Very Noisy Option Prices and Inferences Regarding Option Returns

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Abstract

We show that microstructure biases in the estimation of expected option returns and risk premia are large, in some cases over 50 basis points per day. We propose a new method that corrects for these biases. We then apply our method to real data and produce three main findings. First, the expected returns of straddles and delta-hedged options written on the S&P 500 Index are smaller than previously estimated in the literature. Second, delta-hedged options and straddles written on individual stocks have negative expected returns. Third, the price of individual equity volatility risk is about 45% of the price of market volatility. These findings show that the stylized finding that volatility is not priced in individual stock options is due to microstructure biases.

Keywords: Options; Liquidity; Microstructure bias

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Asset pricing researchers have long noted that measurement errors in prices can result in biased estimates of expected returns and risk premia. Blume and Stambaugh (1983) demonstrate that simple average returns are upward-biased estimates of expected returns due to measurement errors in asset prices, such as errors due to a bid-ask bounce. We call this bias as mean return bias (MR bias). Fama (1984) and Stambaugh (1988) examine a setting in which the measurement errors in the dependent and independent variables of a regression are correlated. As a result, the estimated coefficients are spurious. We refer to this problem as correlated errors-in-variables bias (CEIV bias). Asparouhova, Bessembinder, and Kalcheva (2010, 2013) show that even when the measurement errors in the dependent variables are uncorrelated with the errors in the independent variables, measurement errors in prices will lead to bias in all coefficients in any regression in which the dependent variable is a stock return. We refer to this bias as regression coefficient bias (RC bias).

This paper has three objectives. First, we gauge the size of the MR, CEIV, and RC biases in the estimation of expected option returns and risk premia. Second, we propose methods to adjust for these biases and evaluate them using a realistically calibrated simulation study. Finally, we analyze the extent to which the stylized facts related to the pricing of volatility risk are affected by these MR, CEIV, and RC biases. The economic magnitude of these biases may be so large that they affect some of the stylized facts established in the option literature because, while a number of practices are now commonplace in the estimation of expected returns and risk premia in equity markets, the option literature does not adopt these practices. Indeed, it is now common in the asset pricing literature to use value weighting in addition to equal weighting because MR biases can largely be eliminated with value-weighted returns (see Blume and Stambaugh (1983)). The literature on options, on the other hand, largely analyzes equal-weighted returns, with value-weighted returns typically ignored. Moreover, for many purposes in the equity literature, monthly returns are now preferred over daily returns due to the relative unimportance of microstructure effects. In contrast, the option literature tends to use daily returns, because researchers in this literature focus on delta-hedged returns, and the quality of delta hedging declines quickly with the return horizon. In addition, observation errors are further mitigated in the equity literature by focusing on more liquid stocks, with researchers often excluding those with small share
prices or market capitalizations. Conversely, the prices of many options, particularly those that are out of the money, are small relative to bid-ask spreads and minimum tick sizes, meaning that many prices are likely observed with substantial error.

We start our analysis by showing theoretically that the estimation of expected returns and risk premia in option portfolios poses some challenges that are unique to option-based portfolios. As a result, the methods used in the asset pricing literature to deal with biases in the estimation of expected returns and risk premia are not immediately applicable to options. In particular, option portfolios are often constructed from delta-hedged option positions rather than simple unhedged positions. Errors in stock and option prices not only lead to errors in returns, but also to errors in hedge ratios (deltas), which are highly nonlinear functions of stock and options prices. These errors are correlated, leading to MR, CEIV, and RC biases unique to option-based portfolios, and the biases cannot be solved with simple applications of the methods described in the equity literature. For instance, Asparouhova, Bessembinder, and Kalcheva (2010) show that MR biases in expected returns of stock portfolios can largely be eliminated by weighting observations by lagged gross returns. In the case of option portfolios, this return-weighted averaging helps to decrease the bias in estimated expected returns; however, it does not completely address this bias because, differently from the MR bias in stock portfolios, the MR bias in option portfolios is partially driven by the correlation between errors in returns and errors in hedge ratios. This correlation drives the MR biases in both delta-neutral straddles and delta-hedged calls or puts.

We then use Monte Carlo simulations to analyze the magnitude of these biases in the estimation of expected returns and risk premia of option-based portfolios. Our simulations are based on the Black and Scholes model and on measurement errors that are calibrated to the bid-ask spreads observed in the data. It is important to note that our simulation results are independent of the simulated model. An option pricing model, such as the Black and Scholes model, specifies the risk factors that affect option expected returns, which are estimated with prices that contain measurement errors. The microstructure biases that we examine are due to these measurement errors which are independent of the underlying risk factors driving the true and unobservable expected option returns. Therefore, our simulation
results are not related to underlying simulated option pricing model.\(^1\)

We examine five different types of option portfolios: calls or puts, both unhedged or delta-hedged, and delta-neutral straddles. The results of our simulations indicate that the MR, CEIV, and RC biases can be large in these five types of option portfolios. Specifically, equal-weighted average returns are biased estimates of option and option-based portfolio returns. The MR biases can be as large as 50 basis points (bps) per day. The large and positive MR bias that we find in option returns is consistent with the large bid-ask spreads in option prices. Even though the MR bias is positive in most cases, negative bias is also observed. This occurs because the MR bias in option portfolios is more complex than that described in Blume and Stambaugh (1983) as measurement errors in prices affect not only returns but also the hedge ratios in option portfolios. The simulations also highlight the importance of the CEIV bias. While the CEIV issue has been noted by Fama (1984) and Stambaugh (1988) in regression settings, our results indicate that the CEIV bias can be quite large in option portfolio sorts. In our case, the CEIV issue arises from the common practice of using the same option prices to form portfolios and compute portfolio returns. As the result, any errors in prices will affect both the portfolio assignment and the resulting portfolio returns; consequently, even if returns were unbiased at the individual asset level, they will not be in portfolios. We also examine the RC biases in cross-sectional regressions estimating the volatility risk premia. Specifically, we regress the returns of delta-hedged calls or puts on option betas to the volatility risk ($\beta_\sigma$). We show that the estimated ordinary least squares (OLS) coefficients on these Fama-MacBeth regressions can also have large positive biases.

Having documented that the MR, CEIV, and RC biases are potentially very large in option portfolios and that these biases are not easily solved by simply adopting the solutions offered in the equity literature, we next propose methods to adjust for these biases. We develop a two-step method to adjust for the MR bias. As mentioned before, the MR bias in option portfolios is partially driven by the correlation between errors in returns and errors in hedge ratios. In our first step, we therefore eliminate this correlation by constructing hedge

\(^1\)Another way to see this point is to note that Blume and Stambaugh (1983) show that the MR biases in stock returns are proportional to the variance of the measurement error in the prices. This variance is not related in any with the model that drives the expected stock returns.
ratios that are independent of any measurement error in the price of the option whose return is being computed. To do so, we calculate “synthetic hedge ratios” with synthetic prices based on no-arbitrage relations. The second step is similar to the method used by Asparouhova, Bessembinder, and Kalcheva (2010), who replace equal-weight averaging in portfolio sorts with returns-weighted averaging. We address the CEIV bias in a manner similar to the Black, Jensen, and Scholes (1972) use of pre- and post-ranking betas. Specifically, we use one set of prices to form portfolios and another set of prices to compute portfolio returns. In the regression setting of the RC bias, we first construct the regressors with synthetic measures of delta (the hedge ratio), which are calculated without any price used to compute the return of interest. Then, as in Asparouhova, Bessembinder, and Kalcheva (2010), we replace OLS with returns-weighted Least Square (WLS) in cross-sectional regressions.

We examine our proposed methods in simulations. In most cases, the bias-adjusted method we propose reduces or completely eliminate the MR, CEIV and RC biases. Specifically, the largest MR+CEIV bias we find in simulations is around 70 bps per day. This bias is eliminated with the bias-adjusted method. The bias-adjusted method also addresses the RC bias quite well. For example, a simulated Fama-MacBeth regression indicates that a delta-hedged call option with $\beta_\sigma$ equal to the median value observed in the sample (1.96) has a predicted return equal to 54 bps per day. This is pure bias, as the true expected return is zero. Our bias-adjusted estimate is on average only -1 bps per day.

The final part of our paper is a study of the likely impact of the MR, CEIV, and RC biases on stylized facts in the empirical option literature related to the price of volatility risk in index and individual stock options. We start by showing that microstructure biases dampen the economic significance of the estimated price of volatility risk. Coval and Shumway (2001) show that equal-weighted returns of straddles are negative and economically large. Bakshi and Kapadia (2003a) show that the mean return of delta-hedged options on the S&P 500 are negative and statistically significant. These results indicate that the price of index volatility risk is economically significant. Our results suggest that equal-weighted returns are upward-biased estimates of straddles and delta-hedged returns. In fact, we show that once we adjust for biases, the mean return of straddles decreases from about -26 to -30 bps per day, while the mean delta-hedged call return decreases from -45 to -148 bps per day.
These results indicate that the price of index volatility risk is more economically significant
than previously estimated.

We also challenge another stylized finding from the literature, namely that while delta-
hedged options written on the S&P 500 have negative expected returns, delta-hedged options
written on individual stocks are often found to not have expected returns significantly dif-
ferent from zero (e.g. Bakshi and Kapadia (2003b) and Diessen, Maenhout, and Grigory
(2009)). Specifically, we examine the returns of delta-hedged call and put options as well as
straddles on individual stocks. Once we adjust for various biases, we find that the expected
returns of most groups of straddles and delta-hedged options written on individual stocks
are negative and significant. In addition, Fama-MacBeth regressions adjusted for RC bias
reveal that the price of volatility risk in individual equity options is negative and around
45% of the price of risk in the S&P 500 options.

We contribute to the literature that analyzes microstructure biases in expected returns.
Specifically, our paper is the first to document that these biases are large and important
for option portfolios. Moreover, we show that these biases are complex and require a mul-
tifaceted solution, which we propose. Our approach extends Blume and Stambaugh (1983)
and Asparouhova, Bessembinder, and Kalcheva (2010, 2013), who study the effect of mi-
icrostructure noise in option prices. Dennis and Mayhew (2009) examine the effect of noisy
option prices on the estimation of option pricing models, while Hentschel (2003) studies the
effect of noise on the estimation of implied volatility smile. We differ from these papers
because we focus on the effect of noisy prices on inferences about the expected returns and
risk premia of options.

We also contribute to the literature on expected index option returns. Much of this
work borrows aspects of the empirical design introduced by Coval and Shumway (2001), who
examine the returns on portfolios of options in much the same way that most equity research
is undertaken. A key innovation of Coval and Shumway (2001) is that they focus largely on
option portfolios in which the sensitivity to the underlying security has been hedged away.
We show that portfolios of these delta-neutral positions pose unique challenges for option
pricing researchers and that bias-adjusted estimators result in much more negative estimates
for the expected returns on many types of option portfolios.
Finally, we contribute to the literature that estimates the expected return of individual equity options. While both Coval and Shumway (2001) and ? find that volatility is negatively priced in index equity options, the evidence that that volatility is priced in individual stock options is weak or non-existent. Using a sample of options written on 25 stocks Bakshi and Kapadia (2003b) finds that the return of delta hedged option strategies on individual indexes is a small fraction of that on the S&P 500 options. Diessen, Maenhout, and Grigory (2009) finds no evidence that volatility is priced on individual equity options written on the components of the S&P 100 index. The finding that volatility is not priced in individual equity options is at the odds with the theory because the volatilities of individual stocks are positively correlated with the index volatility therefore individual equity options are, at some extent, substitutes for options written on the index. Our findings, on the other hand, imply that volatility is in fact priced at the individual stock level options which is consistent with this theory.\(^2\)

The remainder of the paper is as follows. Section 1 describes our data. Section 2 describes the different microstructure biases that affect option portfolio returns. In Section 3, we use Monte Carlo simulations to demonstrate that these biases can be large, and Section 4 proposes a general method to deal with these biases. Section 5 shows how adjusting for these biases impacts some of the stylized facts from the empirical literature on options. Section 6 concludes.

1 Data

We focus on options written on the S&P 500 Index or on firms that are members of the S&P 500 Index. Our goal is to examine whether the MR, CEIV, and RC biases are important even in a sample of relatively liquid contracts. We consider all options with maturities between 10 days and six months. Our sample period for most analyses is from January 1996 to August 2015.

Our primary data come from the IvyDB dataset from OptionMetrics, which contains

\(^2\)Diessen, Maenhout, and Grigory (2009) attribute the difference between the pricing of individual equity options and index options to a negative price in correlation risk. Consistent with the idea that correlation risk is priced, we find that the price of volatility in individual equity options is smaller than that in index options.
closing bid-ask quotes on options, closing stock prices, implied volatilities, and option deltas. Implied volatilities and deltas are computed using the binomial tree approach of Cox and Rubinstein (1979), which accounts for dividends and for the potential early exercise in the American options on individual equities. These are equivalent to Black-Scholes values when dividends are zero or early exercise is not optimal. We compute option portfolio returns based on the midpoints of the option bid-ask quotes and the closing values of the underlying prices. Excess returns subtract the return on a riskless asset, which is based on the shortest maturity yield in the IvyDB zero coupon term structure file, and which accounts for the number of calendar days in the return holding period.\(^3\)

As is standard in the literature, we filter observations that violate arbitrage conditions or that appear to be “stub” quotes or data errors. A detailed description of these filters is in the appendix. A final filter is imposed to exclude a number of very illiquid contracts that do not necessarily represent data problems. Specifically, these are contracts for which the bid price is zero or the ask price is more than two times the bid price. This filter is applied only at the beginning of the holding period and not at the end, so it represents a filter that could be applied in real time by an options trader. We exclude these contracts for the same reason that we focus on S&P 500 firms, as we do not want our results to be attributed solely to the most extreme cases of illiquidity.

Some of our results require the bid-ask spread for each stock. We therefore compute each stock’s effective spread using trade and quote data from TAQ. This spread is twice the average absolute percentage difference between the trade price and the pre-trade bid-ask midpoint, where the average is volume-weighted and taken over all trades for a single stock on a single day. These data are only available through the end of September 2014. We include data from all exchanges.\(^4\)

We present summary statistics in Table 1. Given that we have panel data for both options and stocks, we first calculate the cross-sectional moments for each day in our sample and then the mean of these moments across time. For example, to obtain the mean of the option returns, we first calculate average returns over all options existing on each day, and then take

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3 As discussed by French (1983), interest accrues on a calendar-time basis.
4 TAQ trade data are also filtered to eliminate apparent data errors. A description of this filter is in the appendix.
the time-series average. The summary statistics in Table 1 are mean, standard deviation, median, first and third quantile (Q1 and Q3), and skewness of option returns, stock returns, spreads, and observed moneyness as well as $\beta$s. We follow the option literature in defining the moneyness of an option as $\ln(K/S_t)$ where $K$ is the option strike price, and $S_t$ is the observed price of the stock. The $\beta$ of the option with respect to the underlying ($\beta_S$) is $\Delta_t \times S_t/P_t$ where $\Delta_t$ and $P_t$ are the $\Delta$ of the option and its price respectively, both given by IvyDB. The $\beta$ of the option with respect to the volatility of the underlying ($\beta_{\sigma}$) is $\nu_t/P_t$ where $\nu_t$ is the vega of the option, which is also given by IvyDB. Option spreads are in percentage terms relative to quote midpoints.

Table 1 shows that the average return for the call (put) options on individual stocks is 29 bps (-45 bps). For call options written on individual stocks, the average spread is about 15% and for put options the average relative bid-ask spread is approximately 18%. Median moneyness is negative for call options, indicating more in-the-money call options in the sample, and it is zero for put options. The median $\beta_S$ for call (put) options written on individual stocks is 7.60 (-7.57), while the median $\beta_{\sigma}$ is 1.96 (3.23). The summary statistics for options written on the index are somewhat different from those written on individual stocks, but most notably the mean and the median bid-ask spreads for options written on the index are substantially smaller than those of options written on individual stocks. The average stock return is 5 bps. Finally, the effective spread for stocks is very small, less than 0.2% of transaction prices.

2 Option portfolios and sources of bias

In this section, we formalize the MR, CEIV, and RC biases in the context of options. We adopt a framework that is as general as possible so that we may consider unhedged calls and puts, delta-hedged calls and puts, and delta-neutral straddles.

The option portfolios that we analyze consist of positions in multiple options, delta-hedged options, or straddles. These components of the overall portfolio may also consist of multiple positions. For instance, a delta-hedged call consists of a long position in a call and a short position in the underlying stock. A delta-neutral straddle consists of long positions in the call and the put. To avoid confusion, we refer to these components as strategies and
to the broader combination of these strategies as portfolios.

