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Abstract

I investigate the long-run implications of trade and technology diffusion through trade, when firms are heterogeneous and trade is costly. The paper integrates firm heterogeneity and trade into product innovation growth models from endogenous growth theory. Two specifications of the R&D process are considered. In the first, R&D uses labor and intermediate goods; in the second, it uses labor and available technology. I find that under both specifications, exposure to trade increases average productivity. Furthermore, under the first specification exposure to trade always has a positive effect on economic growth, while it has an ambiguous effect on growth under the second.

JEL Classification: F1, F3, L1, O3, and O4

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1 Introduction

In endogenous growth models, technical progress is driven by new designs that result from the research and development efforts of profit maximizing agents (see Romer (1990), Grossman and Helpman (1991a) and (1991b), and Aghion and Howitt (1992)).¹ An important assumption in these models is that firms that use these designs to produce goods face a “common” production technology. Firm-level empirical studies, however, find the existence of large and persistent productivity differences across firms even in the same industries (see Bernard and Jensen (1999) and Clerides et al. (1998)).

The same production technology assumption has an important consequence in trade context. Since firms face the same technology, when exposed to trade all firms will sell their products in foreign markets. Empirical studies, on the other hand, show that even in traded sectors only some firms participate in trade and, more importantly, there is a strong correlation between firms’ foreign market participation and the firms’ productivity levels (see Bernard and Jensen (1999), (2004a), (2004b), Clerides et al. (1998)).

These findings restrict the implications of these models in yet another important dimension: technology diffusion through trade. There is an influential literature in which these endogenous growth models are used to address the effects of technology diffusion through trade on growth.² Since the above empirical findings state that not all firms participate in trade, the technology diffusion through trade will be more limited than what these models imply. Moreover, given the strong correlation between firms’ productivity levels and foreign market participation, the effects of the diffusion process may be more complicated than

¹New designs correspond to new products in the Romer (1990) and the Grossman and Helpman (1991a) models, and to better qualities in the Grossman and Helpman (1991b), and the Aghion and Howitt (1992) models.

² Some of the important contributions are Grossman and Helpman (1989), (1991a), (1991b), Rivera-Batiz and Romer (1991), and Conolly and Valderrama (2005). Most of the studies in this literature, however, consider the problem in the North-South framework, whereas in this paper I study the problem in two symmetric countries. Important exceptions are Grossman and Helpman (chapter 9, (1991a)) and Rivera-Batiz and Romer (1991), who also consider the effect of increased economic integration (through trade) between two symmetric countries. They, however, have not considered firm heterogeneity and costly trade. As I will discuss in section 3, these modifications have important consequences for results.

what these models predict.

This paper investigates the long-run effects of trade and technology diffusion through trade when firms are heterogeneous in their productivity levels and trade is costly.³ Toward this goal, I integrate Melitz's (2003) work on firm heterogeneity and trade into product innovation (i.e., variety-expansion) endogenous growth models developed by Romer (1990) and Rivera-Batiz and Romer (1991). In formulating the product development process, I considered two complementary specifications, each capturing different natures of the R&D process. In the first specification, product development technology is similar to that of final goods production: labor and intermediate goods are used in R&D. Note that in this case, upon opening to trade, imported goods will also be used in R&D; hence, foreign technology will naturally diffuse in the product development process. In the second specification, however, labor and available technology are used in the R&D process. To capture the positive effects of trade on product innovation in this case, I assume that foreign contribution to local technology increases through trade.

The results of this paper can be summarized as follows. Under both specifications, exposure to trade increases average productivity.⁴ Although trade is costly, under the first type of R&D specification this negative effect is dominated by the positive contributions of the technology diffusion and the average productivity gain. Hence, exposure to trade always has a positive effect on economic growth and consumer welfare. On the other hand, when the second specification is used for the R&D process, the positive effects of trade may not be high enough to overcome its costs. In this case, exposure to trade has an ambiguous effect on economic growth and consumer welfare.

As indicated above, this paper uses Melitz's (2003) influential work on firm heterogeneity and trade. His model is capable of generating the several empirical findings and also shows

³There will be two types of trade costs. One is the unit-transportation cost, the other is the market entry fixed (sunk) cost. As will be shown in section 3, it is this fixed cost together with firm heterogeneity that will endogenously divide firms into two groups: those only serve domestic market and those serve both domestic and foreign markets.

⁴This effect is similar to that in Melitz's (2003) and supported by micro-level empirical studies (See Aw et al. (2000), Pavcnik (2002), and especially Tybout (2003) for a survey of this literature).

that the aggregate productivity increase contributes to a welfare gain. This paper differs from his work in two important aspects. First, in this paper there is a sustained growth, whereas in his paper there is no growth.⁵ Second, the dynamic growth framework used here also allows me to investigate the effects of technology diffusion through trade on growth.

In a recent paper, Baldwin and Robert-Nicoud (2005) also study the long-run implications of trade when firms are heterogeneous. They embed the product innovation process, which is similar to the second type of R&D specification in this paper, in Melitz's set-up and find that exposure to trade has *negative* effects on growth and welfare. Although there are several other methodological differences between this paper and theirs, the most important differences are coming from the formulations of the R&D process. They consider neither the possibility that import goods can be used in the development of new products (as in the first type R&D specification of this paper) nor the possibility that trade in goods can enhance the flow of technology (as in the second specification of this paper).⁶ In section 3, I shall show that omissions of these positive effects of trade are indeed the main reason of their *negative* result.

The plan of this paper is as follows. Section 2 introduces a closed economy model. In section 3, I shall open the economy to trade and investigate the implications of further exposure to trade on the economy. To gain a deeper understanding of the model, in this section I will also consider a special case, where firms productivity levels are drawn from a Pareto distribution. Finally, section 4 offers some concluding remarks.

⁵His model assumes that product development is completely internalized by firms themselves, hence firms can not benefit from technologies developed by other (domestic and foreign) producers. It is this assumption that prevents economy from having a sustained growth. Modeling of production and R&D technologies are also different. I closely follow Romer's (1990) model, where there are three types of activities: production of final goods, intermediate (differentiated) goods, and product development process. Melitz's model is an extension of Krugman's (1980) model where differentiated products are final goods.

⁶After this paper was completed, Baldwin and Robert-Nicoud (2006) published a new working paper in which they extended their work by considering these possibilities. Their new results qualitatively are similar to mine.

