Laguna-Krause (2006), based on a cross-section of both developed and

\(^{34}\) RS compute the average inflation rate as 400*[\ln p_t – \ln p_{t-1}]. We translate their inflation standard deviation into one approximately equal to ours by dividing their standard deviations by 400. Across their experiments, their inflation standard deviation ranges from .0050 to .0066.

\(^{35}\) RS use the output gap in percentage points, while we use decimal form. Thus, dividing their output gap standard deviations by 100 produces roughly comparable results.
developing countries, find that, for most countries, monetary policy was more efficient in the 1990s than in the 1980s and that better monetary policy led to a significant improvement in macroeconomic performance.\footnote{In their study, Cecchetti et. al. (2006) estimate inflation variability-output variability tradeoffs derived from a small structural model and an explicit loss function.}

Unfortunately, our technique does not allow us to distinguish between the “good luck” and “good policy” explanations, but we note the fact that the tradeoff can shift substantially over time suggests that central banks cannot regard the tradeoff as fixed over time.

VI. The Counterfactual Experiments and the Lucas Critique

An important issue in any counterfactual experiment is whether the Lucas critique would invalidate the results. With respect to the first experiment, we noted earlier that the 1962:1-1983:9 period over which the model was estimated was a period in which there was a great deal of variation in output and inflation, especially during the immediate post-October, 1979 months at the end of the estimation period for this experiment. Romer-Romer (2002) characterized the estimation period as one in which the norm was substantial fluctuations in monetary policy variables, and Sims-Zha (2006a) found the 1979-83 period to be distinct. The period covered by the estimation sample for our second experiment, ending in December 2000, appears to be more settled in terms of policy, with interest rate targeting effectively governing most of the period, culminating with substantial attention to the Taylor rule and its role in the Great Moderation. As an initial attempt to examine the importance of Lucas-critique problems, following Dufour (1980; 1982), we re-estimated the model over 1962:1-1983:9 and 1980:1-2000:12, adding a 0-1 dummy variable for each month in which instability is suspected. For the first sample, we added a separate dummy variable for each month of the post-October 1979 period included in the sample, 1979:10 through 1983:9, the period of nonborrowed reserve targeting plus an approximate one year transition period from the end of nonborrowed reserve targeting. For the second sample, we included separate monthly dummies for 1980:1 (the first observation in the estimation period) through 1983:12 (roughly the period of nonborrowed reserve targeting in the second sample and an approximate one year transition period from the end of nonborrowed reserve targeting). Following Sims-Zha (2006b), we used the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC) to compare the models with and without the dummies and thereby to provide information about the stability of the model. The AIC indicated the model with dummies was preferred for both samples, hence indicating instability. However, the SIC suggested that the model without the dummies was preferred for both samples. Thus, evidence regarding the stability of the model over the estimation periods is mixed.
While stability tests can usefully shed light on potential Lucas concerns within the estimation period, in our real-time setting the policy maker is concerned in addition whether a proposed policy action will trigger among agents in the economy the perception that a proposed policy would be interpreted as a regime shift. If so, then the variability tradeoffs we presented in Figures 3 and 4 would be of little use in evaluating the available tradeoffs to policy makers. To this end, Leeper-Zha (2003) have introduced a “modesty statistic” intended to evaluate whether a prospective policy initiative is likely to be viewed as a modest policy intervention.

The Leeper-Zha theoretical approach is a Markov-switching model, with each regime a linear model of the economy (a VAR in their case). Within a regime, the effect of a policy intervention is as described by the first term on the right hand side of our equation (10), the impact of the proposed policy relative to the base projection. Specifically, picking a policy sequence \( \{e_{t+1}, e_{t+2}, \ldots, e_{t+n}\} \), computing the expression \( \sum_{i=0}^{n-1} D_i e_{t+i-1} \) and then scaling by \( \sqrt{\sum_{i=0}^{n-1} D_i^2} \) provides their “modesty statistic.”\(^{37}\) Leeper-Zha (2003) note that the “modesty statistic” has a standard normal distribution, so when this statistic is less than 2, the policy innovation embedded in the \( e \) path over \( t+1 \) to \( t+n \) does not induce agents to change their assessments about the policy regime in place.\(^{38}\)