A strategy is composed of $M$ securities, which may include calls, puts, or stocks. The observed return of this strategy between $t$ and $t+1$ can be represented as:

$$R_{t+1} = \frac{\sum_{k=1}^{M} f_{X_k}(X_t)(X_{k,t+1} - X_{k,t})}{g(X_t)},$$

(1)

where $X$ is the $M \times 1$ vector of observed security prices.

To better understand this representation, consider the simplest strategy we analyze, the unhedged call option. $M = 1$ in this case, and the price vector $X$ consists solely of the price of that option, $C_t$. In this case, $f_C(X_t) = 1$ and $g(X_t) = C_t$. Substituting these expressions, the return is clearly $C_{t+1}/C_t - 1$, the simple rate of return.

For a delta-hedged call, $M = 2$. The price vector would consist of the call price and the stock price ($X_{1,t} = C_t, X_{2,t} = S_t$). The $f$ and $g$ functions would be expressed as

$$f_C(X_t) = 1$$

(2)

$$f_S(X_t) = -\Delta^C(S_t, C_t)$$

(3)

$$g(X_t) = C_t,$$

(4)

where $\Delta^C(S_t, C_t)$ denotes the call delta, or the first derivative of the call price with respect to the underlying stock price. Substituting these expressions, we obtain

$$R_{t+1} = \frac{C_{t+1}}{C_t} - 1 - \beta^C(S_t, C_t) \left( \frac{S_{t+1}}{S_t} - 1 \right),$$

where $\beta^C(S_t, C_t) \equiv \Delta^C(S_t, C_t)S_t/C_t$ can be interpreted as a normalized delta, representing the sensitivity of option returns, rather than prices, to the underlying stock return.

2.1 The MR bias and its components

We begin by analyzing two sources of the MR bias in option portfolios. The first source was first identified by Blume and Stambaugh (1983). It is the bias that would result were hedge ratios, such as the weight of the stock in a delta-hedged call, observed without error. The second is the result of these hedge ratios being imperfectly observed, subject to the same measurement errors that affect option and stock prices.
Following Blume and Stambaugh (1983), we will assume that observed prices contain measurement errors, i.e.,

\[ X_{k,t} = \tilde{X}_{k,t}(1 + \epsilon_{k,t}) , \]

where \( \tilde{X}_{k,t} \) represents the true price of security \( k \), and \( \epsilon_{k,t} \) is an error term. Although we do not observe it, our main interest is on true returns, which are expressed as

\[ \tilde{R}_{t+1} = \sum_{k=1}^{M} f_{X_k}(\tilde{X}_t)(\tilde{X}_{k,t+1} - \tilde{X}_{k,t})/g(\tilde{X}_t) . \]  

(5)

To understand the nature of the effects of microstructure biases on options portfolios, we begin with the simple case in which the strategy is an unhedged call option \((M = 1, X_{1,t} = C_t, f_C(X_t) = 1, g(X_t) = C_t)\). In this case, Blume and Stambaugh (1983) show that the average observed return is a biased estimate of the true expected return. Specifically,

\[ E[R_{t+1}] - E[\tilde{R}_{t+1}] \approx E[\epsilon^2_{C,t}] . \]  

(6)

That is, the observed expected return is biased upward by an amount that is approximately equal to the variance of the relative measurement error in prices.

Naturally, the same type of bias is also present in unhedged puts or stocks. However, the variance of the measurement error in option prices is likely to be much larger than that of stock prices because, as Table 1 shows, the relative bid-ask spreads of options are much larger than the those of stocks.

When we consider more complex option strategies, the sources of microstructure biases are not limited to those described in Blume and Stambaugh (1983). For instance, the MR bias of hedged call options \((E[R_{t+1}] - E[\tilde{R}_{t+1}]\)) is approximately

\[ E[\epsilon^2_{C,t}] - \beta^C(\tilde{S}_t, \tilde{C}_t)E[\epsilon^2_{S,t}] + \frac{\partial \beta^C}{\partial S}(\tilde{S}_t, \tilde{C}_t)E[\epsilon^2_{S,t}] . \]  

(7)

We refer to the first two terms in equation (7) as the “Direct” MR bias (DMR). These terms represent the bias that would obtain if the hedge ratio \( \beta^C(\tilde{S}_t, \tilde{C}_t) \) were observed without error. We refer to the last term in equation (7) as the “generalized” MR bias (GMR), which is not present in simple unhedged positions. To see this, note that the first two terms in equation 7 are similar to those in Blume and Stambaugh (1983), and they result from the

\(^5\text{All proofs are in the appendix.}\)
fact that call and stock returns are upward biased due to errors in the respective observed prices. The last term on the right-hand side of this equation stems from the fact that $\beta^C$ (and $\Delta^C$) is also affected by errors in underlying stock prices. The existence of this bias has not been previously identified in the literature. Naturally, the same type of bias is also present in delta-hedged puts.

Our framework also allows us to examine straddles, which are commonly implemented to have a zero Black-Scholes delta. The return of a delta-neutral straddle, using the notation of equation (5), can be represented by setting $M = 2$, $X_1 = C$, $X_2 = P$, and $g(X_t) = 1$. The $f$ functions are given by

$$
 f_C(X_t) = \frac{-\Delta^P(P_t, S_t)}{\Delta^C(C_t, S_t) C_t} \equiv v^C_t \\
 f_P(X_t) = \frac{\Delta^C(C_t, S_t)}{\Delta^C(C_t, S_t) P_t - \Delta^P(P_t, S_t) C_t} \equiv v^P_t
$$

$v^C_t$ and $v^P_t$ represent the fraction of the strategy’s value in the call and put, respectively, and result in the cancellation of the positive delta of the call and the negative delta of the put.

In this case, MR bias $(E[R_{t+1}] - E[\tilde{R}_{t+1}])$ is approximately

$$
 v^C_t E[\epsilon^2_{C,t}] + v^P_t E[\epsilon^2_{P,t}] + \frac{\partial v^C_t}{\partial C} E[\epsilon^2_{C,t}] + \frac{\partial v^P_t}{\partial P} E[\epsilon^2_{P,t}].
$$

As with the bias for delta-hedged returns, this expression contains terms that are analogous to those from Blume and Stambaugh (1983). Specifically, the first two terms follow from that paper and would comprise the entire bias were $v^C_t$ and $v^P_t$ not measured with error. The sum of these terms are the DMR bias in the case of straddles. However, if option prices are measured with error, then those errors will also impact $v^C_t$ and $v^P_t$, leading to the latter two terms in equation (10). The sum of these last two terms is the GMR in straddles.

The new source of bias represented in the last term in equation (7) or the last two terms in equation (10) is easily seen to be the effect of error-induced correlation between prices and hedge ratios. For delta-hedged calls, for example, a proportional error $\epsilon_{S,t}$ in the time-$t$ stock price will change the observed hedge ratio $-\beta^C(S_t, C_t)$ by approximately $-\frac{\partial \beta^C}{\partial S}(\tilde{S}_t, \tilde{C}_t) \epsilon_{S,t}$. The same error will also affect next-period observed returns, changing them by approximately $-\epsilon_{S,t}$. The last term in equation (7) is simply the expectation of the product of these two effects.
The two components of the MR bias pertain to returns on individual strategy returns but have a clear effect on the returns on equally weighted portfolios, as emphasized by Blume and Stambaugh (1983). That paper shows that some biases can be reduced by weighting observations proportionally by their lagged gross returns, lagged prices, or, in the case of stocks, their lagged market capitalization. Intuitively, this is because weighting by returns or prices induces a negative covariance between the portfolio weights and the next-period observed returns of the strategies. This covariance approximately offsets the positive bias in equation (6), resulting in a weighted portfolio return that is approximately unbiased.

Unfortunately, this type of bias adjustment is incomplete in the setting we analyze because of the GMR bias. Equation (7) shows a component of the bias in observed returns that depends on $\beta^C$, which is a highly non-linear function of the underlying stock price and of the call price. Similarly, for straddles, the latter two terms of equation (10) are highly nonlinear in call and put prices and are unlikely to be eliminated by simple weighting schemes such as those based on past returns. Thus, it is not immediately clear whether simple weighting schemes can effectively deal with biases in observed option returns. We address this issue in our simulations in Section 4.

2.2 The CEIV bias

The CEIV bias arises in two versions. The first is in settings in which the researcher is interested in computing average returns on sorted portfolios, each consisting of many strategies. The other is when the researcher uses cross-sectional regressions to examine the relation between strategy returns and some variable of interest. An example in which this bias is present is when portfolios are formed on the basis of Black-Scholes deltas or when individual strategy returns are regressed on deltas.

A problem arises in both cases because the deltas are calculated based on option and stock prices that contain measurement errors. Errors in stock prices have a direct impact on delta, as they are an input into the delta formula. Option price errors also influences delta because volatility is a required input to the delta formula, and because options researchers typically compute the delta of a given option based on that option’s implied volatility, e.g.,

$$\sigma_{i,t}^{\text{implied}} = \sigma_{i,t}^{\text{implied}}(C_t, S_t)$$

Note the addition of the index $i$, which we add because we are now
considering portfolios of multiple strategies. Thus, observed deltas depend on stock prices and, through implied volatility, on option prices as well. Deltas will therefore be infected by measurement errors in both, i.e.,
\[ \Delta C_i,t = \Delta^C(\sigma_{i,t}^{implied} S_{i,t}) = \Delta^C(\tilde{C}_{i,t}(1 + \epsilon_{C_{i,t}}), \tilde{S}_{i,t}(1 + \epsilon_{S_{i,t}})). \]

In portfolio sorts, when we combine strategies into delta-sorted portfolios, errors in deltas will lead to the misclassification of some strategies. If our intent was to form a portfolio out of all strategies with deltas between \( \Delta_{\text{min}} \) and \( \Delta_{\text{max}} \), for example, then misclassification can be seen as the difference between indicators based on the observed (\( \Delta \)) and the true (\( \tilde{\Delta} \)) deltas:
\[ 1(\Delta_{\text{min}} \leq \Delta_{i,t} < \Delta_{\text{max}}) \neq 1(\Delta_{\text{min}} \leq \tilde{\Delta}_{i,t} < \Delta_{\text{max}}). \]

An equal-weighted portfolio of all strategies with deltas in that range has a return that can be written as a function of individual strategy returns and these indicator functions:
\[
R_{P,t+1} = \frac{\sum_{i=1}^{N} R_{i,t} 1(\Delta_{\text{min}} \leq \Delta_{i,t} < \Delta_{\text{max}})}{\sum_{i=1}^{N} 1(\Delta_{\text{min}} \leq \Delta_{i,t} < \Delta_{\text{max}})}
\]

In this equation, \( R_{i,t} \) denotes the return on the \( i^{th} \) strategy contained in the portfolio. \( N \) is the number of all strategies in the dataset in period \( t \).

When computing portfolio returns, the CEIV bias is the result of errors in observed prices causing errors in the indicator functions, inducing a spurious covariance between \( R_{i,t} \) and \( 1(\Delta_{\text{min}} \leq \Delta_{i,t} < \Delta_{\text{max}}) \). This covariance has a one-to-one impact on the bias in average portfolio returns.

This is not the only known bias caused by portfolio misclassification. Indeed, at least since Black, Jensen, and Scholes, researchers have used one set of data to estimate pre-ranking betas, used solely for classification, and another set of data to estimate the post-ranking betas used in asset pricing tests. In the Black, Jensen, and Scholes (1972) setting, however, the observation errors in the sorting variable (beta) are not correlated with errors in returns (Shanken (1992)). In our setting, errors are present in the sorting variable, but the same errors are also embedded in the returns themselves, substantially increasing the effect that those errors have on portfolio returns.

An analogous bias exists in cross-sectional regressions, e.g.,
\[ R_{i,t+1} = a + b \Delta_{i,t} + \epsilon_{i,t}, \]
when the observed delta (\( \Delta_{i,t} \)) is also affected by measurement errors in stock and option prices. This bias is analogous to that in Fama (1984) and Stambaugh (1988). Note that this regression bias also cannot be addressed using standard methods for dealing with errors in
variables (e.g., WLS) because the errors that affect our explanatory variable are correlated with the error in returns.

2.3 The RC bias

Asparouhova, Bessembinder, and Kalcheva (2010) point out that microstructure biases affect not only the estimation of expected portfolio returns but also the cross-sectional regression analysis of returns. Asparouhova, Bessembinder, and Kalcheva (2010, 2013) focus on regressions using stock returns, but the same type of bias is present in regressions that use option returns. To see this, assume that the true expected return of an option strategy satisfies the relation

\[ E_t[\tilde{R}_{it+1}] = b'\tilde{\theta}_i \]

where \( E_t[\tilde{R}_{it+1}] \) is the true expected return for strategy \( i \) at time \( t + 1 \), \( \tilde{\theta}_i \) is a vector that consists of ones and \( L \) true characteristics at time \( t \) for option portfolio \( i \), and \( b \) is the vector that consists of a constant term and slope coefficients on the option-portfolio characteristics. We normally estimate the vector \( b \) with the regression

\[ R_{it+1} = b'\theta_t + \eta^i_{t+1}, \quad (12) \]

where \( R_{it+1} \) and \( \theta_t \) are strategy-level returns and characteristics, both calculated with option and stock prices observed with measurement errors. \( \eta^i_{t+1} \) is the regression residual.

The OLS estimates of \( b \) in regression (12) are biased. To see this effect, define \( \hat{\theta}_t = [\hat{\theta}_t, \cdots, \hat{\theta}_N] \), which is a \((L+1)\times N\) matrix. Similarly, define \( \hat{R}_{t+1} = [\hat{R}_{t+1}, \cdots, \hat{R}_N]' \), which is \( N \times 1 \) matrix. The estimated coefficient of the cross-sectional regression estimated with variables without measurement error is \((\hat{\theta}_t\hat{\theta}_t')^{-1}(\hat{\theta}_t\hat{R}_t)\), and it is consistent. In comparison, the estimator based on variables with measurement errors, \((\theta_t\theta_t')^{-1}(\theta_tR_t)\), is biased even when the independent variables are observed without measurement errors, that is when \( \theta_t = \tilde{\theta}_t' \). From equation (6) or (7), \( R_t = \tilde{R}_t + Z_t \), where \( Z_t \) is the bias in the observed return due to measurement errors. Therefore, the OLS estimate of \( b \) based on observed returns when the independent variables are free of measurement errors is \((\hat{\theta}_t\hat{\theta}_t')^{-1}(\hat{\theta}_t\hat{R}_t) = (\theta_t\theta_t')^{-1}(\tilde{\theta}_t\tilde{R}_t) + (\theta_t\theta_t')^{-1}(\theta_tZ_t)\). Asparouhova, Bessembinder, and Kalcheva (2010) show that \((\theta_t\theta_t')^{-1}(\tilde{\theta}_tZ_t) \neq 0\), where \( 0 \) is an \((L + 1) \times 1\) vector of zeros. Hence, the estimated coefficients are biased.

Asparouhova, Bessembinder, and Kalcheva (2010, 2013) analyze this bias in the case
that the strategy returns $R_{i,t+1}$ are simple rates of return on stocks. In their setting, the bias can be largely eliminated by using weighted least squares, with gross returns weights, to run each cross-sectional regression. Specifically, the WLS estimate of $b$ is equal to

$$(\theta_t W_t \theta'_t)^{-1}(\theta_t W_t \tilde{R}_t) + (\theta_t W_t \theta'_t)^{-1}(\theta_t W_t Z_t)$$

and

$$(\theta_t W_t \theta'_t)^{-1}(\theta_t W_t Z_t)$$

converges to 0 with gross returns weights ($W_t$).

While WLS estimate with gross returns weights addresses the RC bias in the case of stocks, it is not clear if it addresses this bias in the case of option strategies. As we show in Section 2.1, measurement errors in prices affect the returns of option strategies both directly as in Blume and Stambaugh (1983) and indirectly through the effect on the hedge ratios. In fact, as we do for the MR bias, we divide the RC bias in two components: One component is the direct RC bias (DRC) and is due to the effect of measurement errors on the returns of the securities in the strategy, the other component is the generalized RC bias (GRC) and results from the fact that the hedge ratios in option strategies are calculated with prices that have measurement errors. While it is likely that a WLS regression with a simple weighting mechanism addresses the DRC bias, it is unlikely that it addresses the GRC bias since hedge ratios are highly non-linear functions of prices. We examine the importance of each of these two components of the RC bias in Section 3.