2 Specification of the Model

2.1 Consumer Behavior

I begin with a description of consumers behavior. I assume that consumers have identical preferences and that they maximize utility over an infinite horizon. Intertemporal utility takes the following form

$$U_t = \int_t^\infty e^{-\rho(\tau-t)} \ln C(\tau) d\tau, \quad (2.1)$$

where ρ is the subjective discount rate and $C(\tau)$ represents consumption at time τ . The natural logarithm of the consumption measures instantaneous utility at a moment in time. Households can freely borrow and lend at the instantaneous interest rate $r(\tau)$. Every consumer maximizes his intertemporal utility described by (2.1) subject to an intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]} p_Y(\tau) C(\tau) d\tau \leq A(t), \quad (2.2)$$

where $R(\tau) \equiv \int_0^\tau r(s) ds$ represents the cumulative interest factor up to time τ , p_Y is the price of consumption (or final) good Y (hence $p_Y(\tau)C(\tau)$ represents the flow of spending at time τ), and $A(t)$ denote the present value of the stream of factor incomes plus the value of initial assets at time t . To simplify the notation, I suppress the time arguments and I shall do so in subsequent analysis as long as it causes no confusion. I also hereafter normalize p_Y to one for all t . With this assumption, the intertemporal optimization problem yields that the consumption, C , must grow according to

$$\frac{\dot{C}}{C} = r - \rho. \quad (2.3)$$

2.2 Producer Behavior

There are two types of manufacturing activities: production of consumption goods and production of intermediate goods that have already been invented. I follow Romer (1990) by writing the production function for consumption goods as

$$Y = AL_Y^{1-\alpha} \int_{j \in \mathcal{J}} q(j)^\alpha dj, \quad (2.4)$$

where Y is the final output, L_Y denotes labor input, $q(j)$ represents the amount of intermediate good j used in production, $\mathcal{J}$ denotes the mass of available intermediate goods, and A and α are constants with $0 < \alpha < 1$.

For expositional simplicity, it is more convenient to write the above production function as $Y = AL_Y^{1-\alpha} Q^\alpha$, where Q denotes the aggregate manufacturing index for intermediate goods and is given by

$$Q = \left[\int_{j \in \mathcal{J}} q(j)^\alpha dj \right]^{\frac{1}{\alpha}}.$$

I further assume that the product market for consumption goods is competitive. The profit maximizing strategy yields that

$$L_Y = \frac{(1-\alpha)Y}{w} \quad \text{and} \quad Q = \frac{\alpha Y}{P}, \quad (2.5)$$

where w is wage rate and P denotes the aggregate price index associated with Q , and it is given by

$$P = \left[\int_{j \in \mathcal{J}} p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}, \quad (2.6)$$

where $p(j)$ is the price of brand j intermediate good and $\sigma = 1/(1-\alpha) > 1$ is the elasticity of substitution between any two brands. With these aggregates Q and P , the optimal quantity and expenditure levels for individual brands are given by

$$q(j) = Q \left[\frac{p(j)}{P} \right]^{-\sigma} \quad \text{and} \quad e(j) = E \left[\frac{p(j)}{P} \right]^{1-\sigma}, \quad (2.7)$$

where $E = PQ$ is the aggregate expenditure on intermediate goods.

Intermediate goods are produced by a continuum of monopolists, each choosing to produce a different variety. Endogenous growth models assume that these monopolists have identical production technology. As argued in the introduction, firm-level empirical studies have shown the existence of large and persistence productivity differences even in a narrowly

defined industries. To capture this aspect of reality, following Melitz (2003), I assume that firms have different productivity levels indexed by $\varphi > 0$. More specifically, for a firm with productivity φ to produce q units of intermediate goods, q/φ units of final goods must be forgone.⁷ I also assume that goods are nondurables, i.e. they depreciate fully after use.⁸ Regardless of its productivity level, each firm faces a residual demand curve described in (2.7). Profit maximizing behavior yields the following price rule:

$$p(\varphi) = \frac{1}{\alpha\varphi}. \quad (2.8)$$

Given this pricing rule, firm profit is then

$$\pi(\varphi) = e(\varphi) - q/\varphi = e(\varphi)/\sigma, \quad (2.9)$$

where $e(\varphi)$ is expenditure on the firm's product (i.e. firm's revenue). Using this pricing rule in (2.7) and (2.9):

$$q(\varphi) = Q(P\alpha\varphi)^\sigma, \quad e(\varphi) = E(P\alpha\varphi)^{\sigma-1}, \quad \text{and} \quad \pi(\varphi) = \frac{E}{\sigma}(P\alpha\varphi)^{\sigma-1}. \quad (2.10)$$

These equations further imply that

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^\sigma \quad \text{and} \quad \frac{\pi(\varphi_1)}{\pi(\varphi_2)} = \frac{e(\varphi_1)}{e(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}. \quad (2.11)$$

Hence, a more productive firm will have a lower price, will produce more output, and will earn higher profit than a less productive firm.

2.3 Aggregation

Let $\mu(\varphi)$ denotes the distribution of productivity levels over a subset of $(0, \infty)$ and n be the mass of firms. Note that $\mu(\varphi)d\varphi$ is the fraction of firms who have productivity level of φ

⁷As has been emphasized by Romer (1990), this does not mean that consumption goods are directly converted into intermediate goods. It simply implies that the production function of a firm with productivity level φ is given by $y(\varphi) = \varphi Y$, i.e. inputs needed to produce one unit of consumption are shifted from the production of consumption goods into the production of φ units of intermediate goods.

⁸Assuming no depreciation, as Romer (1990) does, will only complicate the analysis, without affecting any of the results.

and $n\mu(\varphi)d\varphi$ is the total number of firms who have productivity level of φ . Thus, aggregate price P is given by

$$P = \left[\int_0^\infty p(\varphi)^{1-\sigma} n\mu(\varphi)d\varphi \right]^{\frac{1}{1-\sigma}}. \quad (2.12)$$

Using the pricing rule (2.8), this can be written $P = n^{1/(1-\sigma)}p(\tilde{\varphi})$, where

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1} \mu(\varphi)d\varphi \right]^{\frac{1}{\sigma-1}}, \quad (2.13)$$

where $\tilde{\varphi}$ is a weighted average of the firm productivity levels φ and is independent of the number of firms n . As argued by Melitz (2003), $\tilde{\varphi}$ also represents aggregate productivity because it completely summarizes the information in the distribution of productivity levels $\mu(\varphi)$ relevant for all aggregate variables. Furthermore, by using (2.11) it is straightforward to show that other aggregate variables are given by

$$P = n^{\frac{1}{1-\sigma}}p(\tilde{\varphi}), \quad Q = n^{\frac{1}{\alpha}}q(\tilde{\varphi}), \quad E/\sigma = \Pi = n\pi(\tilde{\varphi}), \quad \text{and} \quad Y = nAL_Y^{1-\alpha}q(\tilde{\varphi})^\alpha, \quad (2.14)$$

where Π denotes the aggregate profit of intermediate-good sector.

2.4 Dynamic Structure of Industry and Value of Firm

Following Melitz (2003), it is assumed that each operating firm faces a constant probability δ in every period of a bad shock that would force the firm to exit. The profits recorded in (2.9) are one component of the return to the owners of an operating firm. For the monopolists the important thing is net present discounted value of profits. As is well known in this literature, these net present discounted values can be found by solving the following dynamic programming problem:

$$\pi_t(\varphi) = (r + \delta)\nu_t(\varphi) - \dot{\nu}_t(\varphi), \quad (2.15)$$

where $\nu(t)$ denote the *expected* value of a claim to the stream of profits that accrues to a typical firm operating at time t . Above $r + \delta$ reflects the fact that in each period the firm can be hit by a bad shock and, hence, be out of business. Assuming that bubbles do not

emerge in this dynamic equilibrium setting, this implies that the stock market value at time t of a firm is given by

$$\nu_t(\varphi) = \int_t^\infty e^{-[R(s)-R(t)]-\delta(s-t)} \pi_s(\varphi) ds, \quad (2.16)$$

where $R(t)$ again represents the cumulative discount factor applicable to profits earned at time t .