We have computed the Leeper-Zha modesty statistic in the context of our counterfactual experiments, modifying their approach by using the randomly drawn disturbances to the other equations under which our policy interventions are computed rather than assuming that the shocks to the non-policy equations are all zero. Under this condition, in the trials underlying our basic results in Figures 3 and 4, the largest computed modesty statistics (in absolute value) for each experiment are presented in Table 4, where in each cell the relevant statistics for the impact of the policy sequence for output, the inflation rate, and the interest rate are given respectively. All the computed statistics are well below 2.0, so that there is no evidence that our proposed inflation bands and the policies needed to achieve them would have triggered Lucas critique-type concerns among agents in the economy. This implies, importantly, added confidence that the tradeoffs presented in Figures 3 and 4 can be reasonably interpreted as valid tradeoffs available to policy makers.

---

\(^{37}\) Consistent with our approach, Leeper-Zha use the \( e \) shock to the policy equation as the policy innovation, and assume as we do that “… although the policy advisor chooses [the \( e \)-innovation], private agents treat it as random.” (Leeper-Zha, p. 1678).

\(^{38}\) Of course, alternative policy regimes can be “close” to each other, so that distinguishing between these regimes may be difficult. Thus, a modesty statistic of less than 2 is necessary but not sufficient to claim that no important Lucas effects are present.
VI. Summary and Conclusion

Our focus in this paper is twofold: (i) illustration of how a VAR model can be used to implement and evaluate inflation forecast targeting and (ii) the derivation in real time of the output-inflation variability tradeoff available to the central bank under inflation forecast targeting and estimation of how this tradeoff has changed over time. Tolerance bands of varying widths around transitional inflation targets constructed to achieve 2% inflation are considered.

Our inflation forecast targeting approach is based on dynamic, stochastic simulations of the average inflation rate over a two-year horizon using the moving average representation of the VAR model. Deviations of the forecast average inflation rate from target generate interventions in the form of changes in the federal funds rate designed to gradually push the forecast inflation rate back to target, and we compute the required adjustments to the federal funds rate.

The technique is illustrated through two counterfactual experiments using real-time data. The first experiment begins in 1983:10 and is based on a VAR estimated over 1962:1-1983:9 whereas the second experiment begins in 2001:1 using a model estimated over 1980:1-2000:12. In terms of technical results, we find: (1) less intervention is needed as the width of the bands increases, and fewer interventions are needed in the 2001 experiment than in the 1983 experiment; (2) more interventions are needed to reduce the inflation rate than are needed to raise the inflation rate, reflecting the inflationary pressures during the periods, which in turn makes targeting exercises important; (3) fewer consecutive interventions are required as the width of the target band increases, and fewer consecutive interventions are needed for the 2001 experiment than the 1983 experiment; and (4) a given inflation variability is associated with lower output and interest rate variability in the 2001 experiment than in the 1983 experiment, i.e. the trade-offs between inflation variability and output variability and inflation variability and interest rate variability shifted favorably between our time periods. As noted earlier, the source of the shift might stem from “good luck” or “good policy”, but a critical lesson is that significant shifts in the trade-offs have occurred over time and hence that the objective trade-offs faced by the central bank should not be assumed to be fixed.
Table 1: Frequency of Policy Interventions

<table>
<thead>
<tr>
<th>A. 1983 Experiment</th>
<th>0% band</th>
<th>1% band</th>
<th>2% band</th>
<th>∞ band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average interventions per 24 month trial</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From above band</td>
<td>24.0</td>
<td>14.8</td>
<td>9.2</td>
<td>13.1</td>
</tr>
<tr>
<td>From below band</td>
<td>11.8</td>
<td>12.5</td>
<td>0.6</td>
<td>9.5</td>
</tr>
<tr>
<td>Average maximum consecutive interventions</td>
<td>24.0</td>
<td>12.0</td>
<td>9.2</td>
<td>0</td>
</tr>
<tr>
<td>Percent of trials with any intervention</td>
<td>100</td>
<td>93.4</td>
<td>73.0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. 2001 Experiment</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average interventions per 24 month trial</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From above band</td>
<td>24.0</td>
<td>17.0</td>
<td>7.0</td>
<td>8.2</td>
</tr>
<tr>
<td>From below band</td>
<td>7.0</td>
<td>8.1</td>
<td>0.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Average maximum consecutive interventions</td>
<td>24.0</td>
<td>7.7</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>Percent of trials with any intervention</td>
<td>100.0</td>
<td>61.2</td>
<td>18.8</td>
<td>0</td>
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</table>