3 Examining the bias with simulations

We examine the magnitude of the MR, CEIV, and RC biases in option portfolios using Monte Carlo simulations. We simulate daily stock returns by assuming that they follow a market model, i.e.,

$$(R^S_t - R^f_t) = \beta_S(R^M_t - R^f_t) + \epsilon^S_t,$$

where $R^S_t$, $R^M_t$, and $R^f_t$ are the stock return, market return, and risk-free rate at time $t$, respectively; $\beta_S$ is the beta for the stock; and $\epsilon^S_t$ is the error in the market model at time $t$. For simplicity, we assume that the risk-free rate is 0. Roughly matching our actual sample, we simulate daily market returns from a normal distribution with a mean of 0.03%, and standard deviation of 1%. We assume a sample of 500 stocks, whose betas are uniformly distributed between 0.5 and 1.5. Each stock’s error term $\epsilon^S_t$ follows a normal distribution, with a mean of zero and a standard deviation that is drawn from a uniform distribution with a lower bound of 1% and an upper bound of 4%. Therefore, the simulated stock returns are constructed by simulated market returns, simulated betas,
and simulated model errors from the market model above. Each stock volatility is obtained from the market model, using the simulated beta, market volatility, and volatility of the error term from the model.

We simulate option prices based on the Black-Scholes model. Required inputs are therefore the stock price, stock volatility, time to maturity, and strike price. Without loss of generality, we assume that all stock prices are $100 at the beginning of the simulation period. At each time $t$, we assume that there are options with three different maturities (22, 44, and 66 days) for each stock. For each maturity, there are options with nine different strike prices $K$, which are set such that the log moneyness of those options, defined as $\ln(K/S)$, are randomly spaced between $-2.5\sigma\sqrt{T}$ and $2.5\sigma\sqrt{T}$.\(^\text{6}\) Therefore, for each underlying stock, there are $9 \times 3 = 27$ call options. Because our simulated sample contains 500 stocks, there are $27 \times 500 = 13,500$ call option prices at each time $t$ in each trial. Similarly, there are 13,500 put option prices at each time.

To gauge the size of the MR, CEIV, and RC biases on option returns, we assume that options and underlying stock prices are observed with errors. Specifically, we assume that, at each time $t$ in the simulated sample, we observe $S = \tilde{S}(1 + \epsilon_S)$, $C = \tilde{C}(1 + \epsilon_C)$, $P = \tilde{P}(1 + \epsilon_P)$ where $\tilde{S}$, $\tilde{C}$, and $\tilde{P}$ are true stock, call and put prices, and $S$, $C$, and $P$ are observed prices. All measurement errors are drawn from symmetric triangular distributions, which are bounded distributions with probability density functions that are piecewise linear, increasing below the median and decreasing above it, reaching zero at either bound. This choice of density reflects a view that price errors are likely bounded by the size of the bid-ask spread. By choosing lower and upper bounds equal to $-1/2$ or $+1/2$ times the relative bid-ask spread, we ensure that the difference between observed prices and true prices is never larger than the spread, with differences closer to zero more likely than those further away.

The simulation of price errors therefore requires a model of relative bid-ask spreads. For stocks, we choose spreads to match the mean and the standard deviation of the cross-section distribution of the effective bid-ask spreads across S&P 500 stocks. Spreads on options are chosen to match a number of patterns we observe in the data, namely that option spreads

\(^{6}\)Random strike prices allow for a more realistic assessment of the bias associated with misclassification of options into delta bins.
are higher when the underlying stock has a higher spread, when the option is further out of
the money, and when the option has a shorter time until expiration. We provide full details
on the model used for this purpose in the Appendix.

We repeat the above simulations 1,000 times. In each trial, we examine the returns of five
types of option portfolios: call options, put options, delta-hedged calls, delta-hedged puts,
and straddles. We examine the results of portfolio sorting and cross-sectional regressions.

3.1 The size of the MR and CEIV biases

Table 2 displays the results of the portfolio sorts. All values are in terms of basis points per
day, and all portfolios are equally weighted. The long call and long put strategies are sorted
by $\beta_S$. The delta-hedged strategies are sorted by $\beta_\sigma$. We sort delta-hedged calls and puts by
their $\beta_\sigma$ because by construction delta hedged portfolios have zero $\beta_S$. Straddles are sorted
by moneyness, defined as the ratio of the strike price to the stock price.

In each panel, we report the true mean return and the biases under different assumptions
about which variables are impacted by measurement error. Specifically, to assess the relative
importance of each component of the MR bias and of the CEIV bias, we begin with a set
of simulations in which only the direct component of the MR bias is present (DMR); hedge
ratios and sort variables are assumed to be known. (See Section 2.1 for a description of the
components of the MR bias.) We then allow for measurement errors in hedge ratios to see
the incremental effect of the second component of the MRMR bias (GMR). The final set of
simulations adds errors to the sort variables, allowing us to gauge the size of the CEIV bias.

Panels A and B report results for unhedged calls or puts sorted on the basis of $\beta_S$. The
first row of Panels A and B report the mean returns without any measurement error. The
second row of each panel displays the total bias that will be present in empirical work. These
results therefore are the sum of the MR bias and the CEIV bias. There are no use of hedge
ratios in unhedged calls and puts strategies. Hence the MR bias is only composed by the
direct effect of measurement errors on returns.

The first notable result is that the MR and the CEIV biases can be enormous. For
instance, the observed equal-weighted average return of deep-out-the-money call options
(high $\beta_S$ call options) is about 31 bps larger than the actual average daily return. This is
more than 50% of the true mean return of about 54 bps. These large biases are especially important for out-of-the-money options.

The third row of Panels A and B reports results based on the known sorting variable (no measurement errors in $\beta_S$). The third row therefore allows us to gauge the size of the MR bias without the influence of the CEIV bias. The MR biases are generally quite large and positive. This is consistent with the result in Blume and Stambaugh (1983) that the MR bias is positive and proportional to the variance in the measurement errors in prices. Moreover, in a pattern that will repeat throughout the rest of the paper, the MR bias is largest for out-of-the-money options (high $\beta_S$ for calls and low $\beta_S$ for puts) and smallest for in-the-money options (low $\beta_S$ for calls and high $\beta_S$ for puts) due to the former having much larger bid-ask spreads as a proportion of their value.

The difference between the second and third rows of Panels A and B allows us to gauge the importance of the CEIV bias. For options that are close to at-the-money (ATM) calls and puts (portfolio 3), the sum of the MR and CEIV biases is close to zero. Because the MR bias is positive, the CEIV bias is negative for these options. To understand why the CEIV bias is negative in this case, take the example of an OTM call option (portfolio 4 in Table 2) that experiences a positive price error. The positive price error increases the option’s implied volatility, which is used to compute its $\beta_S$. The higher the implied volatility, the lower the $\beta_S$ becomes. Hence, this option ends up being misclassified in portfolio 3. Because a positive error in prices tends to reverse, the next period return of this option tends to be negative, resulting in a negative CEIV bias for the call options in portfolio 3.

The results for delta-hedged options are presented in Panels C and D of Table 2. The total bias in delta-hedged mean returns can be many orders of magnitude larger than the true delta-hedge mean return. For instance, the estimated delta-hedged mean return of out-of-the-money calls (high $\beta_S$) is about 70 bps per day ($68 + 2$ bps). In comparison, when all the data are measured without any error this mean return is 2 bps per day. The difference between the second and third rows of Panels C and D is the CEIV bias. The CEIV bias has a moderately larger impact than it did in Panels A and B, but as a whole this bias is roughly the same for unhedged and delta-hedged options. The MR bias in the case of delta-hedge options has two non-zero components because these hedged-option returns are
calculated with deltas that also contain measurement errors. To gauge the impact of each component of the MR bias, the fourth row of Panels C and D displays the results in which we assume that we measure the hedge ratios without any error. This row therefore displays the DMR bias. By comparing the third and fourth rows within each of these panels, we can see that the GMR component (bias due to errors in hedge ratios) is relatively small, at least compared to the DMR bias. The fact that the GMR is relatively small is not surprising because this component is proportional to the variance of the error in stock prices, which Table 1 shows is relatively small.

Straddle portfolios are presented in Panel E. Sorting bias is minuscule when portfolios are formed on the basis of moneyness (strike divided by spot), because moneyness is unaffected by errors in option prices. Hence, results for straddles in Panel E only show the effects of the direct and GMR biases.

Relative to the results in Panels A to D, the bias due to errors in returns shown in Panel E appears to be very small. However, it is important to keep in mind that straddle portfolios are far less risky than most delta-hedged options, with return standard deviations that are similar in magnitude to those of equity portfolios. Thus, a bias of 4 bps per day, or around 10% per year, is very large relative to the annualized standard deviation of 15-25%.

Table 2 also shows that the GMR bias is always negative, as the third row is always smaller than the second. Again, differences between the two rows appear small relative to other numbers in the table, but relative to the riskiness of these portfolios such differences are nontrivial.

Overall, Table 2 shows that microstructure biases are usually positive, a result of the dominant effect of the DMR bias. The magnitude of this bias varies dramatically across portfolios, being most severe for out-of-the-money options. However, other biases are negative in some cases, and for some portfolios these negative biases dominate the DMR bias. The table therefore shows that the microstructure biases in option portfolios cannot be simply described as large and positive; they are more complex than those for equity markets as described by Blume and Stambaugh (1983).
3.2 The size of the RC bias

Table 3 presents the results of regressions using simulated data samples that are affected by errors in stock and options prices. We estimate three different Fama-MacBeth regressions that are motivated by the cross-sectional relationship between option returns and their exposure to the underlying and volatility risks. Specifically, in a general stochastic volatility model (e.g. Duarte and Jones (2010), the instantaneous expected return of a delta-hedged derivative is:

\[ E\left[ \frac{df}{f} - \beta_S \frac{dS}{S} \right] = \beta_\sigma \lambda_\sigma \]  

where \( f \) is the price of the derivative, \( S \) is the underlying price, \( \beta_S \) is the sensitivity of the option return to stock return, \( \beta_\sigma \) is the sensitivity of the option return to changes in volatility, and \( \lambda_\sigma \) is the price of volatility risk. One Fama-MacBeth regression has \( \beta_S \) as the independent variable, another has \( \beta_\sigma \), the third has both. In all cases, the true parameter means are either approximately or exactly zero. This is the case because in the simulated Black and Scholes model delta-hedged options have no exposure to the risk of the underlying asset and the price of volatility risk is zero. Theoretically, small deviations from zero are possible because the market betas of delta-hedged returns are not exactly zero over a one-day horizon. Moreover, small deviations from zero may the result of Monte Carlo simulation error.

Panel A of Table 3 presents results based on delta-hedged call and put options in the simulated sample regressed on \( \beta_S \). Panel B presents results based on delta-hedged call and put options in the simulated sample regressed on \( \beta_\sigma \), while Panel C presents the results using both \( \beta_S \) and \( \beta_\sigma \). We find that the biases on estimated coefficients are substantial. The column labeled "RC+CEIV" presents the results in which prices, hedge ratios, and regressors have measurement errors. The coefficient on delta in the specification with RC and CEIV biases in Panel A is around 3.6 (-2.7) for calls (puts). The biases are particularly strong in the \( \beta_\sigma \) coefficient. For instance, the coefficient of \( \beta_\sigma \) is about 27.5 (19.7) in the case of calls (puts). This result implies that positive and large bias on the estimation of the price of volatility risk in a Fama-MacBeth regression. This result implies that a delta-hedged call option with a \( \beta_\sigma \) equal to the median value observed in our sample (1.96 from Table 1) has
a positive expected return of about 54 basis points per day. In truth, this delta-hedged call option expected return is zero.

Similar to the portfolio sorts, we report results in which there is no measurement error to the delta regressor so that we can assess the impact of the CEIV bias in the regressions. These results are displayed in the column labeled "RC". We also estimate regressions in which the regressors and the hedge errors are assumed to be observed without measurement errors. These regressions are presented in the column labeled "DRC". The coefficients in these regressions help us understand the effect of the DRC bias.

In general, the DRC bias, due to the impact of measurement errors on observed returns, is the dominant source of bias in the regression coefficients. Indeed, the coefficients in the columns labeled "DRC" are close to those in the column labeled "RC+CEIV", indicating that the CEIV bias and especially the GMR bias are relatively small compared to DMR bias.

4 Adjusting for the MR, CEIV, and RC biases

In this section, we introduce a generalized and unified framework to adjust the MR, CEIV, and RC biases in option returns. Our approach to bias adjustment relies heavily on the idea of synthetic prices and hedge ratios. These are used to address the spurious correlations that drive two of the three biases we are faced with. We describe these synthetic measures in Section 4.1. With those measures in hand, we tackle the MR bias in Section 4.2. In Section 4.3, we describe how we address the CEIV and RC biases. Finally, Section 4.4 examines the performance of our approach using simulated data.

4.1 Synthetic variables

Spurious correlations, induced by errors in stock and option prices, are responsible for one component of the MR bias, the GMR bias. Equation (7) shows this component of the MR bias in delta-hedged returns. The measurement errors in stock prices affect both the stock return and the hedge ratio (e.g., $\beta^C_S$) that multiplies that return in a delta-hedged option strategy. The CEIV bias is also caused by the spurious correlations between the independent (or sorting) variables and dependent variable in regressions and sorting procedures. We
address these bias by replacing option prices, implied volatilities, underlying prices and hedge ratios ($\Delta$ and $\nu$) by synthetic values calculated with non-arbitrage relations. Naturally, these synthetic values have measurement errors as the original variables do. However, by construction, the measurement errors in these synthetic variables are not correlated with the measurement errors in the returns on the corresponding options strategy.

Specifically, we define the synthetic underlying price $(\bar{S}_{i,t})$ as the median price of stock $i$ or index estimated via put-call parity based on the five call and put pairs with delta closest to 0.5 for calls (-0.5 for puts) of all the options with the shortest time-to-maturity (nine days to 30 days until expiration). We account for dividends if any, using the IvyDB dividend file.\(^7\)

We define synthetic implied volatility for calls and puts, $\bar{\sigma}_C^{j,i,t}$ and $\bar{\sigma}_P^{j,i,t}$ respectively, as the implied volatilities calculated for each call and put with time-to-maturity and strike price $j$ written on stock $i$ using the observed closing mid-quotes of the call and put prices as well as the synthetic underlying price $(\bar{S}_{i,t})$. We compute synthetic call prices $(\bar{C}_{j,i,t})$ using the implied volatility of the corresponding put $(\bar{\sigma}_P^{j,i,t})$ and $(\bar{S}_{i,t})$. The synthetic put prices $(\bar{P}_{j,i,t})$ are computed in an analogous way. For options on individual stocks (index) these implied volatilities, call and put prices are calculated with a 100-period binomial tree (Black and Scholes formula). Dividends and dividend yields are from the IvyDB dividend file.\(^8\)

To compute synthetic Greeks for the call ($\bar{\Delta}_C^{j,i,t}$ and $\bar{\nu}_C^{j,i,t}$), we use the corresponding put implied volatility $(\bar{\sigma}_P^{j,i,t})$ and the synthetic underlying price $(\bar{S}_{i,t})$. The synthetic beta of the call with respect to the underlying is $\bar{\beta}_C^{j,i,t} = \bar{\Delta}_C^{j,i,t} \times \bar{S}_{i,t}/\bar{C}_{j,i,t}$, the synthetic beta with respect to volatility is $\bar{\beta}_{\sigma,C}^{j,i,t} = \bar{\nu}_C^{j,i,t} \times \bar{S}_{i,t}/\bar{C}_{j,i,t}$. The synthetic Greeks and betas for the puts are calculated in an analogous way. In the case of the American options written on individual stocks, we numerically differentiate the option prices calculated with a 100-period binomial tree to obtain $\Delta$ and $\nu$. In the case of the European options written on the S&P

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\(^7\)The put-call parity formula is $P + S - PV(dividends) = C + Ke^{-rT}$ in the case of individual stocks and $P + Se^{-qT} = C + Ke^{-rT}$ where $q$ is the index dividend yield. Naturally put-call parity does not hold in general for the options written on individual stocks since these are American. Note, however, that we are using close to at-the-money options with short maturity in our calculation of synthetic underlying price. The value of early exercise for these options is small and hence put-call parity holds approximately well for these options.

\(^8\)For index options we assume a continuous dividend rate that is constant over the remaining life of the option. We calculate this rate from the IvyDB index dividends file by averaging all daily dividend yields from the day after the observation date to the expiration date.
500 index, we use the closed-form solutions for $\Delta$ and $\nu$ in the Black and Scholes model.

### 4.2 Adjusting for MR bias

To better understand our approach, recall that equation (5) represented option strategy returns in the form of $R_{i,t+1} = \sum_{k=1}^{M} f_{X_k}(X_{k,i,t})(X_{k,i,t+1} - X_{k,i,t})/g(X_{i,t})$. The first step in our bias adjustment procedure is to replace $X_{k,i,t}$ in $f_{X_k}(X_{i,t})$ by its synthetic value. For example, if the first element of the vector $X_{i,t}$ is the stock price, then we replace the observed stock price when calculating $f_{X_1}(X_{i,t})$ with its synthetic value described in the previous section. The actual stock price is still used when calculating $f_{X_2}(X_{i,t})$.