2.5 Innovator Behavior and Equilibrium Dynamics

I now turn to the technology for product development. In formulating the R&D process, I will follow Romer (1990), Rivera-Batiz and Romer (1991), and, to some extent, use insights provided by Baldwin and Robert-Nicoud (2005). I consider two types of R&D specifications, each captures different features of R&D, and they, therefore, complement each other. In the first specification, production of new designs is similar to that of final products: labor and intermediate goods are productive in research. In particular, the final goods are used (or more appropriately forgone) in development of new products. In the second specification, on the other hand, labor and available technology (or stock of knowledge) are used in the innovation process. In this case, the number of products n is used as a proxy for the level of technology. Following Rivera-Batiz and Romer (1991), I will call these two specifications lab-equipment and knowledge-driven specifications of R&D, respectively.

I assume that when an entrepreneur devotes a finite amount of resources to R&D for an infinitesimal time period, it can incrementally expand the mass of available products. I also assume that the R&D sector is perfectly competitive and they finance the up-front product development costs by issuing equity. This perfectly competitive sector makes and sells blueprints for new varieties.

The variety-innovation process, however, is stochastic under both specifications. Innovators first invest f_e (f_e/n) units of final good (labor) to develop a new variety under the first specification (the second specification). Each variety is associated with a productivity level φ , which is randomly drawn from a common distribution $\phi(\cdot)$, which has positive support

over $(0, \infty)$ and has a continuous cumulative distribution Φ . After learning the associated productivity level, the innovator checks the expected value of the firm that may produce this variety. With the associated productivity level φ , if its expected value is greater than the cost of adapting the product in to the market, denoted by f_d (wf_d/n), then the innovator will spend f_d (f_d/n) additional units of the final good (labor). Otherwise, the innovators will destroy it, which will cost them f_e (wf_e/n).

With this interpretation, there are two conditions. First, among the observed productivity levels, there is now a cut off productivity level φ_* where

$$\nu(\varphi_*) = \begin{cases} f_d & \text{Lab-equipment R\&D} \\ wf_d/n & \text{Knowledge-driven R\&D} \end{cases} \quad (2.17)$$

Only $\varphi \geq \varphi_*$ stays in the market, others will not be introduced. With this cut off productivity level and distribution function $\Phi(\cdot)$, the aggregate productivity index now is given by

$$\tilde{\varphi}(\varphi_*) = \left[\frac{1}{1 - \Phi(\varphi_*)} \int_{\varphi_*}^{\infty} \varphi^{\sigma-1} \phi(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}. \quad (2.18)$$

Second, since the R&D sector is competitive, the expected cost should equal expected profit. Now what is the ex-ante expected cost? Note that f_d/n (wf_d/n) occurs if productivity level is greater than φ_d . Thus, expected cost $\bar{C}$ is given by

$$\bar{C} = \begin{cases} f_e + (1 - \Phi(\varphi_*))f_d & \text{Lab-equipment R\&D} \\ w[f_e + (1 - \Phi(\varphi_*))f_d]/n & \text{Knowledge-driven R\&D} \end{cases} \quad (2.19)$$

What is the expected benefit $\bar{B}$? This is the ex-ante value of an average firm:

$$\bar{B} = (1 - \Phi(\varphi_*)) \left[\frac{1}{1 - \Phi(\varphi_*)} \int_{\varphi_*}^{\infty} \nu(\varphi) \phi(\varphi) d\varphi \right], \quad (2.20)$$

where $1 - \Phi(\varphi_*)$ is the ex-ante probability of successfully entering into the market and the term in bracket is the average value of firm. Thus, in competitive equilibrium

$$\frac{1}{1 - \Phi(\varphi_*)} \int_{\varphi_*}^{\infty} \nu(\varphi) \phi(\varphi) d\varphi = \begin{cases} \bar{f} & \text{Lab-equipment R\&D} \\ w\bar{f}/n & \text{Knowledge-driven R\&D,} \end{cases} \quad (2.21)$$

where $\bar{f} = f_d + \frac{f_e}{1-\Phi(\varphi_*)}$. The left hand side of (2.21) is the average value of a successful entrant, and the right hand side is its average development cost. Thus, to expect to “produce” a new variety innovator has to use $\bar{f}$ ($\bar{f}/n$) units of final goods (labor).

In previous section it is assumed that in each period an operating firm can be hit by a bad shock and be out of the market. To replace the dying varieties, the R&D sector should produce a measure of new varieties equal to δn . Let R_e (L_e) denote the total amount of final goods (labor) used by entrepreneurs in R&D, then the expected number of changes in the number of products is given by

$$\dot{n} = \begin{cases} R_e/\bar{f} - \delta n & \text{Lab-equipment R\&D} \\ nL_e/\bar{f} - \delta n & \text{Knowledge-driven R\&D,} \end{cases} \quad (2.22)$$

Before going further it is important to emphasize that the knowledge-driven specification is closely related to Melitz’s (2003) model. If technology was treated as a private capital (hence, there will be $\bar{f}$ instead of $\bar{f}/n$) and there was no change in the number of firms (i.e. $\dot{n} = 0$), then there would be no growth in the economy. In this case, (2.22) implies that $n = L_e/(\delta\bar{f})$ and the qualitative implications of this model will be similar with that in Melitz (2003).

The dynamic evolution of this economy is now described by (2.3) and (2.22) together with three other conditions described in (2.16), (2.17), and (2.21). Given the stochastic nature of the problem, analysis of this system is quite complex. I, therefore, confine myself to the steady-state equilibrium analysis, where the variables have constant growth rates. The steady-state analysis (see Appendix A.1 for details) yields that the equilibrium cutoff level φ_* is a *constant* and the *same* under both types of R&D specifications. The growth rate of the economy, denoted by g_a , is given by

$$g_a = \begin{cases} \alpha^\sigma \bar{\varphi}_*^{\sigma-1} L/\bar{f} - \rho - \delta & \text{Lab-equipment R\&D} \\ \xi L/\bar{f} - (1-\xi)\rho - \delta & \text{Knowledge-driven R\&D,} \end{cases} \quad (2.23)$$

where $\xi = \alpha/(1+\alpha)$.

3 Open Economy

I now consider the impact of trade in intermediate goods in a world that is composed of two countries of the kind just analyzed. Firms wishing to export face per-unit costs (such as transport and tariffs) that do not depend on firm characteristics such as productivity. Per unit trade costs are modeled in the standard iceberg formulation, whereby $\tau > 1$ units of a good must be shipped in order for one unit to arrive at its destination. Because countries are symmetric, they have the same prices for final goods, which is again normalized to one, and the same mass of firms n .