Table 2: Detailed Analysis of Shocks

<table>
<thead>
<tr>
<th>A. 1983 Experiment</th>
<th>1% Band</th>
<th>2% Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Residuals</td>
<td>Policy Residuals</td>
<td>Random Residuals</td>
</tr>
<tr>
<td>5th percentile</td>
<td>-.007</td>
<td>-.006</td>
</tr>
<tr>
<td>95th percentile</td>
<td>.009</td>
<td>.020</td>
</tr>
<tr>
<td>Mean Policy Residual</td>
<td>.0045</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. 2001 Experiment</th>
<th>1% Band</th>
<th>2% Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Residuals</td>
<td>Policy Residuals</td>
<td>Random Residuals</td>
</tr>
<tr>
<td>5th percentile</td>
<td>-.006</td>
<td>-.004</td>
</tr>
<tr>
<td>95th percentile</td>
<td>.008</td>
<td>.013</td>
</tr>
<tr>
<td>Mean Policy Residual</td>
<td>.0043</td>
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Table 3: Standard Deviations of Key Variables

<table>
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<tr>
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<th>0% band</th>
<th>1% band</th>
<th>2% band</th>
<th>∞ band</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. 1983 Experiment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>.01984</td>
<td>.01428</td>
<td>.01047</td>
<td>.00819</td>
</tr>
<tr>
<td>Inflation</td>
<td>.00605</td>
<td>.00883</td>
<td>.01178</td>
<td>.01492</td>
</tr>
<tr>
<td>Interest rate</td>
<td>.04519</td>
<td>.02636</td>
<td>.01586</td>
<td>.01167</td>
</tr>
<tr>
<td><strong>B. 2001 Experiment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>.01692</td>
<td>.01117</td>
<td>.00967</td>
<td>.00963</td>
</tr>
<tr>
<td>Inflation</td>
<td>.00435</td>
<td>.00614</td>
<td>.00723</td>
<td>.00733</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>.04022</td>
<td>.01489</td>
<td>.01076</td>
<td>.01039</td>
</tr>
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</table>

Table 4: Modesty Statistics

<table>
<thead>
<tr>
<th></th>
<th>0% band [y, p, i]</th>
<th>1% band [y, p, i]</th>
<th>2% band [y, p, i]</th>
<th>∞ band [y, p, i]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1983 Experiment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.92, .53, .71</td>
<td>.81, .48, .68</td>
<td>.70, .49, .65</td>
<td>.75, .61, .52</td>
<td></td>
</tr>
<tr>
<td><strong>2001 Experiment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.54, .71, .76</td>
<td>1.25, .66, .65</td>
<td>1.06, .70, .53</td>
<td>1.12, .77, .38</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2: Actual Inflation, Base Projections, & Target Bands.
References


Appendix: Implementing Inflation Target Simulations

In this appendix, we provide technical detail on computation of the FAIR and how we compute the policy actions needed to maintain it on or inside a target band. Let the elements $k$ and $j$ in the vector $y_j$ represent the federal funds rate and inflation, respectively. Consider the $j^{th}$ equation in text equation system (3) when $n=1$, which is the one-period-ahead inflation equation:

$$y_{j,t+1} = \sum_{k=1}^{N} d_{0,k} e_{i,t+1} + BP_{j,1}$$  \hspace{1cm} (A1.1)