More generally, let $X_{k,-i,t}$ be a vector composed of observed prices for all elements other than $k$ and with $X_{k,-i,t}$ equal to the synthetic price of the $k^{th}$ element of the price vector. That is, $X_{k,-i,t}$ is identical to $X_{i,t}$ except for its $k^{th}$ element. Therefore, the first step of the proposed method is to replace equation (5) with

$$
\bar{R}_{i,t+1} = \sum_{k=1}^{M} \frac{f_{X_k}(X_{k,-i,t})(X_{k,i,t+1} - X_{k,i,t})}{g(X_{i,t})}.
$$

(14)

Note that there is no change to the denominator from equation (5) to equation (14).

The second step is to form portfolios using weights that are equal to

$$
w^i = \frac{g(X_{i,t})}{g(X_{i,t-1})} \left/ \sum_{j=1}^{M} \frac{g(X_{j,t})}{g(X_{j,t-1})} \right.
$$

(15)

In some cases, $g(X_{i,t})$ is simply the price of an asset, making this choice identical to the gross-returns weighting of Asparouhova, Bessembinder, and Kalcheva (2010, 2013).

We prove in the appendix that under certain conditions, $\sum_{i=1}^{N} w^i \bar{R}_{i,t+1}$ is an approximately unbiased estimator of the expected option portfolio returns $E[R_{P,t+1}]$. One condition is that the selection of the options that comprise this portfolio is itself not contaminated by error. We discuss how to ensure that this condition holds in the following section.

For delta-hedged options, the intuition for why this two-step method works can be traced back to the origins of the biases at work. Section 2 points out that part of the bias in the returns of delta-hedged option strategies is the result of the fact that the weight of the stock needed to hedge the option is affected by the same errors that impact the stock’s rate of
return. This spurious covariance is responsible for the term $-\frac{\partial \beta}{\partial S}(\tilde{S}_t, \tilde{C}_t) E[\epsilon^2_{S,t}]$ in equation (7).

The first step of the proposed method addresses this source of bias by calculating the option’s delta using the synthetic stock price, whose measurement errors should be uncorrelated with those impacting the observed stock price. In the notation of equation (5), this step entails replacing $f_S(X_t) = -\Delta^C(S_t, C_t)$ with $f_S(X_t^S) = -\Delta^C(\tilde{S}_t, C_t)$.

The second step of the proposed method addresses the same types of biases that are presented in Blume and Stambaugh (1983) and Asparouhova, Bessembinder, and Kalcheva (2010, 2013). Because $g(X_t) = C_t$ for delta-hedged calls (and $g(X_t) = P_t$ for puts), the second step is identical to the gross-return weighting of Asparouhova, Bessembinder, and Kalcheva (2010, 2013).

For straddles, step one involves replacing $f_P(X_t)$ and $f_C(X_t)$ with

$$f_P(C_t, \tilde{P}_t, S_t) = \frac{\Delta^C(C_t, S_t)}{\Delta^C(C_t, S_t) P_t - \Delta^P(C_t, S_t) C_t}$$

$$f_C(\tilde{C}_t, P_t, S_t) = \frac{-\Delta^P(P_t, S_t)}{\Delta^C(P_t, S_t) P_t - \Delta^P(P_t, S_t) \tilde{C}_t}$$

(16) (17)

where $P_t = C_t + e^{-rT}K - S_t$, $\tilde{C}_t = P_t - e^{-rT}K + S_t$, $\Delta^P(C_t, S_t) = \Delta^C(C_t, S_t) - 1$, and $\Delta^C(P_t, S_t) = \Delta^P(P_t, \tilde{S}_t) + 1$. For step two, given that $g(X_t) = 1$ for straddles, the portfolio remains equally weighted.$^9$

In this scheme, the position in the put ($f_P(C_t, \tilde{P}_t, S_t)$) depends only on the prices of the call and the underlying security. The position in the call ($f_C(\tilde{C}_t, P_t, S_t)$) depends only on the prices of the put and the underlying security. The result is that the straddle return can be written as

$$f_P(C_t, \tilde{P}_t, S_t) (P_{t+1} - P_t) + f_C(\tilde{C}_t, P_t, S_t) (C_{t+1} - C_t).$$

(18)

Because noise in prices does not produce a bias in price differences, only in returns, both $P_{t+1} - P_t$ and $C_{t+1} - C_t$ are unbiased. Furthermore, the measurement error in $P_{t+1} - P_t$ is

$^9$The synthetic deltas used to calculate straddle returns do not need to use synthetic underlying prices. Because the stock price is not a component of $X_t$, using the observed stock price to calculate hedge ratios will not induce a bias. In the following section, we require a synthetic delta that is not computed using observed stock prices. For consistency, we therefore define synthetic deltas here as depending on the synthetic rather than the observed stock price.
uncorrelated with that of $f_P(C_t, \bar{P}_t, S_t)$, as the latter expression is computed without using observed put prices. Similarly, the error in $C_{t+1} - C_t$ is uncorrelated with $f_C(\bar{C}_t, P_t, S_t)$.

### 4.3 Adjusting for the CEIV and RC biases

The CEIV bias arises when errors in prices simultaneously drive errors in strategy returns ($X_t$) and sorting variables or regressors ($\theta_{i,t}$). To adjust for this bias, we must eliminate this spurious correlation. To do so, we replace $\theta_{i,t}$ with its synthetic counterpart $\bar{\theta}_{i,t}$. The key requirement of $\bar{\theta}_{i,t}$ is that it is calculated independently of any elements of $X_t$.

As an example of how this is implemented, suppose that we are examining delta-hedged call options and wish to form portfolios on the basis of delta. We should therefore use a synthetic delta, which is based solely on the price of the put and the synthetic price of the stock. This measure is not contaminated by errors in the call or the stock, which are elements of $X_t$, but it is in theory identical to the observed delta computed from stock and call prices.

For straddles, requiring that $\bar{\theta}_{i,t}$ be independent of $X_t$ turns out to be more restrictive, because $X_t$ includes both the call and put prices for straddles. Thus, we cannot use a synthetic call delta based on the price of the put, or vice versa. Thus, our straddle portfolios are sorted on the basis of moneyness, $K/S_t$, rather than delta, as the former can be computed independent of any option prices.

To adjust for the RC bias in cross-sectional regressions, when $\theta_{i,t}$ is measured without error, we first replace the strategy returns defined in equation (5) with those defined in equation (14). This step eliminates the spurious correlation between security returns and hedge ratios. We then replace the OLS regression with a WLS regression, using the weights in equation (15). Doing so addresses the RC in the same way proposed by Asparouhova, Bessembinder, and Kalcheva (2010, 2013).

To adjust for the RC bias in cross-sectional, when $\theta_{i,t}$ is measured with error that is correlated with errors in the dependent variable, we first adjust for the CEIV bias. Specifically, we replace $\theta_{i,t}$ with its synthetic counterpart $\bar{\theta}_{i,t}$. Then we repeat the approach described in the previous paragraph. In the appendix, we show, that under this condition, the WLS approach proposed in the previous section is an approximately unbiased estimator of the true regression coefficient vector.
4.4 Examining the bias-adjustment method with simulations

Using Monte Carlo simulation, we now assess the proposed method for bias adjustment. We contrast its performance with the method suggested by Asparouhova, Bessembinder, and Kalcheva (2010), in which observations are weighted by lagged gross returns. In the case of unhedged calls and puts, these weights are therefore \( C_t/C_{t-1} \) and \( P_t/P_{t-1} \), respectively. We use the same weights for delta-hedged option returns as well, effectively ignoring errors that arise from the stock hedge term. For straddles, the weight is \( C_t/C_{t-1} \times P_t/P_{t-1} \), which would eliminate microstructure bias were the hedge ratios observed without error.\(^{10}\)

Table 4 reports biases for unhedged and delta-hedged calls and puts as well as delta-neutral straddles. Similar to the results from Table 2, call and put portfolios are sorted by \( \beta_S \) and \( \beta_\sigma \). Specifically, the unadjusted and gross return-weighted results are based on portfolios sorted on observed \( \beta_S \) and \( \beta_\sigma \), while the sorts used for the proposed method are based on synthetic \( \beta_S \) and \( \beta_\sigma \).

Overall, the table shows that our proposed bias-adjustment method successfully decreases the biases in mean returns. For instance, Panel C shows biases in unadjusted average returns of delta-hedged calls that are as large as 68 bps per day. In comparison, the biases under our proposed method are generally much smaller, with the largest bias among unhedged returns being just -9 bps and statistically not significant.

Asparouhova, Bessembinder, and Kalcheva (2010) propose using gross-weighting returns to address the MR bias in equity returns, the results in Table 4 indicate that in many cases gross returns weighting still results in biased estimates of mean returns in the case of options. In particular, out-the-money calls (high \( \beta_S \)) and puts (low \( \beta_S \)) have large biases that are not corrected by returns weighting. This is because the CEIV bias is large for these options. This bias is only corrected by our use of the synthetic delta as the sorting variable.

Panel E of Table 4 presents the results for straddles. Under the simulated model, straddles should have zero mean returns. The results in Panel E indicate that MR+CEIV biases are substantial and positive. Gross return weighting reduces these biases, causing them to

\[^{10}\text{The straddle return is } v^C(C_t/C_{t-1} - 1) + v^P(P_t/P_{t-1} - 1). \text{ Weighting it by } C_t/C_{t-1} \times P_t/P_{t-1} \text{ will produce a quantity that is proportional to } v^C(P_t/P_{t-1})(C_t - C_{t-1}) + v^P(C_t/C_{t-1})(P_t - P_{t-1}). \text{ As long as } v^C \text{ and } v^P \text{ are observed without error, the bias in this expression is approximately zero. If the true expected returns on the call and the put are zero, the bias is exactly zero.}\]
change sign and become negative and statistically significant. The proposed method almost completely eliminates the biases, with all of them below 0.4 bps in magnitude.

Table 5 presents the results of bias adjustment in regressions similar to those in Table 3. In general, the results in Panels A, B, and C which examine all delta-hedged puts and calls indicate that our method dramatically reduces biases in estimated coefficients. We are especially interested on the biases of the coefficients in the $\beta_\sigma$ given the stylized fact in the literature that volatility is not priced in individual stocks options (e.g., Boehmer, Grammig, and Theissen (2007)). The RC+CEIV biases are very large and positive with OLS regressions. For instance the coefficients on $\beta_\sigma$ are about 27 for call and 20 for puts. WLS regressions decrease this bias quite a bit with a statistically significant coefficient in Panel B of about 11 (7) for call (put) options. Our bias adjustment results in a coefficient of about -0.6 that is not statistically different from zero at the usual significance levels.

5 Are the MR, CEIV and RC biases consequential for stylized results in the options literature?

Table 6 reports average unadjusted and bias-adjusted returns on portfolios of call and put options written on the S&P 500 Index. We include both unhedged and delta-hedged strategies. The results in Panel A indicate the unadjusted average call returns are often different from the unadjusted averages. For example, the return for the high $\beta_S$ call option (deep OTM) in Panel A is about five times smaller in magnitude when we apply our bias adjustment (12.1 bps vs. 1.5 bps). Results for unhedged puts in Panel B are similar, aside from their negative betas tending to make average returns negative. Bias-adjusted returns on deep OTM options (low $\beta_S$) are far more negative than unadjusted averages. As with calls, these results are almost certainly the result of MR bias being larger for options that are more deeply OTM, as these options tend to have much higher relative bid-ask spreads.

An important stylized fact in the options literature is that the price of index volatility risk is negative. In particular, Bakshi and Kapadia (2003a) show that the return of delta-hedge options written on the S&P 500 have a negative mean return. While, Coval and Shumway (2001) show that the expected returns of straddles on index options are negative and large.

Our bias adjustment indicates that the price of index volatility risk is negative and even
smaller than that initially estimated in Coval and Shumway (2001) and Bakshi and Kapadia (2003a). We examine the effect of the MR+CEIV biases on Bakshi and Kapadia (2003a) findings in Panels C and D of Table 6. These panels display the results of bias adjustment to delta-hedged calls and puts. Bias adjustment more than triples the magnitude of the deep OTM delta-hedged call portfolio (high $\beta_\sigma$). The effects of bias adjustment on delta-hedged put portfolios are somewhat less dramatic but still very economically important. Panel A of Table 7 presents empirical results for straddles similar to those in Coval and Shumway (2001). Consistent with Coval and Shumway (2001), we find that the daily returns of straddles are negative and large. Indeed, for our entire sample period, we find that equal weighted straddle mean returns are about -26 bps per day. Once we adjust for biases, this average return is even smaller, about -30 bps.

The current literature in options also suggests that options on individual stocks have different return patterns from those written on the indexes. Specifically, the literature suggests that volatility risk in individual stock options is not priced. For instance, Bakshi and Kapadia (2003b) find that the average delta hedged returns of options written on individual stocks are not as negative as those written on the index, while Diessen, Maenhout, and Grigory (2009) find no statistically significant difference between the implied and the actual volatility of individual stocks.

Using our bias adjustment, we find that the price of volatility risk in individual stock options is not nearly as different from the price of volatility on the S&P 500 Index as previously suggested in the literature. Panel B of Table 7 shows the returns of straddles written on individual stocks. Without the bias adjustment the average return is about -9 bps per day which is close to a third of the average unadjusted return of straddles on the S&P 500 in Panel A (-25 bps). Adjusting for biases has a significant effect on the average returns of straddle on individual equities. Indeed, the adjusted return is approximately -14 bps per day which is almost half of the bias adjusted return of straddles on the index (-30 bps). Table 8 presents the mean return of calls, puts and delta-hedged options written on individual stocks sorted by $\beta_S$ and $\beta_\sigma$. The results in Panels A and B indicate that as in the case of index options, unadjusted average call and put returns are often different from the adjusted averages. Moreover, delta-hedged calls and puts have in general negative mean
returns after adjusting for biases. This is again consistent with the idea that volatility is negatively priced in individual equity options as it is in indexes options.

To further examine the effect of biases adjustments in the price of volatility risk in options written on the index and on individual stocks, we examine the effect of bias adjustment in the price of volatility risk with Fama-MacBeth regressions similar to those in Tables 3 and 5. Table 9 shows the results of regressing delta-hedged call and put returns on $\beta_S$ and $\beta_\sigma$.

The results in Table 9 indicate that adjusting for the RC+CEIV biases makes a large difference in the estimation of price of volatility among options written on individual stocks. Panel A of Table 9 shows that without bias adjustment the coefficient on $\beta_\sigma$ is positive and significant for call and not statistically different from zero for puts and in the specification with both puts and calls. This finding would be consistent with the stylized fact in the options literature that volatility is not priced at the individual stocks level. However, the coefficients estimated with bias adjustments show a different picture. For instance, in the specification with both calls and puts the estimated price of volatility risk is -10 with t-statistics of -11.

Table 10 show the results of similar Fama-MacBeth regressions on options written on the S&P 500. As in Table 9, bias adjustments have a negative effect on the estimated price of volatility risk. Specifically, the specification with both put and call options shows a price of index volatility risk of about -4.4 without bias adjustment. On the other hand, adjusting for bias results in a price of volatility risk of about -22 in Panel A of Table 10. A comparison between the results in Tables 9 and 10 reveals that after adjusting for microstructure biases we conclude that the price of volatility risk in individual equity options is about 45% (-10/-22) of the price of volatility risk in the S&P 500 options.

6 Conclusion

In this paper, we show that the estimation of expected returns and risk premia on option-based portfolios suffers from biases similar to those discussed by Blume and Stambaugh (1983), Fama (1984), Stambaugh (1988), and Asparouhova, Bessembinder, and Kalcheva (2010, 2013). While the solutions proposed in those papers were excellent starting points in our work, they are not directly applicable to option portfolios because errors in prices induce correlated errors in hedge ratios, sorting variables, and regressors, which render existing
We build on this prior work by proposing a new method that addresses the problems created by these spurious correlations. Using simulations, we show that our method is effective in reducing biases in the estimation of expected returns of option portfolios and in estimating regression coefficients involving option strategy returns.