Therefore, each firm's pricing rule in its domestic market is still given by $p_d(\varphi) = 1/\alpha\varphi$. Firms who export will set higher prices in the foreign markets that reflect the increased marginal cost τ of serving these markets: $p_x(\varphi) = \tau/\alpha\varphi = \tau p_d(\varphi)$. Thus, revenues earned from domestic sales and export sales to any given country are, respectively, $e_d(\varphi) = E(P\rho\varphi)^{\sigma-1}$ and $e_x(\varphi) = \tau^{1-\sigma}e_d(\varphi)$, where E and P again denote the aggregate expenditure and price index of intermediate goods in each country. Since $\pi(\varphi) = e(\varphi)/\sigma$, the combined profit of a firm, $\pi(\varphi)$, is given by:

$$\pi(\varphi) = \begin{cases} \pi_d(\varphi) & \text{if it does not export,} \\ \pi_d(\varphi) + \pi_x(\varphi) = (1 + \tau^{1-\sigma})\pi_d(\varphi) & \text{if it exports.} \end{cases} \quad (3.1)$$

3.1 Innovator Behavior

I now turn to the technology for product development, which is similar with that in the closed economy. Note that now a firm may serve in the foreign market too; in this case, however, the inventor should devote *additional* resources for modifying product to meet the foreign market specifications.⁹ Specifically, under the lab-equipment specification, after developing the new variety, which costs f_e units of final goods, the innovator checks the associated productivity level φ . With this productivity level, if the firm's expected value is greater than f_d , then the innovator will spend f_d units of output to serve this into the

⁹Existence of such sunk market entry costs have been well documented by econometric studies, see Roberts and Tybout (1997) and Bernard and Jensen (2004b).

domestic market. If the productivity level is high enough to also cover foreign market adaptation costs, then the innovator will spend f_x units of output, in addition to f_d , to serve this product into the foreign market. Otherwise, it will destroy it, which will cost the firm f_e .

Under the second specification, however, the foreign market adaptation cost will be wf_x/K_n , where w is wage rate and K_n denotes available technology.¹⁰ In section 2.5, I assumed that two countries are completely isolated and, therefore, indexed the available technology by n , i.e. $K_n = n$. Now they engage in trade and it is plausible to think that the foreign contribution to local technology increases with trade. Grossman and Helpman (1991a), for example, argue that international trade in tangible commodities improves the flow of intangible ideas in the following ways. First, through imports new products will be available in the local markets. The local researchers will gain new insights from inspecting and using these goods. Second, exposure to international trade will increase the number of personal contracts between domestic and foreign producers, which then enhances the exchange of knowledge between countries. Finally, foreign purchasing agents may suggest new ways to develop new products (or to improve manufacturing process).¹¹

Following their insights, in formulating the knowledge-driven R&D specification, I will assume that the amount of technology transferred from the other country is a function of total trade (import plus export) with other country. More specifically, let $\mathcal{VT}$ be the value of total trade (imports plus exports) of home country and $\mathcal{VQ}$ the value of intermediate goods produced at home country (symmetry assumption ensures that these will be identical across countries). The the fraction of technology transferred to home country will be given

¹⁰Now innovators invest f_e/K_n units of labor to develop a product, and if it has high enough productivity, then they use f_d/K_n units of labor to meet domestic market specifications.

¹¹Using these insights, Grossman and Helpman (Chapter 6, (1991a)) present a technology diffusion model for a small open economy, where the total amount of technology transferred from the rest of the world is a function of trade volume and the number of goods available in the rest of the world. Under a knife-edge condition about the parameters of the model, they show that it is possible to have a sustained growth; hence, exposure to trade increases economic growth.

by

$$\Psi = \Psi \left(\frac{\nu T}{\nu Q} \right), \quad (3.2)$$

where Ψ is an increasing function.¹² Hence, $K_n = (1 + \Psi)n$.

Similar to the closed economy case, there are again two conditions. First, note that a firm with the associated productivity φ will serve in the domestic market if $\nu_d(\varphi) \geq f_d$, where ν_d is the value generated by domestic sales. And it will export to the other country if $\nu_x(\varphi) \geq f_x$, where ν_x is the value generated by foreign sales. Thus, now there are two cutoff levels, one for domestic market and other for foreign market:

$$\nu_i(\varphi_i) = \begin{cases} f_i & \text{Lab-equipment R\&D} \\ w f_i / K_n & \text{Knowledge-driven R\&D} \end{cases} \quad (3.3)$$

where $i = d, x$. With these cutoffs, the value of a firm will be given by

$$\nu(\varphi) = \begin{cases} \nu_d(\varphi) & \text{if } \varphi_d \leq \varphi \leq \varphi_x \\ \nu_d(\varphi) + \nu_x(\varphi) & \text{if } \varphi_x \leq \varphi \end{cases} \quad (3.4)$$

Second, the R&D sector is competitive; hence, using the same arguments that I used in closed economy case, it is straightforward to show that

$$\frac{1}{1 - \Phi(\varphi_d)} \int_{\varphi_d}^{\infty} \nu(\varphi) \phi(\varphi) d\varphi = \begin{cases} \tilde{f} & \text{Lab-equipment R\&D} \\ w \tilde{f} / K_n & \text{Knowledge-driven R\&D,} \end{cases} \quad (3.5)$$

where $\tilde{f}$ is given by

$$\tilde{f} = f_d + \frac{1}{1 - \Phi(\varphi_d)} f_e + \frac{1 - \Phi(\varphi_x)}{1 - \Phi(\varphi_d)} f_x. \quad (3.6)$$

As in the closed economy case described by (2.22), the left hand side of (3.5) represents the average value of a successful entrant and the right hand side represents its average development cost. With this interpretation, the expected number of new products is given by

¹²Alternatively, it can be assumed that the fraction of technology transferred from the other country is a function of the fraction of foreign firms that export to home. Analysis based on this specification doesn't change the conclusions, and it is available from the author upon request.

$$\dot{n} = \begin{cases} R_e/\tilde{f} - \delta n & \text{Lab-equipment R\&D} \\ (1 + \Psi)nL_e/\tilde{f} - \delta n & \text{Knowledge-driven R\&D,} \end{cases} \quad (3.7)$$

where R_e (L_e) again denotes total amount of final goods (labor) devoted to R&D.

3.2 Aggregation and Equilibrium Dynamics

Consider the aggregate price index P . Note that when trade is allowed in a given country not only domestic but also foreign producers will sell their goods. Then P is given by

$$P = \left[\frac{1}{1 - \Phi(\varphi_d)} \int_{\varphi_d}^{\infty} p(\varphi)^{1-\sigma} n \phi(\varphi) d\varphi + \frac{1}{1 - \Phi(\varphi_x)} \int_{\varphi_x}^{\infty} (\tau p(\varphi))^{1-\sigma} n_x \phi(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}},$$

where n_x is the mass of foreign firms who have sales at home market, i.e. $n_x = \zeta_x n$ with $\zeta_x = [1 - \Phi(\varphi_x)]/[1 - \Phi(\varphi_d)]$. Thus, $N = (1 + \zeta_x)n$ represents the total mass of varieties available to consumers in each country.