Under the assumption that policy makers are concerned with a 24 month average inflation rate, for periods 2 through 24, the analogous equations are

$$y_{j,t+2} = \sum_{k=1}^{N} d_{0,k} e_{i,t+2} + \sum_{i=1}^{N} d_{1,k} e_{i,t+1} + BP_{j,2}$$  \hspace{1cm} (A1.2)

$$
\vdots
$$

$$y_{j,t+24} = \sum_{k=1}^{N} d_{0,k} e_{i,t+24} + \sum_{i=1}^{N} d_{1,k} e_{i,t+23} + \cdots + \sum_{i=1}^{N} d_{23,k} e_{i,t+1} + BP_{j,24}$$  \hspace{1cm} (A1.24)

Summing equations (A1.1) through (A1.24) and then averaging yields

$$\frac{1}{24} (y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+24}) = \frac{1}{24} \left( \sum_{k=1}^{N} d_{0,k} e_{i,t+1} + \sum_{i=1}^{N} d_{0,k} e_{i,t+2} + \sum_{i=1}^{N} d_{1,k} e_{i,t+1} + \cdots + \sum_{i=1}^{N} d_{23,k} e_{i,t+1} + BP_{j,1} + BP_{j,2} + \cdots + BP_{j,24} \right)$$

$$= \frac{1}{24} \left\{ \sum_{i=1}^{N} d_{0,i} e_{i,t+1} + d_{0,k} e_{i,t+1} + \sum_{i=1}^{N} d_{1,i} e_{i,t+2} + d_{1,k} e_{i,t+1} + \sum_{i=1}^{N} d_{23,i} e_{i,t+1} + BP_{j,1} + BP_{j,2} + \cdots + BP_{j,24} \right\}$$

We next show how to compute the current period policy shock needed to attain the FAIR. Define

$$Y_{j,t+1} = \frac{1}{24} (y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+24})$$

$Y_{j,t+1}^*$ to be the forecast inflation rate and let the targeted, average inflation rate be

$$Y_{j,t+1}^* = \frac{1}{24} (y_{j,t+1} + y_{j,t+2} + \cdots + y_{j,t+24})^*$$

Assume for now that the goal is to achieve this target exactly; that is,
assume for now that the width of the inflation band is zero. Then conditional on \( \epsilon_{i,t+1}, i \neq k \), as well as on
\( \epsilon_{i,t+2}, \epsilon_{i,t+3}, \ldots, \epsilon_{i,t+24} \), there is a value for the current policy innovation, \( \epsilon_{k,t+1} \), that will achieve this inflation target. Specifically, we solve the previous equation for the policy innovation undertaken at the beginning of period \( t+1 \) designed to attain the target:

\[
\epsilon_{k,t+1} = \left( \sum_{j=0}^{23} d_{j,k} \right)^{-1} \left\{ (y_{j,t+1} + y_{j,t+2} + \ldots + y_{j,t+24})^* - \sum_{i=1}^{N} d_{i,0} \epsilon_{i,t+1} - \sum_{i=1}^{N} d_{i,0} \epsilon_{i,t+2} - \sum_{i=1}^{N} d_{i,1} \epsilon_{i,t+4} - \ldots - \right. \\
\left. \sum_{i=1}^{N} d_{i,0} \epsilon_{i,t+23} - \ldots - \sum_{i=1}^{N} d_{i,1} \epsilon_{i,t+24} \right\} 
\]

We next relax the assumption that the average inflation rate is targeted exactly, and show how to pursue a policy objective of constraining inflation to lie within a given, predetermined bandwidth. For period \( t+1 \), we want the inflation rate within the pre-specified band \( Y_{t+1}^* \pm \tau \) where \( \tau \) is half the bandwidth.\(^1\) It may be that no policy intervention is needed, which will occur when the shocks to the economic system are such that

\[
Y_{t+1}^* - \tau < Y_{t+1} < Y_{t+1}^* + \tau.
\]

If, on the other hand,

\[
Y_{t+1} < Y_{t+1}^* - \tau
\]

or if

\[
Y_{t+1} > Y_{t+1}^* + \tau,
\]

a policy intervention is needed to return the inflation rate either to the edge of the band or to some pre-specified value interior to it. For instance, if the policy choice is to return to the edge of the band, then the policy innovation is computed by replacing the term \( (y_{j,t+1} + y_{j,t+2} + \ldots + y_{j,t+24})^* \) in equation (A2) with

\[
(y_{j,t+1} + y_{j,t+2} + \ldots + y_{j,t+24})^* \pm \tau = Y_{t+1}^* \pm \tau ,
\]

depending on whether the FAIR is computed to be above or below the tolerance range.