We find that adjusting for biases in the estimation of expected returns is consequential for some of the most well-known stylized facts in the empirical option literature. Specifically, we find that the daily returns of straddles and delta-hedged options written on the S&P 500 are even more negative than of what is commonly estimated when not using bias adjustments (e.g., Coval and Shumway (2001) and Bakshi and Kapadia (2003a)). We also find that the returns of straddles and delta-hedged options written on individual stocks are not as different from those written on the S&P 500 index as suggested in Bakshi and Kapadia (2003b). Moreover, we find in Fama-MacBeth regressions that the price of volatility risk in individual stock options is about 40% of the price of volatility risk in S&P 500 options. Our findings indicate that the price of volatility in individual equity options is not as different from that in index options as previously suggested in the literature (e.g. Diessen, Maenhout, and Grigory (2009)).
References


Table 1: **Summary statistics** This table presents mean, standard deviation, median, first and third quantile (Q1 and Q3), and skewness of option returns, stock returns, relative spreads, moneyness, $\beta_S$, and $\beta_\sigma$. Returns are displayed in basis points. The relative bid-ask spread of stock $i$ is defined in a given day as the volume weighted average of the relative bid-ask spread of all transactions during the day. It is calculated with TAQ data. The relative option spread is given by $\frac{\text{ask}_{i,j} - \text{bid}_{i,j}}{\frac{\text{ask}_{i,j} + \text{bid}_{i,j}}{2}}$, where $\text{ask}_{i,j}$ and $\text{bid}_{i,j}$ are the closing ask and bid prices on option $j$ written on stock $i$. These closing prices are from IvyDB. Bid-ask spreads are displayed in percentages. The moneyness of option $j$ on stock $i$ is defined as $\log(\frac{K_j}{S_i})$, where, $S_i$ is the observed closing stock price, and $K_j$ is the strike price. To obtain each statistic, we first calculate the cross-sectional values for each day in our sample and take the average over time. $\beta_S$ and $\beta_\sigma$ are the $\beta$s of the options with respect to the underlying price and the volatility of the underlying respectively.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Q1</th>
<th>Q3</th>
<th>Skewness</th>
</tr>
</thead>
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<tr>
<td><strong>A: Call Options on SP500 stocks (Avg. number of options per day: 10,390)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>2047.08</td>
<td>-788.97</td>
<td>651.94</td>
<td>4.41</td>
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<td>19.89</td>
<td>3.68</td>
<td>15.17</td>
<td>2.60</td>
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<td>Moneyness</td>
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<td>-0.04</td>
<td>0.24</td>
<td>-0.19</td>
<td>0.09</td>
<td>-0.67</td>
</tr>
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<td>$\beta_S$</td>
<td>9.63</td>
<td>7.60</td>
<td>7.24</td>
<td>4.61</td>
<td>12.54</td>
<td>3.25</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
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<td>1.96</td>
<td>5.48</td>
<td>0.45</td>
<td>6.39</td>
<td>2.54</td>
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<tr>
<td><strong>B: Put Options on SP500 stocks (Avg. number of options per day: 11,182)</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Return</td>
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<td>-72.90</td>
<td>1762.59</td>
<td>-799.99</td>
<td>597.61</td>
<td>3.63</td>
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<td>Bid-ask Spread</td>
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<td>8.34</td>
<td>22.04</td>
<td>4.27</td>
<td>20.39</td>
<td>2.19</td>
</tr>
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<td>Moneyness</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.24</td>
<td>-0.15</td>
<td>0.13</td>
<td>-0.03</td>
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<tr>
<td>$\beta_S$</td>
<td>-8.91</td>
<td>-7.57</td>
<td>6.07</td>
<td>-11.53</td>
<td>-4.74</td>
<td>-2.66</td>
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<tr>
<td>$\beta_\sigma$</td>
<td>4.61</td>
<td>3.23</td>
<td>4.61</td>
<td>0.81</td>
<td>7.43</td>
<td>2.05</td>
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<tr>
<td><strong>C: Call Options on SP500 index (Avg. number of options per day: 207)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return</td>
<td>24.79</td>
<td>20.21</td>
<td>932.25</td>
<td>-465.19</td>
<td>498.35</td>
<td>0.08</td>
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<tr>
<td>Bid-ask Spread</td>
<td>8.45</td>
<td>3.71</td>
<td>11.20</td>
<td>1.87</td>
<td>9.81</td>
<td>2.15</td>
</tr>
<tr>
<td>Moneyness</td>
<td>-0.06</td>
<td>-0.04</td>
<td>0.11</td>
<td>-0.12</td>
<td>0.03</td>
<td>-0.71</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>18.81</td>
<td>14.17</td>
<td>14.06</td>
<td>8.26</td>
<td>25.32</td>
<td>1.34</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
<td>7.93</td>
<td>3.41</td>
<td>9.61</td>
<td>0.91</td>
<td>12.12</td>
<td>1.49</td>
</tr>
<tr>
<td><strong>D: Put Options on SP500 index (Number of options per day: 197)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return</td>
<td>-39.85</td>
<td>-34.45</td>
<td>818.37</td>
<td>-544.42</td>
<td>460.24</td>
<td>-0.33</td>
</tr>
<tr>
<td>Bid-ask Spread</td>
<td>9.56</td>
<td>6.92</td>
<td>8.31</td>
<td>4.08</td>
<td>12.06</td>
<td>1.93</td>
</tr>
<tr>
<td>Moneyness</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.09</td>
<td>-0.10</td>
<td>0.03</td>
<td>-0.39</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-19.31</td>
<td>-17.70</td>
<td>7.90</td>
<td>-24.74</td>
<td>-13.12</td>
<td>-0.63</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
<td>8.39</td>
<td>7.87</td>
<td>5.16</td>
<td>3.71</td>
<td>12.59</td>
<td>0.33</td>
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<tr>
<td><strong>E: Stocks (Avg. number of stocks per day: 404)</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Return</td>
<td>4.96</td>
<td>1.05</td>
<td>195.76</td>
<td>-91.66</td>
<td>97.25</td>
<td>0.26</td>
</tr>
<tr>
<td>Bid-ask Spread</td>
<td>0.21</td>
<td>0.17</td>
<td>0.15</td>
<td>0.13</td>
<td>0.24</td>
<td>4.80</td>
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</table>
Table 2: **Biases in simulated option portfolio sorts.** This table presents the results of 1000 simulation trials of stocks and options returns. The simulations are based on the CAPM and the Black and Scholes model with added measurement errors to the underlying stocks and option prices. The relative measurement errors are drawn from symmetric triangular distributions with support between ±0.5 times the relative bid-ask spread. Panels A and B present equal-weighted average returns of call and puts sorted by their $\beta_S$ with respect to the underlying ($\beta_S$). Panels C and D present equal-weighted average returns of delta-hedged calls and puts sorted by their $\beta_S$ with respect to the volatility of the underlying ($\beta_\sigma$). Panel E displays the mean returns of straddles sorted by moneyness (the log of the ratio of the stock to the strike price). All returns are in basis points. The row labeled ‘True mean’ shows the mean returns of each strategy under the assumption that stock and option returns are observed without any error due bid-ask spreads. The row labeled ‘MR+CEIV bias’ shows the bias in the equal-weighted mean returns in the case that errors in stock and option prices due to bid-ask spreads affect the calculation of the hedge ratios and sorting variables ($\beta_S$) as well as option and stock returns. The row labeled ‘MR bias’ shows the results in the case that errors in stock and option prices affect the calculation of the hedge ratios ($\Delta$) as well as option and stock returns but do not affect the sorting variables. The row labeled ‘DMR bias’ shows the results in the case that errors in stock and option prices affect the calculation of option and stock returns but do not affect the hedge ratios and sorting variables. T-statistics are within parentheses.

**A: Call options**

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>True mean</td>
<td>10.45(1.01)</td>
<td>19.69(1.17)</td>
<td>26.93(1.06)</td>
<td>37.21(0.99)</td>
<td>54.11(0.88)</td>
</tr>
<tr>
<td>MR+CEIV bias</td>
<td>-2.17(-18.18)</td>
<td>-0.20(-0.93)</td>
<td>0.16(0.47)</td>
<td>10.32(18.18)</td>
<td>31.32(43.02)</td>
</tr>
<tr>
<td>MR bias</td>
<td>0.32(5.76)</td>
<td>0.61(8.01)</td>
<td>4.30(22.15)</td>
<td>10.88(32.45)</td>
<td>18.67(32.69)</td>
</tr>
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</table>

**B: Put options**

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>True mean</td>
<td>-57.92(-0.95)</td>
<td>-32.94(-0.95)</td>
<td>-20.73(-0.92)</td>
<td>-13.25(-0.96)</td>
<td>-6.47(-0.91)</td>
</tr>
<tr>
<td>MR+CEIV bias</td>
<td>41.46(64.66)</td>
<td>12.02(23.06)</td>
<td>-0.61(-1.89)</td>
<td>-0.27(-1.57)</td>
<td>-1.96(-19.60)</td>
</tr>
<tr>
<td>MR bias</td>
<td>29.14(53.43)</td>
<td>15.32(44.56)</td>
<td>4.74(24.27)</td>
<td>0.89(11.66)</td>
<td>0.56(9.35)</td>
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</table>

**C: Delta-hedged calls**

<table>
<thead>
<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>True mean</td>
<td>0.01(0.16)</td>
<td>0.02(0.09)</td>
<td>0.35(0.30)</td>
<td>0.21(0.06)</td>
<td>2.13(0.23)</td>
</tr>
<tr>
<td>MR+CEIV bias</td>
<td>6.26(96.11)</td>
<td>-5.35(-65.68)</td>
<td>-1.20(-8.81)</td>
<td>0.31(0.76)</td>
<td>68.35(88.84)</td>
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<tr>
<td>MR bias</td>
<td>0.41(6.70)</td>
<td>0.59(7.43)</td>
<td>2.05(15.36)</td>
<td>16.50(41.45)</td>
<td>48.82(66.54)</td>
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<tr>
<td>DMR bias</td>
<td>0.37(6.11)</td>
<td>0.53(6.42)</td>
<td>2.33(14.74)</td>
<td>16.66(40.44)</td>
<td>50.90(64.18)</td>
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**D: Delta-hedged puts**

<table>
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<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>True mean</td>
<td>0.01(0.17)</td>
<td>0.05(0.17)</td>
<td>0.36(0.31)</td>
<td>0.13(0.04)</td>
<td>1.13(0.13)</td>
</tr>
<tr>
<td>MR+CEIV bias</td>
<td>7.15(108.96)</td>
<td>-5.91(-74.07)</td>
<td>-1.07(-8.16)</td>
<td>1.85(5.57)</td>
<td>48.53(80.00)</td>
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<tr>
<td>MR bias</td>
<td>0.48(7.79)</td>
<td>0.92(11.67)</td>
<td>1.73(13.34)</td>
<td>12.06(37.91)</td>
<td>35.35(59.76)</td>
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<tr>
<td>DMR bias</td>
<td>0.46(7.55)</td>
<td>0.86(10.59)</td>
<td>2.01(12.48)</td>
<td>12.16(37.21)</td>
<td>36.38(58.00)</td>
</tr>
</tbody>
</table>

**E: Straddles**

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>True mean</td>
<td>0.17(0.17)</td>
<td>0.26(0.16)</td>
<td>0.21(0.13)</td>
<td>0.16(0.11)</td>
<td>0.19(0.27)</td>
</tr>
<tr>
<td>MR bias</td>
<td>4.70(41.98)</td>
<td>2.64(25.48)</td>
<td>1.03(11.73)</td>
<td>3.73(33.37)</td>
<td>5.41(53.33)</td>
</tr>
<tr>
<td>DMR bias</td>
<td>6.31(54.46)</td>
<td>4.01(38.23)</td>
<td>1.99(22.81)</td>
<td>5.56(48.42)</td>
<td>7.19(67.87)</td>
</tr>
</tbody>
</table>

34
Table 3: Biases in simulated Fama-MacBeth regressions. This table presents the results of 1000 simulation trials of stocks and options returns. The simulations are based on the CAPM and Black and Scholes model with added measurement errors to the underlying stocks and option prices. The relative measurement errors are drawn from symmetric triangular distributions with support between ±0.5 times the relative bid-ask spread. The table displays the average across simulations of the estimated coefficients and t-statistics of OLS regressions. The dependent variables are the returns of delta-hedged calls and puts respectively. The independent variables are option $\beta$s with respect to the underlying ($\beta_S$) and the volatility of the underlying ($\beta_{\sigma}$). The column labeled ‘True mean’ shows the mean coefficient under the assumption that stock and option returns are observed without any error due bid-ask spreads. The column labeled ‘RC+CEIV’ shows the mean coefficients in the case that errors in stock and option prices affect the calculation of the hedge ratios and independent variables as well as option and stock returns. The column labeled ‘RC’ shows the results in the case that errors in stock and option prices affect the calculation of the hedge ratios ($\Delta$s) as well as option and stock returns but do not affect the independent variables. The column labeled ‘DRC’ shows the results in the case that errors in stock and option prices affect only the calculation of option and stock returns but do not affect the hedge ratios and independent variables.

<table>
<thead>
<tr>
<th></th>
<th>Calls</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RC+CEIV</td>
<td>RC</td>
<td>DRC</td>
<td></td>
<td>RC+CEIV</td>
<td>RC</td>
<td>DRC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True Mean</td>
<td></td>
<td></td>
<td></td>
<td>True Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.53</td>
<td>-21.66</td>
<td>-7.52</td>
<td>-7.53</td>
<td>-0.49</td>
<td>-13.54</td>
<td>-3.95</td>
<td>-3.95</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>0.10</td>
<td>3.58</td>
<td>2.15</td>
<td>2.15</td>
<td>0.09</td>
<td>-2.67</td>
<td>-1.59</td>
<td>-1.58</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(0.17)</td>
<td>(85.30)</td>
<td>(54.50)</td>
<td>(52.36)</td>
<td>(0.16)</td>
<td>(-74.03)</td>
<td>(-46.10)</td>
<td>(-44.68)</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.04</td>
<td>-10.11</td>
<td>-1.12</td>
<td>-1.18</td>
<td>-0.15</td>
<td>-6.92</td>
<td>-0.49</td>
<td>-0.55</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(0.10)</td>
<td>(-65.94)</td>
<td>(-8.00)</td>
<td>(-8.34)</td>
<td>(-0.37)</td>
<td>(-52.98)</td>
<td>(-4.00)</td>
<td>(-4.43)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>0.54</td>
<td>27.48</td>
<td>17.10</td>
<td>17.10</td>
<td>0.56</td>
<td>19.66</td>
<td>12.24</td>
<td>12.23</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(0.16)</td>
<td>(94.19)</td>
<td>(63.35)</td>
<td>(61.28)</td>
<td>(0.17)</td>
<td>(82.39)</td>
<td>(54.31)</td>
<td>(52.91)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.69</td>
<td>-5.42</td>
<td>3.49</td>
<td>3.53</td>
<td>-0.44</td>
<td>-6.02</td>
<td>1.37</td>
<td>1.40</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(-0.23)</td>
<td>(-14.27)</td>
<td>(9.29)</td>
<td>(9.19)</td>
<td>(-0.19)</td>
<td>(-22.71)</td>
<td>(5.24)</td>
<td>(5.30)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-0.31</td>
<td>32.58</td>
<td>22.11</td>
<td>22.22</td>
<td>0.16</td>
<td>20.88</td>
<td>14.75</td>
<td>14.86</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(-0.42)</td>
<td>(64.03)</td>
<td>(45.42)</td>
<td>(44.79)</td>
<td>(0.21)</td>
<td>(53.21)</td>
<td>(38.73)</td>
<td>(38.49)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>0.15</td>
<td>-0.92</td>
<td>-0.91</td>
<td>-0.93</td>
<td>-0.07</td>
<td>0.22</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>$\beta_{\sigma}$</td>
<td>(0.28)</td>
<td>(-13.16)</td>
<td>(-13.08)</td>
<td>(-13.07)</td>
<td>(-0.14)</td>
<td>(3.85)</td>
<td>(7.98)</td>
<td>(8.26)</td>
</tr>
</tbody>
</table>
Table 4: Correcting for biases in simulated option portfolio sorts. This table presents the results of 1000 simulation trials of stocks and options returns. The simulations are based on the CAPM and the Black and Scholes model with added measurement errors to the underlying stocks and option prices. The relative measurement errors are drawn from symmetric triangular distributions with support between $\pm 0.5$ times the relative bid-ask spread. Panels A and B present equal-weighted average returns of calls and sorted by their $\beta$s with respect the underlying. Panels C and D present equal-weighted average returns of delta-hedged calls and puts sorted by their $\beta$s with respect to the volatility of the underlying. Panel E displays the mean returns of straddles sorted by moneyness (the log of the ratio of the stock to the strike price). The row labeled ‘MR+CEIV bias’ shows the bias in the equal-weighted mean returns in the case that errors in stock and option prices affect the calculation of the hedge ratios and sorting variables as well as option and stock returns. The row labeled ‘Gross returns weighting’ shows the biases in the return-weighted mean returns – sorting the options by observed $\beta$s and calculating average returns using option raw returns as weights. The row labeled ‘Proposed Method’ shows the bias in the method presented in the paper to adjust for the MR and CEIV biases. All returns are in basis points and t-statistics are within parentheses.