Using the same function defined in (2.17), let $\tilde{\varphi}_d = \tilde{\varphi}(\varphi_d)$ and $\tilde{\varphi}_x = \tilde{\varphi}(\varphi_x)$ denote the average productivity levels of all firms and exporting firms, respectively. Let $\tilde{\varphi}_o$ be the average productivity defined as:

$$\tilde{\varphi}_o = \left(\frac{1}{N} [n\tilde{\varphi}_d^{\sigma-1} + n_x(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}] \right)^{\frac{1}{\sigma-1}},$$

which “reflects the combined combined market share of all firms and the output shrinkage linked to exporting” (Melitz (2003)). Similar to the closed economy case, the aggregate variables are now given by:

$$P = N^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_o), \quad Q = N^{\frac{1}{\alpha}} q(\tilde{\varphi}_o), \quad \Pi = N\pi_d(\tilde{\varphi}_o), \quad \text{and} \quad Y = NAL_Y^{1-\alpha} q(\tilde{\varphi}_o)^\alpha. \quad (3.8)$$

Furthermore, using the definitions of $\mathcal{VT}$ and $\mathcal{VQ}$ it is straightforward to show that

$$\frac{\mathcal{VT}}{\mathcal{VQ}} = \frac{2\zeta_x (\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}}{(1 + \zeta_x)\tilde{\varphi}_o^{\sigma-1}}. \quad (3.9)$$

I will again only consider the steady-state equilibrium and let g_o denote the growth rate of the number of products produced in any country. Similar to the closed economy case (see Appendix A.2 for details), in steady-state the cutoffs φ_d and φ_x will be constants and

identical under both type of R&D specifications. More importantly, however, $\varphi_d > \varphi_*$, i.e. the cutoff level for domestic market entry is now higher than that in the closed economy. This will further imply that the average productivity will be higher in open economy. The growth rate g_o will be given by

$$g_o = \begin{cases} \alpha^\sigma (1 + \zeta_x) \tilde{\varphi}_o^{\sigma-1} L / \tilde{f} - \rho - \delta & \text{Lab-equipment R\&D} \\ \xi(1 + \Psi) L / \tilde{f} - (1 - \xi)\rho - \delta & \text{Knowledge-driven R\&D,} \end{cases} \quad (3.10)$$

where again $\xi = \alpha / (1 + \alpha)$. Subtracting (2.23) from this equation yields that

$$g_o - g_a = \begin{cases} \frac{\alpha^\sigma L}{\tilde{f}} \left[\frac{(1 + \zeta_x)(\tilde{\varphi}_o / \tilde{\varphi}_*)^{\sigma-1}}{\tilde{f} / \bar{f}} - 1 \right] & \text{Lab-equipment R\&D} \\ \frac{\xi L}{\tilde{f}} \left[\frac{(1 + \Psi)}{\tilde{f} / \bar{f}} - 1 \right] & \text{Knowledge-driven R\&D.} \end{cases} \quad (3.11)$$

Before discussing the sign of growth differences, notice the striking similarities between the expressions in brackets. In the first expression there is $(1 + \zeta_x)$ instead of $1 + \Psi$ and $(\tilde{\varphi}_o / \tilde{\varphi}_*)^{\sigma-1}$ basically reflects the effects of intermediate goods used in development of new products. The sign of $g_o - g_a$, then, depends on the signs of the expressions in brackets. Since $\varphi_d > \varphi_*$, it follows that $\tilde{f} / \bar{f} > 1$. Since $(1 + \Psi) > 1$, the ratio $\frac{1 + \Psi}{\tilde{f} / \bar{f}}$ can be either greater than 1, or small than 1, or equal to 1. Thus, the sign of the second expression is ambiguous. On the other hand, the numerator in the lab equipment specification is always greater than the denominator, i.e. $(1 + \zeta_x)(\tilde{\varphi}_o / \tilde{\varphi}_*)^{\sigma-1} > \tilde{f} / \bar{f}$ (see Appendix 2 for proof).

To sum up, exposure to contributes to the average productivity gain. It *increases* economic growth under the lab-equipment R&D specification, while it has an *ambiguous* effect on economic growth under the knowledge-driven R&D specification. Since, in this model, consumption has the same growth rate with output, the above result further implies that in the long-run moving from the autarky to trade will have a positive (ambiguous) effect on consumer welfare under the lab-equipment (knowledge-driven) R&D specification.

It will be interesting to compare these results with those in other studies. Rivera Batiz and Romer (1991) also consider two types of R&D specifications with no firm heterogeneity and trade costs. Under the lab-equipment R&D specification, they find that opening

to trade increases the growth rate of the economy. This paper delivers the same conclusion under a different mechanism: exposure to trade increases average productivity and this, in turn, contributes to a gain in economic growth. Thus, the model presented here shows a benefit from trade that has not been theoretically investigated before. Under the knowledge-driven specification, however, they find that exposure to trade does not contribute to economic growth, unless there is an economic integration through which ideas can flow across countries.¹³ This paper, however, explicitly links the degree of integration with trade and it shows that exposure to trade has an ambiguous effect on growth.

As indicated in the introduction, Baldwin and Robert-Nicoud (2005) also study the long-run implications of trade on economic growth when firms are heterogeneous. However, they consider neither the possibility that imported goods can be used in development of new products nor the possibility that trade in goods can enhance the flow of technology. They only consider the knowledge-driven R&D specification and assume that a country *always* transfers a constant fraction of technology from the other country and there will be no additional transfers, even if countries trade with each other. In this case, the expression in parenthesis will be $\bar{f}/\tilde{f} - 1$, which is always negative; hence, they conclude that exposure to trade will worsen the growth rate of the economy. But, this result depends on their assumption about the nature of the technology diffusion.¹⁴

An equally important question now is how further exposure effects average productivity, economic growth, and consumer welfare. Although I can analyze this problem in a general setting, as in the above analysis, to gain more intuition about the model I confine myself

¹³Specifically, they assume that after exposure to trade, the total available technology will be given by $K_n = (1 + \lambda)n$, where λ measures the degree of integration between countries. Grossman and Helpman (chapter 9, (1991a)) also consider exposure to trade between two symmetric economies under knowledge driven specification. They show that trade increases economic growth by eliminating duplications of ideas.

¹⁴Results of this paper will not change, even if I assume that a country always transfers an additional constant fraction of technology from the other country. To see this, let γ denote such fraction. The amount of technology transferred through trade will then be $\Psi(1 - \gamma)n$, which implies that $K_n = (1 + \gamma)n$, if the economy is closed; and $K_n = [1 + \gamma + (1 - \gamma)\Psi]n$, if it is open. It is straightforward to show that this modification will imply that $g_o - g_a = \frac{\alpha(1+\gamma)L}{\tilde{f}} \left[\frac{1 + [(1-\gamma)/(1+\gamma)]\Psi}{\bar{f}/\tilde{f}} - 1 \right]$. Although the effect of technology transferred through trade is weakened, the sign of this expression is again ambiguous, and hence the above conclusion still holds.

to a specific case, where productivity levels are drawn from a Pareto distribution.