\(^1\) As specified, the band is symmetric. If the policy maker were to set policy actions to return inflation to a particular path strictly within the band, then asymmetric bands would also be of interest. For example, the policy maker might respond to a given upward shock to the inflation rate, but not to a downward shock of the same absolute value, as in an opportunistic disinflation policy. It is straightforward to allow for asymmetric bands.
The policy action undertaken in period t+1 implies a subsequent path for the system’s variables, and later evaluation of policy actions must take t+1 policy into account; again, the policy approach implies history dependence. Given this policy action, the average, prospective inflation for the h-period horizon covering periods t+2 through t+25 may be computed similarly to the discussion in equations (A1.1) through (A1.24):

$$\frac{1}{24} (y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+25}) =$$

$$= \frac{1}{24} \left( \sum_{j=1}^{N} d_{0,j} e_{j,t+2} + d_{0,j} e_{k,t+1} + \sum_{j=1}^{N} d_{1,j} e_{j,t+2} + d_{1,j} e_{k,t+1} + \cdots + \sum_{j=1}^{N} d_{j-1,j} e_{j,t+2} + d_{j-1,j} e_{k,t+1} + \sum_{j=1}^{N} d_{j,j} e_{j,t+2} + d_{j,j} e_{k,t+1} + \sum_{j=1}^{N} d_{j+1,j} e_{j,t+2} + d_{j+1,j} e_{k,t+1} + \cdots \right)$$

$$\sum_{j=1}^{N} d_{0,j} e_{j,t+25} + \sum_{j=1}^{N} d_{1,j} e_{j,t+23} + \cdots + \sum_{j=1}^{N} d_{22,j} e_{j,t+2} + d_{22,j} e_{k,t+1} + \sum_{j=1}^{N} d_{23,j} e_{j,t+2} + d_{23,j} e_{k,t+1} + \sum_{j=1}^{N} d_{24,j} e_{j,t+2} + d_{24,j} e_{k,t+1} + \sum_{j=1}^{N} d_{25,j} e_{j,t+2} + d_{25,j} e_{k,t+1} + \sum_{j=1}^{N} d_{26,j} e_{j,t+2} + d_{26,j} e_{k,t+1} + \cdots + BP_{j,2} + BP_{j,3} + \cdots BP_{j,25} \right)$$

To attain the target inflation rate exactly, solve for $\hat{e}_{k,t+2}$ conditional on $\hat{e}_{k,t+1}$:

$$\hat{e}_{k,t+2} = \left( \sum_{j=0}^{23} d_{j,j} \right)^{-1} \{ (y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+25})^* - \sum_{j=1}^{N} d_{0,j} e_{j,t+2} - \sum_{j=1}^{N} d_{1,j} e_{j,t+1} - \cdots \}$$

$$\sum_{j=1}^{N} d_{0,j} e_{j,t+25} - \sum_{j=1}^{N} d_{1,j} e_{j,t+23} - \cdots - \sum_{j=1}^{N} d_{22,j} e_{j,t+2} - \sum_{j=1}^{N} d_{23,j} e_{j,t+1} - \sum_{j=1}^{N} d_{24,j} e_{j,t+2} - d_{24,j} e_{k,t+1} - \sum_{j=1}^{N} d_{25,j} e_{j,t+2} - \sum_{j=1}^{N} d_{26,j} e_{j,t+2} - d_{26,j} e_{k,t+1} - \cdots + BP_{j,2} + BP_{j,3} + \cdots BP_{j,25} \}$$

(A3)

If the bandwidth is nonzero, then analogous to the earlier discussion, replace \( (y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+25})^* \) with \( (y_{j,t+2} + y_{j,t+3} + \cdots + y_{j,t+25})^* \pm \epsilon = Y_{j,t+2} \pm \epsilon. \)

Note that, generalizing equations like (A2) or (A3) to period t+j, computation of the t+j period policy shock needed to attain the FAIR for the subsequent 24 months would include two kinds of terms: policy interventions

\(^{2}\) Note that in equations (A2) and (A3), our policy actions generally respond to all the information in the model. In contrast, policy actions based on the well-known Taylor rule only respond to, say, information on output (relative to potential) and deviations of inflation from target.
needed return the average inflation rate to the band and shocks from the random draw for those periods in which no
intervention is needed.