A: Call options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR+CEIV bias</td>
<td>-2.17(-18.18)</td>
<td>-0.20(-0.93)</td>
<td>0.16(0.47)</td>
<td>10.32(18.18)</td>
<td>31.32(43.02)</td>
</tr>
<tr>
<td>Gross returns weighting</td>
<td>-2.47(-11.02)</td>
<td>-0.77(-1.71)</td>
<td>-4.22(-4.36)</td>
<td>0.67(0.38)</td>
<td>13.53(4.17)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.47(0.53)</td>
<td>-1.60(-1.66)</td>
<td>0.17(0.10)</td>
<td>1.02(0.38)</td>
<td>-2.61(-0.58)</td>
</tr>
</tbody>
</table>

B: Put options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR+CEIV bias</td>
<td>41.46(64.66)</td>
<td>12.02(23.06)</td>
<td>-0.61(-1.89)</td>
<td>-0.27(-1.57)</td>
<td>-1.96(-19.60)</td>
</tr>
<tr>
<td>Gross returns weighting</td>
<td>10.57(3.24)</td>
<td>-3.51(-2.20)</td>
<td>-4.90(-6.05)</td>
<td>-0.96(-3.10)</td>
<td>-2.40(-17.67)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>-2.01(-0.40)</td>
<td>0.10(0.04)</td>
<td>1.95(1.36)</td>
<td>0.93(1.22)</td>
<td>-4.51(-0.89)</td>
</tr>
</tbody>
</table>

C: Delta-hedged calls

<table>
<thead>
<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR+CEIV bias</td>
<td>6.26 (96.11)</td>
<td>-5.35 (-65.68)</td>
<td>-1.20 (-8.81)</td>
<td>0.31 (0.76)</td>
<td>68.35 (88.84)</td>
</tr>
<tr>
<td>Gross returns weighting</td>
<td>5.90 (90.76)</td>
<td>-5.91 (-69.56)</td>
<td>-2.93 (-16.55)</td>
<td>-15.92 (-25.25)</td>
<td>19.85 (14.33)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>2.24 (0.46)</td>
<td>0.21 (0.27)</td>
<td>0.65 (0.57)</td>
<td>4.16 (0.52)</td>
<td>-9.38 (-1.35)</td>
</tr>
</tbody>
</table>

D: Delta-hedged puts

<table>
<thead>
<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR+CEIV bias</td>
<td>7.15 (108.96)</td>
<td>-5.91 (-74.07)</td>
<td>-1.07 (-8.16)</td>
<td>1.85 (5.57)</td>
<td>48.53 (80.00)</td>
</tr>
<tr>
<td>Gross returns weighting</td>
<td>6.74 (103.41)</td>
<td>-6.57 (-80.25)</td>
<td>-2.85 (-16.93)</td>
<td>-9.39 (-17.30)</td>
<td>13.63 (11.46)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>-4.42 (-0.93)</td>
<td>-0.30 (-0.39)</td>
<td>0.55 (0.65)</td>
<td>9.61 (1.57)</td>
<td>-4.18 (-0.59)</td>
</tr>
</tbody>
</table>

E: Straddles

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR bias</td>
<td>4.70(41.98)</td>
<td>2.64(25.48)</td>
<td>1.03(11.73)</td>
<td>3.73(33.37)</td>
<td>5.41(53.33)</td>
</tr>
<tr>
<td>Gross returns weighting</td>
<td>-1.66(-4.20)</td>
<td>-1.59(-3.78)</td>
<td>-1.03(-4.24)</td>
<td>-1.90(-6.04)</td>
<td>-1.78(-2.75)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.37(0.83)</td>
<td>0.27(0.54)</td>
<td>0.32(0.73)</td>
<td>0.37(1.12)</td>
<td>0.34(0.55)</td>
</tr>
</tbody>
</table>
Table 5: Correcting for biases in simulated Fama-MacBeth regressions. This table presents the results of 1000 simulation trials of stocks and options returns. The simulations are based on the CAPM and Black and Scholes model with added measurement errors to the underlying stocks and option prices. The relative measurement errors are drawn from symmetric triangular distributions with support between ±0.5 times the relative bid-ask spread. The dependent variables are the returns of delta-hedged calls and puts respectively. The independent variables are the \(\beta_S\) with respect the underlying (\(\beta_S\)) and the volatility of the underlying (\(\beta_{\sigma}\)). The column labeled ‘RC+CEIV’ shows the average across simulations of the estimated coefficients and t-statistics of OLS regressions under the assumption that errors in stock and option prices affect the calculation of the hedge ratios and independent variables as well as option and stock returns. The row labeled ‘WLS’ shows shows the average across simulations of the estimated coefficients and t-statistics of WLS regressions where the weights are gross option returns. The row labeled ‘Proposed Method’ shows the bias in the method presented in the paper to adjust for the RC and CEIV biases.

<table>
<thead>
<tr>
<th></th>
<th>Calls</th>
<th></th>
<th>Puts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RC+ CEIV</td>
<td>Proposed Method</td>
<td>RC+ CEIV</td>
<td>Proposed Method</td>
</tr>
<tr>
<td>A: (\beta_S) as independent variable</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-21.66</td>
<td>-14.82</td>
<td>1.15</td>
<td>-13.54</td>
</tr>
<tr>
<td></td>
<td>(-70.10)</td>
<td>(-24.69)</td>
<td>(0.19)</td>
<td>(-57.96)</td>
</tr>
<tr>
<td>(\beta_S)</td>
<td>3.58</td>
<td>1.51</td>
<td>-0.10</td>
<td>-2.67</td>
</tr>
<tr>
<td></td>
<td>(85.30)</td>
<td>(18.05)</td>
<td>(-0.17)</td>
<td>(-74.03)</td>
</tr>
<tr>
<td>B: (\beta_{\sigma}) as independent variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-10.11</td>
<td>-9.17</td>
<td>0.58</td>
<td>-6.92</td>
</tr>
<tr>
<td></td>
<td>(-65.94)</td>
<td>(-40.02)</td>
<td>(0.17)</td>
<td>(-52.98)</td>
</tr>
<tr>
<td>(\beta_{\sigma})</td>
<td>27.48</td>
<td>10.72</td>
<td>-0.55</td>
<td>19.66</td>
</tr>
<tr>
<td></td>
<td>(94.19)</td>
<td>(21.36)</td>
<td>(-0.16)</td>
<td>(82.39)</td>
</tr>
<tr>
<td>C: (\beta_S) and (\beta_{\sigma}) as independent variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-5.42</td>
<td>-9.84</td>
<td>1.28</td>
<td>-6.02</td>
</tr>
<tr>
<td></td>
<td>(-14.27)</td>
<td>(-14.57)</td>
<td>(0.22)</td>
<td>(-22.71)</td>
</tr>
<tr>
<td>(\beta_S)</td>
<td>32.58</td>
<td>9.99</td>
<td>0.32</td>
<td>20.88</td>
</tr>
<tr>
<td></td>
<td>(64.03)</td>
<td>(18.24)</td>
<td>(0.43)</td>
<td>(53.21)</td>
</tr>
<tr>
<td>(\beta_{\sigma})</td>
<td>-0.92</td>
<td>0.13</td>
<td>-0.15</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(-13.16)</td>
<td>(1.20)</td>
<td>(-0.28)</td>
<td>(3.85)</td>
</tr>
</tbody>
</table>
Table 6: **Equal weighted and bias adjusted mean returns of options on the S&P 500 index.** Panels A and B present estimated average returns of calls and puts sorted by their *betas* with respect to the underlying price ($\beta_S$). Panels C and D present estimated average returns of delta-hedged portfolios of calls and puts sorted by their *betas* with respect to the underlying volatility ($\beta_{\sigma}$). For each day, we sort the options in the S&P 500 index by their $\beta$s into five groups. The row labeled ‘unadjusted’ displays the time-series mean of the daily equally-weighted average return. The row labeled ‘bias adjusted’ displays the time-series mean of the daily average return adjusted for the MR and CEIV biases using the methods described in the paper. H-L is the average return of the high-$\beta$ group minus the average return of the low-$\beta$ group. ALL is the average return for all options in the panel. The standard error for the average return is calculated with Newey-West standard errors up to five lags. All returns are in basis points. We present the t-statistics in parentheses.

### A: Call Options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>18.83(2.17)</td>
<td>29.26(2.14)</td>
<td>37.13(1.85)</td>
<td>30.74(1.05)</td>
<td>12.14(0.26)</td>
<td>-5.55(-0.14)</td>
<td>25.37(1.08)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>15.23(1.71)</td>
<td>27.02(1.96)</td>
<td>32.37(1.61)</td>
<td>16.47(0.57)</td>
<td>1.54(0.03)</td>
<td>-12.13(-0.31)</td>
<td>18.54(0.80)</td>
</tr>
</tbody>
</table>

### B: Put options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>-92.95(-1.54)</td>
<td>-56.03(-1.45)</td>
<td>-29.78(-0.99)</td>
<td>-16.77(-0.73)</td>
<td>-6.12(-0.37)</td>
<td>85.61(1.88)</td>
<td>-40.11(-1.22)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>-109.14(-1.90)</td>
<td>-51.34(-1.32)</td>
<td>-31.13(-1.06)</td>
<td>-9.00(-0.36)</td>
<td>-11.76(-0.64)</td>
<td>96.16(2.28)</td>
<td>-45.15(-1.38)</td>
</tr>
</tbody>
</table>

### C: Delta-hedged Calls

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>-1.69(-1.30)</td>
<td>-7.18(-2.92)</td>
<td>-18.78(-3.83)</td>
<td>-55.13(-4.87)</td>
<td>-142.84(-4.95)</td>
<td>-141.13(-5.00)</td>
<td>-45.43(-2.90)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>-3.86(-2.92)</td>
<td>-5.16(-0.98)</td>
<td>-36.08(-3.65)</td>
<td>-221.80(-4.89)</td>
<td>-516.21(-4.24)</td>
<td>-512.28(-4.22)</td>
<td>-148.35(-4.78)</td>
</tr>
</tbody>
</table>

### D: Delta-hedged Puts

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>3.03(0.85)</td>
<td>-5.80(-1.05)</td>
<td>-24.11(-2.76)</td>
<td>-50.63(-3.41)</td>
<td>-86.65(-3.07)</td>
<td>-89.82(-3.43)</td>
<td>-33.15(-2.79)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>-3.95(-0.77)</td>
<td>-11.15(-1.69)</td>
<td>-24.94(-2.55)</td>
<td>-45.58(-2.85)</td>
<td>-183.98(-4.90)</td>
<td>-180.17(-4.30)</td>
<td>-54.05(-3.69)</td>
</tr>
</tbody>
</table>
Table 7: **Equal weighted and bias adjusted mean returns of straddles on the S&P 500 index and on stocks in the S&P 500 index.** This table presents estimated average returns of straddles. Panel A presents the results for straddles on the S&P 500 index and Panel B presents the results for straddles on stocks in the S&P 500 index. For each day, we sort the straddles by their moneyness into five groups. The row labeled ‘unadjusted’ displays the time-series mean of the daily equally-weighted average return. The row labeled ‘bias adjusted’ displays the time-series mean of the daily average return adjusted for the MR and CEIV biases using the method described in the paper. ALL is the average return of all the straddles in each panel. Returns are displayed in basis points. The standard error for the average return is calculated using Newey-West standard errors up to five lags. We present the t-statistics in parentheses.

### A: Straddles on S&P500 index

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>-19.94(-4.24)</td>
<td>-29.32(-5.19)</td>
<td>-27.68(-4.67)</td>
<td>-27.41(-4.42)</td>
<td>-25.05(-4.37)</td>
<td>-25.90(-4.78)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>-14.61(-1.45)</td>
<td>-33.26(-4.62)</td>
<td>-32.98(-4.93)</td>
<td>-36.19(-5.18)</td>
<td>-31.39(-4.50)</td>
<td>-29.80(-5.12)</td>
</tr>
</tbody>
</table>

### B: Straddles on stocks in the S&P500 index

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unadjusted</td>
<td>-16.78(-6.62)</td>
<td>-10.10(-3.34)</td>
<td>-2.77(-0.83)</td>
<td>-3.29(-1.01)</td>
<td>-10.84(-4.05)</td>
<td>-8.75(-3.10)</td>
</tr>
<tr>
<td>Biased adjusted</td>
<td>-25.81(-9.59)</td>
<td>-14.70(-4.71)</td>
<td>-4.70(-1.37)</td>
<td>-5.82(-1.72)</td>
<td>-17.16(-6.34)</td>
<td>-13.63(-4.70)</td>
</tr>
</tbody>
</table>
Table 8: **Equal weighted and bias adjusted mean returns of options on individual stocks in the S&P500 index.** Panels A and B present estimated average returns of calls and puts sorted by their $\beta$ with respect to the underlying ($\beta_S$). Panels C and D present estimated average returns of delta-hedged portfolios of calls and puts sorted by their $\beta$ with respect to underlying volatility ($\beta_\sigma$). For each day, we sort the options on individual stocks in the S&P 500 index by their $\beta$s into five groups. The row labeled ‘unadjusted’ displays the time-series mean of the daily equally-weighted average return. The row labeled ‘bias adjusted’ displays the time-series mean of the daily average return adjusted for the MR and CEIV biases using the methods described in the paper. H-L is the average return of the high-$\beta$ group minus the average return of the low-$\beta$ group. The standard error for the average return is calculated with Newey-West standard errors up to five lags. ALL is the average return for all the options in the sample. All returns are in basis points. We present the t-statistics in parentheses.

### A: Call Options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unadjusted</strong></td>
<td>-5.89(-0.99)</td>
<td>11.35(1.36)</td>
<td>25.29(2.32)</td>
<td>40.60(2.92)</td>
<td>92.35(4.68)</td>
<td>98.23(6.31)</td>
<td>32.75(2.87)</td>
</tr>
<tr>
<td><strong>Biased adjusted</strong></td>
<td>-2.53(-0.43)</td>
<td>12.89(1.53)</td>
<td>27.53(2.51)</td>
<td>42.46(3.01)</td>
<td>52.49(2.66)</td>
<td>55.02(3.53)</td>
<td>27.20(2.37)</td>
</tr>
</tbody>
</table>

### B: Put options

<table>
<thead>
<tr>
<th>$\beta_S$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unadjusted weighted</strong></td>
<td>-55.40(-2.24)</td>
<td>-50.55(-3.08)</td>
<td>-41.41(-3.23)</td>
<td>-32.26(-3.33)</td>
<td>-24.68(-4.20)</td>
<td>30.72(1.52)</td>
<td>-40.87(-3.00)</td>
</tr>
<tr>
<td><strong>Biased adjusted</strong></td>
<td>-107.89(-4.40)</td>
<td>-58.34(-3.55)</td>
<td>-40.17(-3.14)</td>
<td>-27.75(-2.80)</td>
<td>-22.12(-3.60)</td>
<td>85.77(4.33)</td>
<td>-49.95(-3.67)</td>
</tr>
</tbody>
</table>

### C: Delta-hedged Calls

<table>
<thead>
<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unadjusted</strong></td>
<td>6.69(14.55)</td>
<td>-6.88(-5.47)</td>
<td>-7.04(-2.49)</td>
<td>-12.02(-1.91)</td>
<td>19.58(1.43)</td>
<td>12.89(0.96)</td>
<td>0.07(0.01)</td>
</tr>
<tr>
<td><strong>Biased adjusted</strong></td>
<td>-3.62(-7.70)</td>
<td>-2.61(-2.10)</td>
<td>-5.13(-1.43)</td>
<td>-6.20(-0.66)</td>
<td>-32.65(-2.06)</td>
<td>-29.04(-1.85)</td>
<td>-9.30(-1.60)</td>
</tr>
</tbody>
</table>