3.3 An Example: Pareto Distribution and Closed Form Solutions

Following Helpman, Melitz, and Yeaple (2004), Melitz and Ottaviano (2005), and many others in this literature, I assume that the productivity levels are drawn from a Pareto distribution:

$$\Phi(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^k, \quad \text{for } \varphi \geq b > 0,$$

where k is the shape parameter and b is scale parameter that bounds the support $[b, +\infty)$ from below. This distribution has finite variance if and only if $k > 2$. I assume that $k + 1 > \sigma$, which ensures that the integrals in aggregate variables converge. With this distributional assumption, I can get closed form solutions for the variables. Using (2.18) with $\phi(\varphi) = kb^k \varphi^{-k-1}$, I get $\tilde{\varphi}_i^{\sigma-1} = k(\varphi_i)^{\sigma-1}/(k+1-\sigma)$, where $i = *, d, x$. Also, note that Pareto distribution implies that $\zeta_x = (\varphi_d/\varphi_x)^k$ and with (A.8) in Appendix A.2, this will imply that $\zeta_x = \tau^{-k} (f_x/f_d)^{-k/(\sigma-1)}$. To sum up:

$$\frac{\tilde{\varphi}_*}{\varphi_*} = \frac{\tilde{\varphi}_d}{\varphi_d} = \frac{\tilde{\varphi}_x}{\varphi_x} = \left(\frac{\beta}{\beta-1}\right)^{1/(\sigma-1)} \quad \text{and} \quad \zeta_x = \left[\frac{\varphi_d}{\varphi_x}\right]^k = \frac{\Omega}{T},$$

where, following Baldwin and Robert-Nicoud (2005), I define $\beta = k/(\sigma-1) > 1$, $T = f_x/f_d$, and $\Omega = \tau^{-k} T^{1-\beta}$. Notice that $\Omega \in [0, 1]$ and when τ and/or T decrease, Ω increases; hence, higher value of Ω corresponds to a more open economy. Using these equations together with the reduced form equilibrium conditions for cutoffs described by (A.4), (A.7), and (A.8) in Appendix A, it is straightforward to show that the equilibrium cutoffs are given by

$$\varphi_* = b \left[\frac{f_d}{(\beta-1)f_e} \right]^{\frac{1}{k}}, \quad \varphi_d = b \left[\frac{(1+\Omega)f_d}{(\beta-1)f_e} \right]^{\frac{1}{k}}, \quad \text{and} \quad \varphi_x = b \left[\frac{(1+\Omega)f_x}{\Omega(\beta-1)f_e} \right]^{\frac{1}{k}},$$

where note that $\varphi_d = (1+\Omega)^{1/k} \varphi_*$.

Using these cutoff levels and (3.9):

$$(1 + \zeta_x) \left(\frac{\tilde{\varphi}_o}{\tilde{\varphi}_*} \right)^{\sigma-1} = (1 + \Omega)^{1+1/\beta}, \quad \tilde{f} = (1 + \Omega)\bar{f}, \quad \text{and} \quad \frac{\mathcal{VT}}{\mathcal{VQ}} = \frac{2\Omega}{1 + \Omega}.$$

With these equations, (3.11) becomes

$$g_o - g_a = \begin{cases} \frac{\alpha^\sigma L}{f} [(1 + \Omega)^{1/\beta} - 1] & \text{Lab-equipment R\&D} \\ \frac{\xi L}{f} \left[\frac{1+\Psi}{1+\Omega} - 1 \right] & \text{Knowledge-driven R\&D.} \end{cases}$$

Under the lab-equipment specification, an increase in Ω increases $g_o - g_a$. Thus, further exposure to trade also increases the growth rate of the economy.

Under the knowledge-driven specification, however, analysis is more complicated and it depends on functional form of Ψ . I consider the following simple specification for Ψ :

$$\Psi(\Omega) = \kappa \left(\frac{\Omega}{1 + \Omega} \right)^\theta, \quad \text{for } 0 \leq \Omega \leq 1,$$

where κ and θ are positive constants. Since $\Psi(\Omega)$ is the fraction of foreign technology transferred to home country, it must be the case that $\Psi \leq 1$. Furthermore, notice that Ψ is an increasing function of Ω . Thus, κ can not be greater than 2^θ ; i.e., $\kappa \in (0, 2^\theta]$. With this specification:

$$\frac{1 + \Psi}{1 + \Omega} - 1 = \frac{\kappa \left(\frac{\Omega}{1 + \Omega} \right)^\theta - \Omega}{1 + \Omega} \equiv G(\Omega).$$

The sign of $G(\Omega)$ depends on $\kappa[\Omega/(1 + \Omega)]^\theta - \Omega$: if it is positive (negative), then $G(\Omega)$ will also be positive (negative). To get more intuition, first assume that $\theta = 1$. In this case, $G(\Omega) = [\Omega/(1 + \Omega)][\kappa/(1 + \Omega) - 1]$ and it is always non-positive, when $\kappa \leq 1$. If $\kappa > 1$, then $G(\Omega)$ will be non-negative for $\Omega \leq \kappa - 1$ and it will be negative for $\Omega > \kappa - 1$ (in this case, notice that $G(\Omega)$ is concave over $[0, \kappa - 1]$). Thus, the effects of trade on growth is crucially depend on the parameter κ . Figure 1 represents the graph of $G(\Omega)$ for different values of κ . This figure together with above discussion imply that exposure to trade has an ambiguous effect on growth. Furthermore, if $\kappa > 1$ and $\Omega < \kappa - 1$, then further exposure to trade also has an ambiguous effect on growth.

What happens if $\theta \neq 1$. Although analysis become more complicated in this case, the basic conclusions remain the same: further exposure (and further exposure) to trade has an ambiguous effect on growth. Figure 2 depicts the $G(\Omega)$ for different combinations of (θ, κ) .

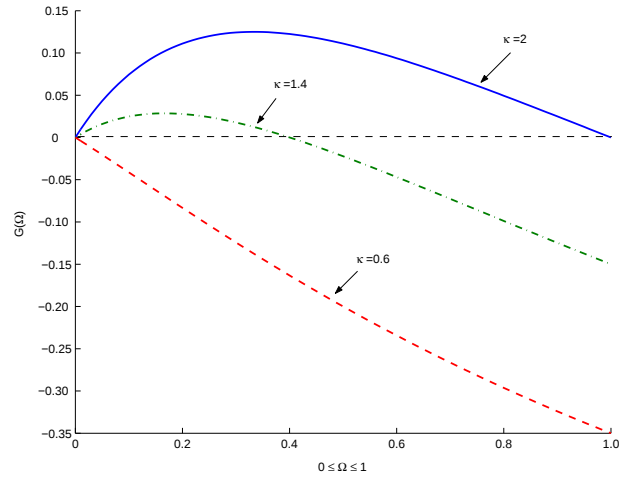


Figure 1: The Graph of $G(\Omega)$ under different κ .