For the various bandwidths of each experiment, we specify a target path and specify a band around this path. Since we sample from the estimated residuals, we do not impose any arbitrary assumptions about the
probability density generating the shocks to the economy. For each trial, computed values for the system variables
are those the economy will follow using the assumed policy interventions that keep the FAIR inside the designated
band, given the shocks to the other equations.3

Data Appendix

1. Real time real GDP data are from the routput.xls file from the qvad folder available for download from the
Philadelphia Federal Reserve Bank. The relevant columns of this file are: routput83q4 for the sample that ends in
1983:9 and routput01q1 for the sample that ends in 2000:12. Data in column routput01q1 were known in the first
quarter of the respective years, and we assume this data was known by the Fed at the beginning of the relevant
counterfactual experiments. Data in column routput83q4 were known in the fourth quarter of 1983. Since our 1983
counterfactual begins in November 1983, the Fed may not have had all the information in this column at the
beginning of the counterfactual. However, we wanted to begin the counterfactual a year after the end of reserve
targeting, and this was the closest approximation to real time GDP data we could obtain for November 1983.

As noted in the text, for each sample various filters were used to construct a potential GDP series, and each output
gap measure was then constructed as actual real GDP minus the filtered potential GDP. The quarterly real time
output gaps were then interpolated to monthly data using the distrib.src procedure in RATS 6.02b.

2. Real time personal consumption expenditure deflator data were taken from various issues of the Survey of
Current Business.
Current Business, Table 7.1. The data in these tables were quarterly, and were interpolated to monthly using the
distrib.src procedure in RATS 6.02b.
   c. We note that the data set for 1980:1-2000:12 is not totally a pure real time data set since data at the very end of
2000 was pulled from the earliest Survey of Current Business in 2001.

3. The federal funds rate is taken from the Global Insight Basic database, series fyff, and the commodity price index
is the Commodity Research Bureau spot market index for all commodities (Global Insight Basic database, series
psccom). These series are not revised and hence the data pulled from the Global Insight databases were used in the
real-time estimations.

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3 While it is possible to do so, we do not take into account the possibility that the model coefficients may be
estimated imprecisely.
Appendix Figure 1: Impulse Response Functions


Appendix Figure 4 (Unemployment Rate Model): Actual Inflation, Base Projections, & Target Bands


Appendix Figure 7: Inflation-Unemployment Rate Standard Deviation Tradeoffs Over Time: 2% Target

Appendix Figure 8: Inflation-Interest Rate Standard Deviation Tradeoffs Over Time: 2% Target
### Table A1: Frequency of Policy Interventions

<table>
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<th>1% band</th>
<th>2% band</th>
<th>∞ band</th>
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<tbody>
<tr>
<td><strong>A. 1983 Experiment</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Average interventions per 24 month trial</td>
<td>24.0</td>
<td>14.1</td>
<td>10.7</td>
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<tr>
<td>From above band</td>
<td>15.9</td>
<td>13.6</td>
<td>10.6</td>
<td>NA</td>
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<tr>
<td>From below band</td>
<td>8.1</td>
<td>0.5</td>
<td>0.1</td>
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<tr>
<td>Average maximum consecutive interventions</td>
<td>24.0</td>
<td>12.9</td>
<td>10.2</td>
<td>0</td>
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<tr>
<td>Percent of trials with any intervention</td>
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<td>94.3</td>
<td>78.2</td>
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<td><strong>B. 2001 Experiment</strong></td>
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<td>1.8</td>
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<tr>
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<td>8.0</td>
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<tr>
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### Table A2: Standard Deviations of Key Variables

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<tr>
<td>Output</td>
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<td>.01083</td>
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<tr>
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