### D: Delta-hedged Puts

<table>
<thead>
<tr>
<th>$\beta_\sigma$</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>H-L</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unadjusted</strong></td>
<td>6.04(11.49)</td>
<td>-8.54(-6.18)</td>
<td>-10.65(-3.31)</td>
<td>-36.39(-6.11)</td>
<td>-3.35(-0.37)</td>
<td>-9.39(-1.06)</td>
<td>-10.57(-2.77)</td>
</tr>
<tr>
<td><strong>Biased adjusted</strong></td>
<td>-3.85(-6.57)</td>
<td>-3.22(-2.13)</td>
<td>-6.63(-1.81)</td>
<td>-20.32(-3.21)</td>
<td>-66.78(-7.23)</td>
<td>-62.93(-6.95)</td>
<td>-18.45(-4.70)</td>
</tr>
</tbody>
</table>
Table 9: OLS and bias-adjusted Fama-MacBeth regressions of returns of delta-hedged options written on S&P 500 stocks. This table presents the results of Fama-MacBeth regressions of delta-hedged call and put returns on the underlying and volatility $\beta_s$. The options are written on individual stocks in the S&P 500 index. Panel A displays the results with $\beta_s$ as independent variable. The columns labeled ‘Call’ display the results for a sample that includes only call options. The columns labeled ‘Put’ display the results for a sample that includes only put options. The columns labeled ‘All Options’ display the results for a sample that includes all put and call options. The columns labeled ‘OLS’ show the estimated coefficients and t-statistics of OLS regressions. The columns labeled ‘Bias Adj.’ show the the estimated coefficients and t-statistics of using the method presented in the paper to adjust for the RC and CEIV biases. The standard errors are calculated using the Fama-MacBeth approach with Newey-West standard errors up to five lags. We present the t-statistics in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
<th>All Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Bias Adj.</td>
<td>OLS</td>
</tr>
<tr>
<td>Intercept</td>
<td>-9.65</td>
<td>7.64</td>
<td>-3.87</td>
</tr>
<tr>
<td></td>
<td>(-5.35)</td>
<td>(2.76)</td>
<td>(-1.57)</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>2.61</td>
<td>-11.07</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(2.21)</td>
<td>(-4.98)</td>
<td>(0.88)</td>
</tr>
</tbody>
</table>

A: $\beta_s$ as independent variable

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
<th>All Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Bias Adj.</td>
<td>OLS</td>
</tr>
<tr>
<td>Intercept</td>
<td>-22.75</td>
<td>7.61</td>
<td>-11.97</td>
</tr>
<tr>
<td></td>
<td>(-6.34)</td>
<td>(1.45)</td>
<td>(-3.54)</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>2.77</td>
<td>0.17</td>
<td>-1.53</td>
</tr>
<tr>
<td></td>
<td>(4.41)</td>
<td>(0.14)</td>
<td>(10.14)</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
<td>-0.28</td>
<td>-11.48</td>
<td>-0.58</td>
</tr>
<tr>
<td></td>
<td>(-0.33)</td>
<td>(-3.30)</td>
<td>(-0.66)</td>
</tr>
</tbody>
</table>

B: $\beta_s$ and $\beta_\sigma$ as independent variables
Table 10: **OLS and bias-adjusted Fama-MacBeth regression of delta-hedged options written on the S&P 500 index.** This table presents the results of Fama-MacBeth regressions of delta-hedged call and put returns on the underlying and volatility $\beta_s$. The options are written on the S&P 500 index. Panel A displays the results with $\beta_S$ as independent variable. The columns labeled ‘Call’ display the results for a sample that includes only call options. The columns labeled ‘Put’ display the results for a sample that includes only put options. The columns labeled ‘All Options’ display the results for a sample that includes all put and call options. The columns labeled ‘OLS’ show the estimated coefficients and t-statistics of OLS regressions. The columns labeled ‘Bias Adj.’ show the estimated coefficients and t-statistics of using the method presented in the paper to adjust for the RC and CEIV biases. The standard errors are calculated using the Fama-MacBeth approach with Newey-West standard errors up to five lags. We present the t-statistics in parentheses.

### A: $\beta_\sigma$ as independent variable

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
<th>All Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Bias Adj.</td>
<td>OLS</td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.62</td>
<td>56.28</td>
<td>10.20</td>
</tr>
<tr>
<td></td>
<td>(-0.78)</td>
<td>(3.15)</td>
<td>(1.04)</td>
</tr>
<tr>
<td></td>
<td>(-2.89)</td>
<td>(-3.29)</td>
<td>(-3.56)</td>
</tr>
</tbody>
</table>

### B: $\beta_S$ and $\beta_\sigma$ as independent variables

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
<th>All Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Bias Adj.</td>
<td>OLS</td>
</tr>
<tr>
<td>Intercept</td>
<td>27.78</td>
<td>12.72</td>
<td>49.58</td>
</tr>
<tr>
<td></td>
<td>(4.20)</td>
<td>(0.53)</td>
<td>(2.83)</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-3.58</td>
<td>6.28</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>(-4.32)</td>
<td>(1.73)</td>
<td>(4.47)</td>
</tr>
<tr>
<td>$\beta_\sigma$</td>
<td>0.21</td>
<td>-27.00</td>
<td>-4.75</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(-3.06)</td>
<td>(-3.15)</td>
</tr>
</tbody>
</table>
Internet Appendix: Very Noisy Option Prices and Inferences Regarding Option Returns

Jefferson Duarte, Christopher S. Jones, and Junbo L. Wang*

May 2019

*Duarte is with the Jesse H. Jones School of Business at Rice University (jefferson.duarte@rice.edu). Jones is with the Marshall School of Business at University of Southern California (christopher.jones@marshall.usc.edu). Wang is with the Ourso College of Business at Louisiana State University (junbowang@lsu.edu).
Appendix A  Bias in expected observed delta-hedged call and straddle returns

We make the following assumptions to examine the bias in delta-hedged call and straddle returns:

**Assumption 1** Errors in prices are independent in time-series, among all portfolio i’s and across instruments. In addition, the errors are independent of true stock prices.

**Assumption 2** Assume that $\Delta \tilde{C}_{t+1} = C_{t+1} - C_t \to 0$, $\Delta \tilde{P}_{t+1} = P_{t+1} - P_t \to 0$ and that $\Delta \tilde{S}_{t+1} = S_{t+1} - S_t \to 0$ as $\Delta t \to 0$, i.e., time period is short so the change in stock and option prices is small.

### A.1 Delta-hedged return

The observed return for a delta-hedged call is expressed as

$$R_{t+1} = \frac{C_{t+1} - C_t}{C_t} - \Delta^C(S_t, C_t) \frac{S_t S_{t+1} - S_t}{S_t} \quad (1)$$

Let $\tilde{S}_t$ be the true price of the stock and $\epsilon_{S,t}$ be the relative price error, i.e., $S_t = \tilde{S}_t(1 + \epsilon_{S,t})$. Analogously, let $C_t = \tilde{C}_t(1 + \epsilon_{S,t})$, and let $\tilde{R}_{t+1}$ be the actual delta-hedged return. Following the same derivation as in Blume and Stambaugh (1983) for stock returns,

$$E[\frac{C_{t+1} - C_t}{C_t}] = E[\frac{\tilde{C}_{t+1} - \tilde{C}_t}{\tilde{C}_t}] + E[\epsilon_{C,t}^2]. \quad (2)$$

Hence, the expected value of the first term of the delta-hedged return in equation 1 is a biased estimate of the actual expected return of the call option.

To deal with the second term on the right-hand side of equation 1, recall that $\beta^C(S_t, C_t) = \Delta^C(S_t, C_t) \frac{\tilde{S}_t}{\tilde{C}_t}$. Define

$$f(S_t, C_t, S_{t+1}) = \beta^C(S_t, C_t) \frac{S_{t+1} - S_t}{S_t}, \quad (3)$$

its Taylor expansion up to the second order can be written as:

$$f(S_t, C_t, S_{t+1}) = f(\tilde{S}_t, \tilde{C}_t, \tilde{S}_t) + \frac{\partial f}{\partial S_t} \tilde{S}_t \epsilon_{S,t} + \frac{\partial f}{\partial C_t} \tilde{C}_t \epsilon_{C,t} + \frac{1}{2} \frac{\partial^2 f \tilde{S}_t^2}{\partial \epsilon_{S,t}^2} \epsilon_{S,t}^2$$

$$+ 1 \frac{\partial^2 f}{\partial C_t^2} (\tilde{C}_t \epsilon_{C,t})^2 + \frac{\partial^2 f}{\partial S_t \partial C_t} \epsilon_{S,t} \epsilon_{C,t} + \frac{\partial^2 f}{\partial S_t \partial S_{t+1}} \epsilon_{S,t} \epsilon_{S,t+1} + o(\epsilon_{S,t}^2) + o(\epsilon_{C,t}^2) \quad (4)$$
Taking expectations in the equation above, noticing that \( E[\epsilon_{S,t}], E[\epsilon_{C,t}], E[\epsilon_{C,t}] \) are zero, together with Assumption 2, we obtain

\[
E[f(S_t, C_t, S_{t+1})] = E[f(\tilde{S}_t, \tilde{C}_t, \tilde{S}_{t+1})] + E\left[ \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \tilde{S}_t \epsilon_{S,t}^2 \right] + E\left[ \frac{1}{2} \frac{\partial^2 f}{\partial C^2} \tilde{C}_t \epsilon_{C,t}^2 \right] + o(\epsilon_{S,t}^2) + o(\epsilon_{C,t}^2).
\]

(5)

Substituting \( f(S_t, C_t, S_{t+1}) \) for its definition (equation 3) and taking derivatives, we arrive at the following:

\[
E[f(S_t, C_t, S_{t+1})] = \beta^C(\tilde{S}_t, \tilde{C}_t)(\mu_S + E[\tilde{S}_{t+1}]E[\epsilon_{S,t}^2]) - \frac{\partial \beta^C}{\partial S}(\tilde{S}_t, \tilde{C}_t)E[S_{t+1}]E[\epsilon_{S,t}^2] + \frac{\mu_s}{2} (\frac{\partial^2 \beta^C}{\partial S^2}(\tilde{S}_t, \tilde{C}_t)S_t^2 E[\epsilon_{S,t}^2] + \frac{\partial^2 \beta^C}{\partial C^2}(\tilde{S}_t, \tilde{C}_t)C_t^2 E[\epsilon_{C,t}^2]) + o(\epsilon_{S,t}^2) + o(\epsilon_{C,t}^2),
\]

(6)

where \( \mu_s = E[\tilde{S}_{t+1}/S_t] \). Under Assumption 2, \( \mu_s \to 0 \), \( E[S_{t+1}] \to S_t \) and \( E[\tilde{S}_{t+1}/S_t] \to 1 \) as \( \Delta t \to 0 \). Hence, Equations 2 and 6 imply that the bias in the delta-hedged return \( (E[R_{t+1}] - E[\tilde{R}_{t+1}]) \) is approximately:

\[
E[\epsilon_{C,t}^2] - \beta^C(\tilde{S}_t, \tilde{C}_t)E[\epsilon_{S,t}^2] + \frac{\partial \beta^C}{\partial S}(\tilde{S}_t, \tilde{C}_t)S_t E[\epsilon_{S,t}^2]
\]

(7)

The bias for delta-hedged puts is similar.

### A.2 Straddle return

The straddle returns can be written as

\[
R_{t+1} = v_t^C \frac{C_{t+1} - C_t}{C_t} + v_t^P \frac{P_{t+1} - P_t}{P_t}.
\]

(8)

Following the same procedure as in the case of delta-hedged calls, we expand the first part in equation 8 to get:

\[
v_t^C \frac{C_{t+1} - C_t}{C_t} = v_t^C \frac{\tilde{C}_{t+1} - \tilde{C}_t}{\tilde{C}_t} + \frac{\partial v_t^C}{\partial S} \epsilon_{S,t} + \frac{\partial v_t^C}{\partial C} \epsilon_{C,t} + \frac{\partial v_t^C}{\partial P} \epsilon_{P,t} + \frac{1}{2} \frac{\partial^2 v_t^C}{\partial S^2} \epsilon_{S,t}^2 + \frac{\partial^2 v_t^C}{\partial C^2} \epsilon_{C,t}^2 + \frac{\partial^2 v_t^C}{\partial P^2} \epsilon_{P,t}^2 + 2(\frac{\partial^2 v_t^C}{\partial S \partial C} \epsilon_{S,t} \epsilon_{C,t} + \frac{\partial^2 v_t^C}{\partial S \partial P} \epsilon_{S,t} \epsilon_{P,t} + \frac{\partial^2 v_t^C}{\partial C \partial P} \epsilon_{C,t} \epsilon_{P,t})
\]

\[
+ \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \left( \frac{\partial v_t^C}{\partial S} \epsilon_{S,t} + \frac{\partial v_t^C}{\partial C} \epsilon_{C,t} + \frac{\partial v_t^C}{\partial P} \epsilon_{P,t} \right) \epsilon_{C,t} + v_t^C \frac{\tilde{C}_{t+1}}{\tilde{C}_t}((1 + \epsilon_{C,t}^2)(1 - \epsilon_{C,t}^2) - 1).
\]

Given Assumption 1 and Assumption 2, the above equation converges to

\[
v_t^C E[\frac{\tilde{C}_{t+1} - \tilde{C}_t}{\tilde{C}_t}] + v_t^C E[\epsilon_{C,t}^2] + \frac{\partial v_t^C}{\partial C} E[\epsilon_{C,t}^2] + o(E[\epsilon_{S,t}^2]) + o(E[\epsilon_{C,t}^2]) + o(E[\epsilon_{P,t}^2]).
\]
Similarly, we can show that the put part can converge to
\[ v_t^P E[\tilde{P}_{t+1} - \tilde{P}_t] + v_t^P E[\epsilon_{P,t}^2] + \frac{\partial v_t^P}{\partial P} E[\epsilon_{P,t}^2] + o(E[\epsilon_{S,t}^2]) + o(E[\epsilon_{C,t}^2]) + o(E[\epsilon_{P,t}^2]) \, . \]

Combining the call and put parts together, we obtain
\[ R_{t+1} = v_t^C E[\tilde{C}_{t+1} - \tilde{C}_t] + v_t^P E[\tilde{P}_{t+1} - \tilde{P}_t] + v_t^C E[\epsilon_{C,t}^2] + \frac{\partial v_t^C}{\partial C} E[\epsilon_{C,t}^2] + v_t^P E[\epsilon_{P,t}^2] + \frac{\partial v_t^P}{\partial P} E[\epsilon_{P,t}^2] + o(E[\epsilon_{S,t}^2]) + o(E[\epsilon_{C,t}^2]) + o(E[\epsilon_{P,t}^2]) \, . \]  

Equation 9 implies that the sources of biases stem from \( v_t^C E[\epsilon_{C,t}^2] \) and \( v_t^P E[\epsilon_{P,t}^2] \) (which are the biases in option returns), as well as from \( \frac{\partial v_t^C}{\partial C} E[\epsilon_{C,t}^2] \) and \( \frac{\partial v_t^P}{\partial P} E[\epsilon_{P,t}^2] \) (which are the biases arising from the correlation between errors in straddle weights and option returns).

Appendix B  Data filters

As is standard in the literature, we filter observations that violate arbitrage conditions or that appear to be “stub” quotes or data errors. For American options on individual equities, the absence of arbitrage requires that the price of the option be no less than the current exercise value. For index calls and calls on individual equities, the option price must be less than the current underlying price. For index puts and puts on individual equities, the option price must be less than the strike price. We also eliminate apparent stub (i.e., non-competitive) quotes for which the bid-ask spread is more than $10 or more than the price of the underlying stock, which may alternatively indicate undocumented missing value codes (e.g., 999) or data errors. We further discard observations in which the bid is equal to or exceeds the ask. Finally, we eliminate large reversals in option returns (2000% followed by -95% or vice versa) under the assumption that they most likely represent data errors.

We note that about 3% of the options that remain are missing an implied volatility. To retain potentially valid data, we fill in these missing values with those of similar contracts. If a call’s implied volatility is missing, we use the implied volatility of the put with the same underlying, strike price, and expiration date. If the put’s implied volatility is also missing, we fill in the call’s implied volatility with its value on the previous day.

We use TAQ data to calculate underlying stock effective spread. TAQ trade data are also filtered by excluding observations with zero price or zero size, eliminating corrected
orders, and dropping trades with a trade condition in \{B, G, J, K, L, O, T, W, Z\}. For transactions with identical time stamps, we compute volume-weighted average prices with Quote observations eliminated if bid, ask, or bid/ask size is missing, if the ask exceeds the bid, if the bid-ask spread is greater than half of the quote midpoint, or or if the quote condition is in \{4, 7, 9, 11, 13, 14, 15, 19, 20, 27, 28\}.

**Appendix C  A model of bid-ask spreads and simulations for measurement errors in prices**

The simulation of price errors requires a model of relative bid-ask spreads. Our model is calibrated to match the spreads of the stocks and options traded on S&P 500 member firms.