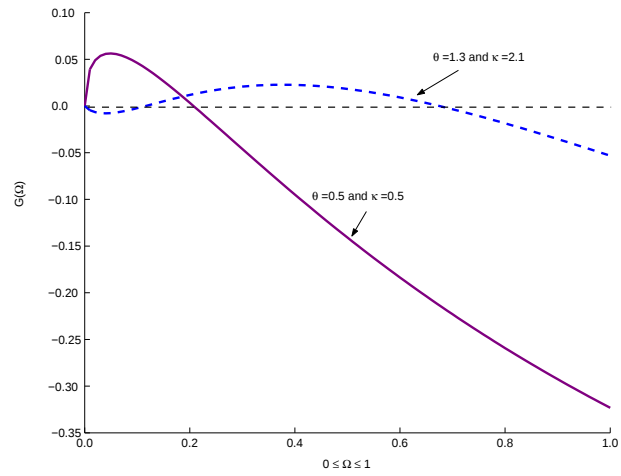


Figure 2: The Graphs of $G(\Omega)$ when $(\theta, \kappa) = (0.5, 0.5)$ and $(\theta, \kappa) = (1.3, 2.1)$.

To sum up: further exposure to trade has a positive effect on economic growth under the lab-equipment specification, while it has ambiguous effects under the knowledge-driven specification. This analysis further implies that in the long-run further exposure to trade will have a positive (ambiguous) effect on consumer welfare under the lab-equipment (knowledge-driven) R&D specification.

4 Concluding Remarks

In this paper, I investigated the long-run effects of trade and technology diffusion through trade when firms are heterogeneous in their productivity levels. I embedded Melitz's (2003) seminal work on firm heterogeneity and trade into product innovation endogenous growth models. I considered two models with two different but complementary specifications of the R&D process. In the first specification, new designs are produced using labor and intermediate goods; whereas in the second specification, labor and available technology are used.

I find that under both specifications, exposure to trade increases average productivity. I also find that although trade is costly, under the first type of R&D specification this negative effect is dominated by the positive contributions of the average productivity gain and technology diffusion through trade. Hence, exposure to trade always has a positive effect on economic growth and consumer welfare. On the other hand, when the second specification is used for the R&D process, the positive effects of trade may not be high enough to overcome its costs. In this case, exposure to trade has an ambiguous effect on economic growth and consumer welfare.

One limitation of the R&D models used here is that they exhibit a scale effect, in the sense that as the labor supply L increases the growth rate of the economy also increases. One way to remove this effect, as pointed out by Barro and Sala-i Martin (2004), is to divide $\bar{f}$ and $\tilde{f}$ by ηL , where η is an appropriately chosen constant. Since ηL itself is constant, the growth rates would be the same as in (2.23) and (3.10), except there will be no L . However,

exposition based on this approach would be unsatisfactory for two reasons. First, this is an ad hoc approach and there is no theoretical justification for this. Second, it would imply that an economy with a few units of labor can produce as many new product as an economy with millions of people. Perhaps the best way to proceed would be to formulate the R&D process in a non-linear fashion such as that in Jones (1995) did. But such analysis is left for a future work.

There are several other directions that the present work can be extended. First, here I only consider one single channel through which technology is diffused. Extending the model to include other channels, especially foreign direct investment, would make it more realistic and would broaden understanding of the process of technology diffusion. Second, here I only assume that there is one sector. A model with two sectors and two factors of production, as in Grossman and Helpman (1991a) would give a better picture of the dynamic comparative advantage of trade. Finally, here I assume that countries are symmetric in all aspects. Allowing differences for country sizes and productivity distributions would be an interesting extension.

A Appendix

In this appendix, I shall prove claims in sections 2 and 3. As said in main text, I confine myself to the steady-state analysis, where all variables have constant growth rates. In steady-state C and Y grow at the same rate and let g_a (g_o) denote this growth rate in closed (open) economy. By (2.3), $r_a = g_a + \rho$ ($r_o = g_o + \rho$).

A.1 Equilibrium Analysis of Section 2

To calculate the growth rate g_a , first note that perfect competition in final goods implies that¹⁵

$$w^{1-\alpha} P^\alpha = 1, \quad (\text{A.1})$$

where recall that the price of final good is normalized to one. This equation further implies that $w = P^{1-\sigma}$. Second, using the optimal quantity function described in (2.7) together with equations in (2.5), the profit equation in (2.9) will be $\pi(\varphi) = \alpha L_Y w [p(\varphi)/P]^{1-\sigma}$. Inserting $w = P^{1-\sigma}$ into this equation implies that $\pi(\varphi) = \alpha L_Y p(\varphi)^{1-\sigma} = \alpha^\sigma L_Y \varphi^{\sigma-1}$. Third, since the total labor supply is fixed, in the steady-state L_Y will be time invariant (in fact, under the lab equipment specification, L_Y always equals L). Moreover, since each firm's productivity level φ does not change over time, this implies that $\pi(\varphi)$ is also time invariant. Hence, (2.16) ensures that

$$\nu(\varphi) = \frac{\pi(\varphi)}{g_a + \rho + \delta}. \quad (\text{A.2})$$

Combining this with (2.17) implies that

$$\pi(\varphi_*) = \begin{cases} (g_a + \rho + \delta) f_d & \text{Lab-equipment R\&D} \\ (g_a + \rho + \delta) w f_d / n & \text{Knowledge-driven R\&D} \end{cases} \quad (\text{A.3})$$

As indicated in the text, only $\varphi \geq \varphi_*$ stays in the market, others will not be introduced, and the aggregate productivity index will given by (2.18). Using (A.2) and (A.3) together

¹⁵Here I also set $A = (1 - \alpha)^{-(1-\alpha)} \alpha^{-\alpha}$.

with (2.11) in equation (2.21), I have

$$H(\varphi_*) = \frac{f_e}{f_d}, \quad \text{with} \quad H(\varphi_*) = [1 - \Phi(\varphi_*)] \left[\left(\frac{\tilde{\varphi}_*}{\varphi_*} \right)^{\sigma-1} - 1 \right], \quad (\text{A.4})$$

where $\tilde{\varphi}_* = \tilde{\varphi}(\varphi_*)$. It is remarkable to notice that under both R&D specifications, I end up with the same equation for the cutoff level φ_* . Using the definition of $\tilde{\varphi}_*$ in (2.18), it is easy to show that $H(\cdot)$ is a monotone-decreasing function.¹⁶ Then, this implies that there exists a unique φ_* that satisfies this equation.

To calculate the growth rate g_a under the lab-equipment case, recall that $\pi(\varphi) = \alpha^\sigma L \varphi^{\sigma-1}$, where I set $L_Y = L$. This together with (A.3) and (A.4) yield that¹⁷

$$g_a = \alpha^\sigma \varphi_*^{\sigma-1} L / f_d - \rho - \delta \quad \Rightarrow \quad g_a = \alpha^\sigma \tilde{\varphi}_*^{\sigma-1} L / \bar{f} - \rho - \delta.$$

To calculate growth rate g_a under the knowledge-driven specification, first note that $\Pi = n\pi(\tilde{\varphi}_*) = E/\sigma = \alpha Y = \alpha w L_Y$, which implies that $\pi(\tilde{\varphi}_*) = \alpha w L_Y / n$. Combining this with the second equation in (A.3) further implies that $\alpha L_Y = (g_a + \rho + \delta) f_d \pi(\tilde{\varphi}_*) / \pi(\varphi_*) = (g_a + \rho + \delta) \bar{f}$, where I used (2.11) together with (A.4). Second, $L_e = (g_a + \delta) \bar{f}$ from (2.22). These two equations together with $L_Y + L_e = L$ yield that

$$g_a = \xi L / \bar{f} - (1 - \xi) \rho - \delta,$$

where $\xi = \alpha / (1 + \alpha)$.