For the underlying stocks, we choose spreads to match the mean and the standard deviation of the cross-section distribution of the effective bid-ask spreads across S&P 500 stocks. We assume that effective spreads is cross-sectionally lognormally distributed. We estimate the mean and standard deviation by computing the empirical moments of the logarithm of the effective spread measure described in the paper. In the simulations, we draw the spread for stock \(i\) from this log-normal distribution. Log stock spread Mean is -6.5674 Standard deviation 0.7308

Spreads on options are chosen to match a number of patterns we observe in the data, namely that option spreads are higher when the underlying stock has a higher spread, when the option is further out of the money, and when the option has a shorter time until expiration. Specifically, we assume that the log of the relative option spreads are normally distributed with mean is determined by the following equation:

\[
\ln \left( \text{option spread}_{i,j,t} \right) = \beta_0 + \beta_1 \times \ln \left( \text{stock spread}_{i,t} \right) + \beta_2 \times \text{number-std}_{i,j,t} + \beta_3 \times |\text{number-std}_{i,j,t}| + \beta_4 \times \sqrt{\tau_{i,j,t}} + \beta_5 \times \sqrt{\tau_{i,j,t}} \times \text{number-std}_{i,j,t} + \beta_6 \times \sqrt{\tau_{i,j,t}} \times |\text{number-std}_{i,j,t}| + \epsilon_i + \epsilon_t + \epsilon_{i,j,t} \tag{10}
\]

where number-std is as the number of standard deviations separating the present values of the strike price and future stock price (current stock price minus dividends), \(\tau_{i,j,t}\) is the time-
to-maturity (in business days) of option \(j\) written on stock \(i\) at time \(t\), \(mbox{stockspread}_{i,t}\) is the spread of the underlying stock \(i\) at time \(t\).\(^1\) We assume that the coefficients \((\beta s)\) are different for puts and calls.

There are at least two notable characteristics of the mean options model in equation 10. First, we specify the mean equation as an error components model. We do so to capture the fact that option bid-ask spreads vary significantly across firms and through time. Second, we also account for nonlinearities in the relation between number-std, time-to-maturity \((\tau_{i,j,t})\) and option relative spreads. We estimate Equation 10 using the Fama-Macbeth regression. We estimate this model using all options written on S&P 500 member stocks that pass our filters, have number-std values between -4 and +3, and have between 10 days and four months (88 days) remaining until expiration. This range of maturities and number-std is sufficient for our Monte Carlo simulations, and it also excludes levels of maturity and number-std that are less frequently observed in the data and prone to outliers. Given the full-sample estimates of the regression coefficients in Equation 10, we compute the regression residuals as

\[
\hat{e}_{i,j,t} = \ln \left( \text{option spread}_{i,j,t} \right) - \text{fitted value}_{i,j,t},
\]

which are estimates of the sum of the firm, time, and idiosyncratic error terms, \(\epsilon_i + \epsilon_t + \epsilon_{i,j,t}\).

We assume that the firm and time components, \(\epsilon_i\) and \(\epsilon_t\), are independent normal draws with zero means and constant variances. Furthermore, we assume that the firm and time components are identical for calls and puts. We therefore estimate the variances of these terms by computing the sample variances of \(\hat{e}_i\) and \(\hat{e}_t\), where \(\hat{e}_i\) is the average of all residuals \(\hat{e}_{i,j,t}\) for all options on firm \(i\), and \(\hat{e}_t\) is the average of all residuals for all options at time \(t\).

Variances of the idiosyncratic errors \(\epsilon_{i,j,t}\) are assumed to depend on the maturity and number-std of the contract as well as the spread of the underlying stock. Given estimates of the firm and time component, we can compute idiosyncratic residuals as

\[
\hat{e}_{i,j,t} = \hat{e}_{i,j,t} - \hat{e}_t - \hat{e}_i.
\]

These residuals are used to compute the dependent variable in the following variance equa-

---

\(^1\)We calculate number-std as \((\ln \text{strike}_{i,j,t} - \ln S_{i,t} + (q_{i,t} - r_{f,t})\tau_{i,j,t}/252) / \text{ATM implied volatility}_{i,t}\). Here, \(S_{i,t}\) is the closing price of stock \(i\) at time \(t\), ATM implied volatility is the volatility of the ATM option written on stock \(i\) with time to maturity \(\tau\), \(q\) is the annualized dividend yield, and \(r\) is the risk-free interest rate.
tion, which is also estimated via Fama-Macbeth:

$$
\epsilon^2_{i,j,t} = \beta'_{0} + \beta'_{1} \times \ln(\text{stock spread}_{i,t}) + \beta'_{2} \times \text{number-std}_{i,j,t} + \beta'_{3} \times |\text{number-std}_{i,j,t}| + \beta'_{4} \times \sqrt{\tau_{i,j,t}} + \beta'_{5} \times \sqrt{\tau_{i,j,t}} \times \text{number-std}_{i,j,t} + \beta'_{6} \times \sqrt{\tau_{i,j,t}} \times |\text{number-std}_{i,j,t}| + \epsilon'_{i,j,t}
$$

(11)

The results of estimation of Regressions 10 and 11 are presented in Table 1. We use these results in the following steps of the Monte-Carlo simulation in the paper.

1. Draw one log relative stock spread for each simulated firm from a normal distribution.
   Calculate the stock spread following the log normal distribution assumption.

2. Draw each idiosyncratic error $\epsilon_{i,j,t}$ from a normal distribution with a zero mean and a standard deviation implied by the evaluation of the variance equation above.

3. Draw each firm error component, $\epsilon_{i}$, from a normal distribution with a zero mean and a constant standard deviation.

4. Compute the log option spread as the sum of the conditional mean (Equation 10) and the error components ($\epsilon_{i} + \epsilon_{i,j,t}$). \(^2\)

Once we have the bid-ask spread for each option and stock in our simulated sample, we simulate the measurement errors in stock and option prices. Specifically, we assume that, at each time $t$ in the simulated sample, we observe $S = \tilde{S}(1 + \epsilon^{S})$, $C = \tilde{C}(1 + \epsilon^{C})$, $P = \tilde{P}(1 + \epsilon^{P})$ where $\tilde{S}$, $\tilde{C}$, and $\tilde{P}$ are true stock, call and put prices, and $S$, $C$, and $P$ are observed prices. All measurement errors are drawn from symmetric triangular distributions, which are bounded distributions with probability density functions that are piecewise linear, increasing below the median and decreasing above it, reaching zero at either bound. This choice of density reflects a view that price errors are likely bounded by the size of the bid-ask spread. By choosing lower and upper bounds equal to $-1/2$ or $+1/2$ times the relative bid-ask spread, we ensure that the difference between observed prices and true prices is never larger than the spread, with differences closer to zero more likely than those further away.

\(^2\)We only simulate cross-sectional returns, the time component ($\epsilon_{t}$) is not simulated.
Let $\epsilon^k$ with $k = S, P$ or $C$ be the triangular distributed relative pricing error for the underlying stock, call or put. By construction, $\epsilon^k$ is bounded between $-\text{Spread}^k/2$ and $\text{Spread}^k/2$ where $\text{Spread}^k$ is the relative spread generated in the simulation using the steps described above. To generate $\epsilon^k$, we generate a random number $\text{rand}^k$ from a standard uniform distribution and use the expression:

$$
\epsilon^k = \begin{cases} 
\frac{\text{Spread}^k}{2} \times (\sqrt{2} \times \text{rand}^k - 1) & \text{if } \text{rand}^k \leq 0.5 \\
\frac{\text{Spread}^k}{2} \times (1 - \sqrt{2} \times (1 - \text{rand}^k)) & \text{if } \text{rand}^k > 0.5
\end{cases}
$$

(12)

Appendix D  The proposed method to adjust biases in portfolios of option returns

In this section, we show that the bias-adjusted portfolio return proposed in the paper converges to zero when the number of portfolios, $N$, converges to infinity and time period ($\Delta t$) converges to zero. Before the proof, we generalize Assumption 2 as follows:

**Assumption 2** Assume that $\Delta \tilde{X}_{k,t+1} = \tilde{X}_{k,t+1} - \tilde{X}_{k,t} \to 0$ as $\Delta t \to 0$.

To prove the claim above, note that

$$
\frac{\sum_{i=1}^{N} \bar{f}_{X}^i \Delta X_{k,t+1}^i + 1}{\sum_{i=1}^{N} g_{i,t-1}} = \frac{\sum_{k=1}^{M} \bar{f}_{X}^k \Delta X_{k,t+1}^k + 1}{\sum_{k=1}^{M} g_{k,t-1}}
$$

For each $k$, it is clear that

$$
\frac{1}{N} \sum_{i=1}^{N} \frac{\bar{f}_{X}^k \Delta X_{k,t+1}^i}{g_{i,t-1}} \to E(\bar{f}_{X}^k) E(\Delta X_{k,t+1}^i) + \text{cov}(\frac{\bar{f}_{X}^k}{g_{i,t-1}}, \Delta X_{k,t+1}^i).
$$

When $\Delta t \to 0$, $E(\Delta X_{k,t+1}^i) = E(\Delta \tilde{X}_{k,t+1}^i) \to 0$ (From Assumption 1 and Assumption 2). In addition, $\frac{\bar{f}_{X}^k}{g_{i,t-1}}$ only depends on errors of $X_{k,t}$ and errors of $X_{k,t-1}$, but not on errors of $X_{k,t}$ and $X_{k,t+1}$. From assumption 1, as $\Delta t \to 0$

$$
cov(\frac{\bar{f}_{X}^k}{g_{i,t-1}}, \Delta X_{k,t+1}^i) = \text{cov}(\frac{\bar{f}_{X}^k}{g_{i,t-1}}, \Delta \tilde{X}_{k,t+1}^i) \to 0.
$$

Therefore,

$$
\frac{1}{N} \sum_{i=1}^{N} \frac{\bar{f}_{X}^k \Delta X_{k,t+1}^i}{g_{i,t-1}} \to 0.
$$

7
Moreover, \( \frac{1}{N} \sum_{i=1}^{N} g_i \) will not be zero. Thus,

\[
\frac{\sum_{i=1}^{N} u_i \tilde{P}_i}{\sum_{i=1}^{N} u_i} \to 0.
\]

Note that the bias is not zero when true price changes is nonzero. To be more specific, for asset \( k \), the bias-adjusted return is

\[
\frac{1}{N} \sum_{i=1}^{N} f_{X_k} \frac{\Delta X_{k,t+1}}{g_l}.
\]

Rewrite \( \frac{f_{X_k} \Delta X_{k,t+1}}{g_l} = F(X_{k,t}^i, X_{k,t-1}^i) \Delta X_{k,t+1}^i \). The second-order Taylor series expansion of the function \( F(X_{k,t}^i, X_{k,t-1}^i) \) at the true asset prices can be written as

\[
F(X_{k,t}^i, X_{k,t-1}^i) = F(\hat{X}_{k,t}^i, \hat{X}_{k,t-1}^i) + V(\hat{X}_{k,t}^i, \hat{X}_{k,t-1}^i)\epsilon_{t-1,t} + (\epsilon_{X}^i) M(\hat{X}_{k,t}^i, \hat{X}_{k,t-1}^i)\epsilon_{t-1,t}.
\]

Here \( \epsilon_{X}^i = [\epsilon_{X_{1,t}^i}, \ldots, \epsilon_{X_{N,t}^i}, \epsilon_{X_{1,t-1}^i}, \ldots, \epsilon_{X_{N,t-1}^i}, \epsilon_{X_{k+1,t}^i}, \ldots, \epsilon_{X_{N,t}^i}] \), a column vector that contains all the pricing errors for time \( t-1 \), and all but \( k \)th pricing error for time \( t \). \( V \) and \( M \) are the vector and matrix of first- and second-order derivatives of function \( F \) with respect to its elements. In comparison, \( \Delta X_{k,t+1} = \Delta \hat{X}_{k,t+1} + \epsilon_{t+1} - \epsilon_{X_{k,t}}. \) From Assumption 1,

\[
\frac{1}{N} \sum_{i=1}^{N} (F(X_{k,t}^i, X_{k,t-1}^i) \Delta X_{k,t+1}^i) \to E(F(\hat{X}_{k,t}^i, \hat{X}_{k,t-1}^i)) E(\Delta \hat{X}_{k,t+1}^i) + E((\epsilon_{X}^i) M(\hat{X}_{k,t}^i, \hat{X}_{k,t-1}^i)\epsilon_{t-1,t}) E(\Delta \hat{X}_{k,t+1}^i) \tag{15}
\]

From this equation, the bias comes from the second component. Note that this component depends on the variances of the relative error terms. In addition, the summary statistics in the paper indicate that the average relative spread for options is about 15% to 18%, so the variances of price error for the average spread are about \( (0.18)^2/4 \approx 0.01 \) if the distribution of error is uniform between bid and ask prices.\(^3\) The stock spread is even smaller, making the variance for the price error even smaller than 0.01. Thus, the bias should be less than \( 0.01 \times Const \times E(\Delta \hat{X}_{k,t+1}^i) \) (here \( Const \) is the norm of \( M \) function), which is a second-order effect. Similarly, we can show that the bias in denominator \( \frac{1}{N} \sum_{i=1}^{N} g_i \) is close to \( E(\frac{g(X_{k,t}^i)}{g(X_{k,t-1}^i)}) \), up to a second-order effect. Therefore, the bias in 13 is also a second-order effect, which

\(^3\)In general, the distribution should be concentrated at the midpoint of the bid and ask prices, making variance even smaller.
should not be large. We also conduct simulations in the paper and show that the magnitude of the bias is very small.

**Appendix E The proposed method to adjust biases in regressions with option returns**

In addition to Assumptions 1 and 2, we need to make the following assumptions:

**Assumption 3** The stock and option characteristics \( \theta_i \) are independent of the errors in observed returns and true stock and option prices. This implies that the null hypothesis \( (b = 0) \) is true.

Let weight \( w^i = \frac{g_i}{g_{i-1}} \), and let weight matrix, \( W_t \), be a diagonal with \((i, i)\) element \( w^i \).

**Assumption 4** The matrix \( \left( \frac{1}{N} \theta_i \theta_i' \right)^{-1} \) converges to an invertible matrix.

We propose the following method to adjust for the bias in the regression coefficients. Note that

\[
\hat{b}_{WLS} = (\theta_i W_t \theta_i')^{-1}(\theta_i W_t \tilde{Y}_i) + (\theta_i W_t \theta_i')^{-1}(\theta_i W_t (Y_t - \tilde{Y}_t)).
\]

The first component of the estimator converges to the true regression coefficients \( b \), which is zero under the null. We need to show that the second component converges to zero. It is clear that

\[
(\theta_i W_t \theta_i')^{-1}(\theta_i W_t (Y_t - \tilde{Y}_t)) = \left( \frac{1}{N} \theta_i W_t \theta_i' \right)^{-1} \left( \frac{1}{N} \theta_i W_t (Y_t - \tilde{Y}_t) \right).
\]

Moreover,

\[
\frac{1}{N} \theta_i W_t (Y_t - \tilde{Y}_t) = \frac{1}{N} \theta_i W_t \tilde{Y}_t - \frac{1}{N} \theta_i W_t \tilde{Y}_t.
\]

From Assumptions 1, 2, and 3,

\[
\frac{1}{N} \theta_i W_t \tilde{Y}_t \to E(\theta_i)E\left( \frac{\sum_{k=1}^M f_{X_k^i}(\tilde{X}_k^i) \Delta \tilde{X}_{k,t+1}^i}{g(\tilde{X}_t^i)} \right),
\]

the above expectation converges to zero as true price changes \( \Delta \tilde{X}_t^i \to 0 \) as \( \Delta t \to 0 \) (Assumption 2). Similarly,

\[
\frac{1}{N} \theta_i W_t \tilde{Y}_t \to E(\theta_i)E\left( \frac{\sum_{k=1}^M f_{X_k^i}(X_k^i) \Delta \tilde{X}_{k,t+1}^i}{g(X_{k-1}^i)} \right).
\]
Using the notation introduced in the previous section, the above equation can be written as

\[
E(\theta^i_t)E(\sum_{k=1}^{M} \frac{\bar{f}_i X_{i,k,t+1}^i}{g_{t-1}^i}) = E(\theta^i_t)(\sum_{k=1}^{M} (E(\frac{\bar{f}_i X_{i,k,t+1}^i}{g_{t-1}^i})E(\Delta X_{k,t+1}^i) + \text{cov}(\frac{\bar{f}_i X_{i,k,t+1}^i}{g_{t-1}^i}, \Delta X_{k,t+1}^i))).
\]

Following the same argument introduced in the previous section, the above equation converges to zero. Together with the Assumption 4,

\[
(\theta^t W_t \theta^t_\prime)^{-1}(\theta^t W_t (\bar{Y}_t - \tilde{Y}_t)) \to 0.
\]

Similar to the case for the average return, the bias in regression is not zero when true price changes is nonzero. We can show that the bias is a second-order effect. The simulations in the paper also suggest that the bias is small.
Table 1: **Option bid-ask spread model estimation results.** This table reports estimates of Fama-MacBeth regressions estimating the mean and the variance of option bid-ask spreads as function of number-std, time-to-maturity $\tau$ and underlying stock effective bid-ask spreads. Number-std is the number of standard deviations separating the present values of the strike price and future stock price (current stock price minus dividends). Newey-West standard errors, using ten lags, are shown in parentheses.

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Standard deviation of $\epsilon_t$: 0.2216
Standard deviation of $\epsilon_t$: 0.0675
References