A.2 Equilibrium Analysis of Section 3

I will use similar arguments as in the previous section. I now want to show that both φ_d and φ_x are constants.

To calculate growth rate g_o , note that (A.1) still holds. Thus, the profit is still given by $\pi_d(\varphi) = \alpha^\sigma L_Y \varphi^{\sigma-1}$. Since $\pi_x(\varphi) = \tau^{1-\sigma} \pi_d(\varphi)$, I have $\pi_x(\varphi) = \alpha^\sigma L_Y (\tau^{-1} \varphi)^{\sigma-1}$. As in the closed economy case, productivity level φ is time invariant and in the steady-state L_Y will

¹⁶ $dH(\varphi)/d\varphi = (1 - \sigma)[H(\varphi) + 1 - \Phi(\varphi)]/\varphi < 0$, since $\sigma < 1$.

¹⁷ Note that A.4 implies that $f_d \left(\frac{\tilde{\varphi}_*}{\varphi_*} \right)^{\sigma-1} = \bar{f} \Rightarrow \frac{\varphi_*^{\sigma-1}}{f_d} = \frac{\tilde{\varphi}_*^{\sigma-1}}{\bar{f}}$.

be a constant fraction of L (again, under the lab-equipment specification $L_Y = L$). Thus, profits $\pi_d(\varphi)$ and $\pi_x(\varphi)$ are also time invariant which together with (2.16) imply that

$$\nu_i(\varphi) = \frac{\pi_i(\varphi)}{g_o + \rho + \delta} \quad \text{for } i = d, x. \quad (\text{A.5})$$

Combining these with equations in (3.3) yields that

$$\pi_i(\varphi_i) = \begin{cases} (g_o + \rho + \delta)f_i & \text{Lab-equipment R\&D} \\ (g_o + \rho + \delta)w f_i / K_n & \text{Knowledge-driven R\&D} \end{cases} \quad (\text{A.6})$$

where $i = d, x$.

From $\pi_x(\varphi) = \tau^{1-\sigma}\pi_d(\varphi)$ and (2.11), I obtain the following relationship between φ_d and φ_x :

$$\frac{\pi_x(\varphi_x)}{\pi_d(\varphi_d)} = \tau^{1-\sigma} \left(\frac{\varphi_x}{\varphi_d} \right)^{\sigma-1} = \frac{f_x}{f_d} \iff \varphi_x = \varphi_d \tau \left(\frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}}, \quad (\text{A.7})$$

where $\varphi_x > \varphi_d$ further implies that $\tau(f_x/f_d)^{\frac{1}{\sigma-1}} > 1$ and I assume that this is the case.

As in the closed economy case, if I use (A.5), (A.6), together with (2.11) in (3.5), I get (upon simplifications):

$$H(\varphi_d) + \frac{f_x}{f_d} H(\varphi_x) = \frac{f_e}{f_d}, \quad (\text{A.8})$$

where $H(\cdot)$ is defined as in (A.4). Equations (A.7) and (A.8) constitute a system of two equations with two unknowns φ_d and φ_x . Notice again that (A.7) and (A.8) are obtained under both specifications, i.e. the cut of levels φ_d and φ_x are identical under both R&D specifications. I have already shown that $H(\varphi)$ is a monotone-decreasing function. Moreover, since according to (A.7) φ_x is an increasing function of φ_d , equation (A.8) together with (A.7) immediately yield a unique solution for (φ_d, φ_x) . To show that $\varphi_d > \varphi_*$, notice that the right hand sides of (A.4) and (A.8) are identical. For each φ , the left hand side of (A.8), however, is greater than that of (A.4). Thus, $\varphi_d > \varphi_*$.

To calculate the growth rate g_o under the lab-equipment specification, note that $\pi_d(\varphi) = \alpha L p(\varphi_d)^{1-\sigma}$ [and $\pi_x(\varphi_x) = \tau^{1-\sigma} \alpha L p(\varphi_x)^{1-\sigma}$]. This together with the first equation in (A.6) yields

$$g_o = \frac{\alpha^\sigma \varphi_d^{\sigma-1}}{f_d} L - \rho - \delta.$$

Since $\varphi_d > \varphi_*$, it easily follows that $g_o > g_a$. Expression for g_o in the main text can be obtained as follows. Note that (A.7) implies $g_o = \alpha^\sigma (\tau^{-1} \varphi_x)^{\sigma-1} L / f_x - \rho - \delta$. These will further imply that

$$(g_o + \rho + \delta) f_d \left(\frac{\tilde{\varphi}_d}{\varphi_d} \right)^{\sigma-1} = \alpha^\sigma L (\tilde{\varphi}_d)^{\sigma-1} \quad \text{and} \quad (g_o + \rho + \delta) f_x \left(\frac{\tilde{\varphi}_x}{\varphi_x} \right)^{\sigma-1} = \alpha^\sigma L (\tau^{-1} \tilde{\varphi}_x)^{\sigma-1}.$$

Combining these equations together with (A.8) and using the definition of $\tilde{\varphi}_o$, I get

$$g_o = \frac{\alpha^\sigma (1 + \zeta_x) \tilde{\varphi}_o^{\sigma-1}}{\tilde{f}} L - \rho - \delta.$$

The growth rate g_o under the knowledge-driven specification will be derived as follows. As in the closed economy case, $\Pi = N \pi_d(\tilde{\varphi}_o) = E/\sigma = \alpha Y/\sigma = \alpha w L_Y$, which further implies that $\pi_d(\tilde{\varphi}_o) = \alpha w L_Y / N$. Combining this with the second equation in (A.6) ensures that $\alpha L_Y = [N/K_n](g_o + \rho + \delta) f_d \pi_d(\tilde{\varphi}_o) / \pi_d(\varphi_d) = [(1 + \zeta_x)/(1 + \Psi)](g_o + \rho + \delta) f_d (\tilde{\varphi}_o / \varphi_d)^{\sigma-1}$, where I use $N = (1 + \zeta_x)$, $K_n = (1 + \Psi)n$, and (2.11). Notice that the definition of $\tilde{\varphi}_o$ from the main text implies that $\tilde{\varphi}_o^{\sigma-1} = (\tilde{\varphi}_d^{\sigma-1} + \zeta_x (\tau^{-1} \tilde{\varphi}_x)^{\sigma-1}) / (1 + \zeta_x)$. Using this together with equations (A.7) and (A.8), I obtain that $f_d (\tilde{\varphi}_o / \varphi_d)^{\sigma-1} = \tilde{f} / (1 + \zeta_x)$; hence, $\alpha(1 + \Psi) L_Y = (g_o + \rho + \delta) \tilde{f}$. Notice that $(1 + \Psi) L_e = (g_o + \delta) \tilde{f}$ from the second equation in (3.7). Again these equations together with $L_Y + L_e = L$ yield that

$$g_o = \xi(1 + \Psi) L / \tilde{f} - (1 - \xi) \rho,$$

where $\xi = \alpha / (1 + \alpha)$.

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